Arvind Borde / PHY 19, Week 9: The Wave Nature of Matter

§9.1 Our First Quantum Steps

By 1920 or so, we had: Planck's explanation (1900) of the blackbody spectrum, with emission and absorption of radiation occurring at specific quantized energies, $E = hf$. Einstein's explanation (1905) of the photoelectric effect with light behaving as a particle. Rutherford's and, especially, Bohr's work on atoms (1911–), especially the theory of the Hydrogen atom.

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Bohr's theory of the H atom had many successes: the prediction of spectral line wavelengths, the ionization energy, etc. But it left a lot of things open:

- Why are some lines are more intense than others?
- How do you handle multielectron atoms?
- What "equation of motion" gives the evolution of the atom in time from an initial state?
- How does the wave-particle duality of light fit in?
- How might you "quantize" other systems?

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§9.2 De Broglie and Electron Waves

DeBroglie was struck by a couple of aspects of these new quantum theories:

1) The particle picture of light had a wave feature, f, built fundamentally into it ($E = hf$).

2) Bohr's energy levels involved whole numbers, and another place that whole numbers appear in physics is in wave behaviors such as interference and normal modes of vibration.

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This led him to propose that the electron had a wave-particle duality to it, just as the quantum of light, the photon, did. The wavelength and frequency he proposed for the electron (and any material particle) were

$$\lambda = \qquad\qquad f =$$

where p and E are the *relativistic* momentum and total energy.

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$$p =$$

$$E =$$

$$\gamma =$$

where m is the particle rest mass, v its speed.

De Broglie proposed that stable orbits of electrons are ones that support them as standing waves.

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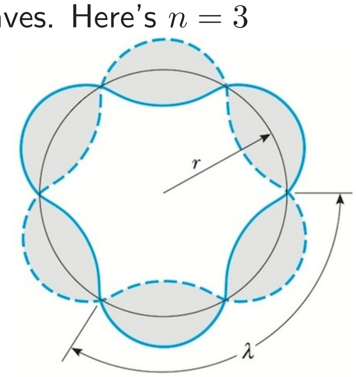
The orbital quantum number n is the number of standing waves. Here's $n = 3$



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ADDITIONAL NOTES

The quantum condition in this picture is that the circumference must accommodate an integral number of waves:

For an electron at non-relativistic speeds, $\lambda = h/m_e v$, by De Broglie's proposal, and we get

$$m_e v r =$$

7 precisely Bohr's angular momentum condition.

(1) What is the (de Broglie) wavelength of a baseball of mass 0.145kg and speed 45m/sec?

$\lambda =$

(2) Why do we never see baseball diffraction or interference?

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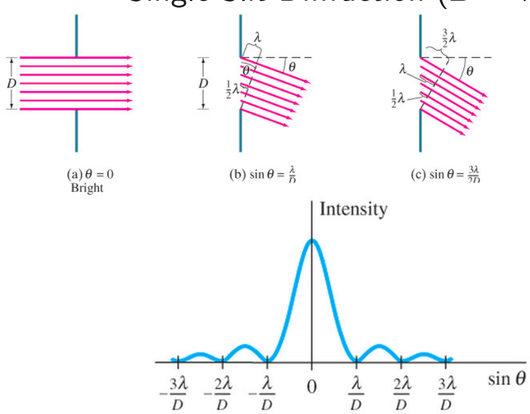
8 ____________

Single Slit Diffraction ($D \sim \lambda$)



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(3) A particle of charge q and mass m is accelerated through a pot. diff. V. What is its De Broglie wavelength, assuming non-relatavistic speeds?

____________________ ____________.

10 ____________ .

(4) What's the wavelength of an electron accelerated from rest by a pot. diff. of $50\,$V?

$\lambda =$

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Since the wavelength of low-speed electrons is comparable to atomic dimensions, we expect to see electron diffraction occuring off atoms.

Observed by George P. Thomson in the U.K., and Davisson and Germer in the U.S. in the 1920s.

Subsequent diffraction experiments established the wave properties of the Hydrogen atom, the Helium

12 aton, and (after it was discovered) the neutron.

ADDITIONAL NOTES

§9.2.1 <u>The electron microscope</u>

The resolution of devices such as microscopes depends on the wavelength of radiation: the shorter the wavelength the better the resolution.

Electrons, even at low speeds, have shorter wavelengths than visible light, and allow us to build higher resolution microscopes (first one in 1931).

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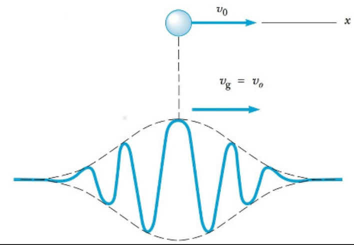
§9.3 Localizing Wave-Particles

A wave, by nature, is spread out in space.

A particle, by nature, is localized.

These are reconciled for wave-particles through __________:



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A wave-packet or a wave group may be constructed by superposing waves of different wavelengths. In order to represent a particle of momentum p_0, the spread of wavelengths is centered on $\lambda_0 = h/p_0$.

An illustration of how wave packets might arise is given by superposing two sine (or cosine) waves of slightly different frequencies.

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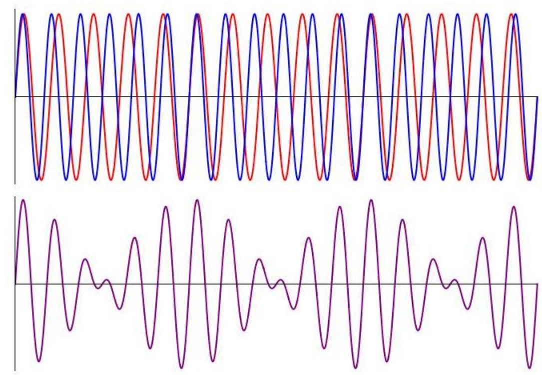

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A cosine wave oscillating in the y direction, and traveling in the positive x direction may be represented as

$$y = A\,cos(kx - \omega t)$$

where $k \equiv 2\pi/\lambda$ is called the wave number, $\omega \equiv 2\pi f$ is the angular frequency, and A, λ and f have their usual meanings.

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A point at fixed phase on the wave, say a particular peak, travels at speed

$$v_p = \lambda f = \frac{\omega}{k}$$

Superposing two such waves of the same amplitude, we get

$$y = A\,cos(k_1 x - \omega_1 t) + A\,cos(k_2 x - \omega_2 t)$$

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ADDITIONAL NOTES

Setting $\Delta\omega = \omega_2 - \omega_1$, $\Delta k = k_2 - k_1$, $k_S = k_1 + k_2$, $\omega_S = \omega_2 + \omega_1$, and using

$$cos(a + b) =$$

we get (after a little enjoyable algebra)

$$y = 2A\,cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right)cos\left(\frac{k_S}{2}x - \frac{\omega_S}{2}t\right)$$

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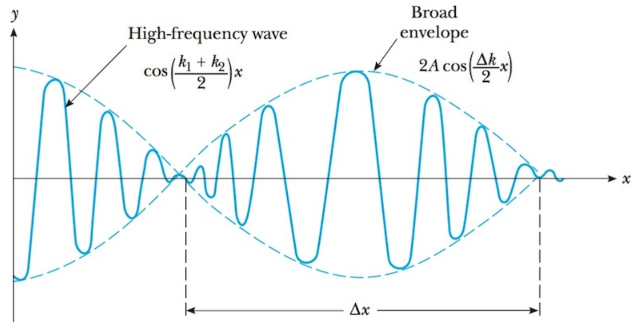
Assuming that $\Delta\omega$ and Δk are small, the spatial effect of the superposition can be pictured as



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(5) How fast do the high frequency waves travel?

$$v_p =$$

(6) How fast does the envelope (group) travel?

$$v_g =$$

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On slide 20, remembering that successive zeros of the cosine function are separated by ____, we see that

$$\frac{\Delta k \Delta x}{2} = \pi$$

or $\Delta k \Delta x = 2\pi$.

Likewise, we can show that $\Delta\omega\Delta t = 2\pi$.

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This has been a simple example of superposing two waves.

It does not give a truly localized wave-packet, one where the amplitude rapidly drops to zero outside a finite spatial region.

To get that we need to superpose a larger number of wavelengths. This requires Fourier analysis.

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In that case, the group velocity becomes

$$v_g = \left.\frac{d\omega}{dk}\right|_{k_0}$$

where k_0 is the peak wave number.

Now, $v_p = \omega/k$ (or $\omega = kv_p$), so

$$v_g = \Big[\qquad\qquad \Big]\Big|_{k_0}$$

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ADDITIONAL NOTES

How does this apply to particle-waves? Consider a particle of mass m and speed v. De Broglie proposed (slide 4) that $\lambda = h/p$ and $f = E/h$.

The phase speed is

$$v_p = f\lambda = \frac{E}{p} = \frac{\sqrt{p^2c^2 + m^2c^4}}{p}$$

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Using $p = h/\lambda = \hbar k$, this becomes

$$v_p = c\sqrt{1 + \left(\frac{mc}{\hbar k}\right)^2}$$

(7) What does this imply about the speed of the individual waves in a packet?

____

But it's v_g that's the speed of the particle.

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(8) What's $k dv_p/dk$?

$$\frac{dv_p}{dk} =$$

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(9) What's $v_p + k dv_p/dk$?

$$v_p\frac{dv_p}{dk} =$$

28 (using slide 26).

(10) From $v_p = E/p$, what is v_p in terms of the particle speed v and c?

$$v_p = \frac{E}{p} = \frac{\gamma mc^2}{\gamma mv} =$$

(11) So, finally, what's v_g?

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These are the ingredients that lead to what we call the wave function, Ψ.

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ADDITIONAL NOTES

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