Arvind Borde / PHY 19, Week 9: The Wavefunction: 1d

§9.1 Basic Ideas

So far

1) GR: ____________________

2) QM: ______________________

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§9.2 The wave function

Postulate:

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We'll first consider a particle in 1d.

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Interpretation: ________________________

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We assume that Ψ is single-valued and continuous.

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§9.2.1 Normalization:

Because of the link with probabilities, we require that

$$\int_{-\infty}^{\infty} |\Psi|^2 = \_\_$$

This allows us to fix C in a wave function represented by $Cf(x)$.

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(1) Find C in the wavefunction $\Psi(x) = Ce^{-|x|/x_0}$, where x_0 is fixed.

$$\int_{-\infty}^{\infty} |\Psi|^2 =$$

So,

$$C =$$

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(2) What's the probability that a particle given by the previous wavefunction will be found in $(-x_0, x)$?

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ADDITIONAL NOTES

§9.2.2 Evolution

In general we'll have $\Psi(x,t)$ and our job will be to determine the evolution of Ψ from a given initial state.

Ψ evolves according to an equation proposed by Erwin Schr̈dingier.

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§9.3 A Free Particle

This is a particle with no forces acting in it. To connect with De Broglie's ideas, we take

$$\Psi(x,t) =$$

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(3) Find $\Psi(x,0)$, if $a(k) = (C\alpha/\sqrt{\pi})e^{-\alpha^2 k^2}$ where C and α are constants.

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§9.4 Forces

In the presence of a force, $F = -dU/dx$, where $U(x)$ is the potential energy, Schrödinger proposed this equation for $\Psi(x,t)$:

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ADDITIONAL NOTES

Solving this, in general, is hard. We will confine ourselves to separable wavefunctions of the form $\Psi(x,t) = \psi(x)\phi(t)$. In this case we get

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§9.4.1 A particle in a box

This a particle confined to move in a fixed space, say $0 \leqslant x \leqslant L$.

The time independent equation becomes:

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§9.4.2 A finite square well

This is particle in "potential well" of depth U. Here the exterior wave function is

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§9.4.3 The harmonic oscillator

The potential energy for a harmonic oscillator is

where m is the mass of the particle and $\omega = \sqrt{K/m}$.

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ADDITIONAL NOTES

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§9.5 Expectation Values

Many quantities we might wish to measure, such as position, have probabilistic values in quantum mechanics. We define the expectation value of a quantity, $f(x)$ as

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We can use this to find the uncertainty in position for a harmonic oscillator:

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§9.6 Observables and Operators

An observable is any particle property that can be measured.

The position and momentum of a particle are observables, as are KE and PE.

In quantum mechanics, we associate an "operator" that acts on functions with each observable.

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ADDITIONAL NOTES

For example, a constant c is an operator, that simply operates on $f(x)$ by $cf(x)$.

Another operator is d/dx.

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The expectation value $\langle Q \rangle$ of an operator $[Q]$ is

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For example, $\langle KE \rangle$ is

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Here are some common operators:

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ADDITIONAL NOTES

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