Arvind Borde / PHY 19, Week 7: Overview; Quantum Theory of Light

§7.1 What is Quantum Theory?

A theory of the very small, developed by many people over many decades.

§7.1.1 1900 Max Planck's quantum hypothesis

$$E = hf$$

for the emission of EM radiation, where f is the frequency of the wave and $h = 6.626 \times 10^{-34}$ J·s is now called Planck's constant.

1

§7.1.2 1905 Albert Einstein explains the photoelectric effect using this quantum hypothesis. He asserts that light sometimes behaves as a particle (which we now call the photon) with momentum given by

$$p = h/\lambda$$

where λ is the wavelength. This momentum is enough to knock electrons out of their host atoms, causing a current to flow.

2

(1) Remembering that $E = pc$ for a photon (from Special Relativity), show that Einstein's expression for the momentum of a photon follows from Planck's expression for quantized energy.

$$hf = E = pc$$

So $$p = \frac{hf}{c} = \frac{h}{\lambda}$$

(because $c = \lambda f$).

3

§7.1.3 1911 Ernest Rutherford shoots alpha particles (Helium nuclei) at a thin gold foil and finds that some come right back at him. He proposes a model of the atom where there's a massive nucleus containing positively charged particles (which he calls protons) and neutral ones (he calls neutrons), with electrons around it, at a distance over $10{,}000$ tines the size of the nucleus.

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But what do these negatively charged electrons do?

They cannot just stay immobile because they would fall into the nucleus.

They cannot orbit the nucleus because they are charged and they would radiate energy away and spiral into the nucleus.

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§7.1.4 1913 Niels Bohr proposes that electrons in atoms have wavelike properties and could exist in atoms only at precise energy levels. Transitions between energy levels explained spectral lines.

6

ADDITIONAL NOTES

§7.1.5 1924 Louis De Broglie proposes all particles have a wavelike nature with wavelength given by

$$\lambda = h/p$$

where p is the momentum. This is called "wave-particle duality." In some contexts things behave like waves, in others like particles.

7

(2) What's the non-relativistic de Broglie λ for an electron ($m = 9 \times 10^{-31}$kg) moving at

(a) 1 m/s?

$$\lambda = \frac{6.626 \times 10^{-34}\ \text{J·s}}{(9 \times 10^{-31}) \cdot (1)} = 7.3 \times 10^{-4}\ \text{m}$$

(b) 1.5×10^8m/s?

$$\lambda = \frac{6.626 \times 10^{-34}\ \text{J·s}}{(9 \times 10^{-31}) \cdot (1.5 \times 10^8)} = 4.9 \times 10^{-12}\ \text{m}$$

8

§7.1.6 1924 Satyendra Nath Bose introduces a statistical theory explaining Planck's theory of radiation. He sends his work to Einstein who has it published and adds an extension of his own.

The particles to which these Bose-Einstein statistics apply are now called bosons. They are the quanta of interactions (photons, gluons, etc.).

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§7.1.7 1925 Werner Heisenberg introduces the first full quantum theory, putting together previous work.

He, Max Born and Pascual Jordan show that this new quantum theory can be naturally formulated using the mathematics of matrices: matrix mechanics.

10

§7.1.8 1926 Erwin Schrödinger introduces a different formulation of quantum mechanics using the mathematics of wave motion: wave mechanics.

Schrödinger's wave mechanics is the most commonly used version of quantum mechanics today.

Every system is describe by a function of space and time, $\psi(x,t)$, called its wave function.

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The wave function obeys an equation we now call Schrödinger's equation. In one space dimension:

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = \frac{-\hbar^2}{2m} \cdot \frac{\partial^2 \psi(x,t)}{\partial x^2} + V(x)\psi(x,t)$$

where $\hbar = h/2\pi$, m is the mass, and $V(x)$ the potential energy.

Schrödinger did not correctly interpret his own wave function.

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ADDITIONAL NOTES

§7.1.9 1926 Max Born provides the correct interpretation of $\psi(x,t)$:

$|\psi(x,t)|^2$ is the probability of finding a particle at a point x at a time t.

13

§7.1.10 1926 Enrico Fermi discovers the relationship between the spin of particles and the statistical laws they follow: fermions have half-integer spin (related to angular momentum in units involving $\hbar$) and bosons have integer spin.

14

§7.1.11 1926 Paul Dirac shows that wave mechanics is mathematically equivalent to matrix mechanics. He also introduced the statistical laws that apply to particles such as electrons. The class of particles to which Fermi-Dirac statistics apply are called fermions. They are the quanta of matter (electrons, quarks, neutrinos, etc.).

15

§7.1.12 1927 Heisenberg introduces the uncertainty principle:

$$\Delta x \Delta p \geqslant \frac{\hbar}{2}$$

where Δx and Δp are the "uncertainties" (related to standard deviations) in position and momentum, respectively. If measure one variable, say x, to high accuracy, the value of the other variable, p, automatically becomes highly uncertain.

16

§7.1.13 1927 Dirac introduces relativistic quantum mechanics and applies it to the electron. This theory makes quantum mechanics consistent with special relativity.

A version that is consistent with general relativity has proved elusive to this day.

17

§7.1.14 1930 Dirac predicts, based on his theory of the electron, that the mathematics dictates the existence of an electron antiparticle – the positron (opposite charge, but otherwise identical).

We now believe every fundamental particle has an antiparticle. Matter-antimatter asymmetry is a puzzle.

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ADDITIONAL NOTES

§7.1.15 1930s and 1940s Lots of developments using QM to explain chemistry, the structure of the nucleus, the structure of stars, etc.

Among the successes, there are puzzles. Applying the ideas of quantum theory to fields leads to infinities in certain calculations, for example.

19

§7.1.16 1948 Sin-Tiro Tomonaga, Julian Schwinger, and Richard Feynman independently show how the infinities that arise when you applied relativistic quantum mechanics to the electromagnetic field could be handled.

Feynman's approach uses "path integrals" – where systems choose the path they take by sniffing out all possible ones.

20

§7.1.17 1960s Schwinger and Sheldon Glashow produce a unified quantum field theory that unifies the weak nuclear force and electromagnetism.

21

§7.1.18 1960s–1970s Quarks are introduced, and the Higgs boson. The work of many people leads to Grand Unified Theories that provide a quantum field theoretical unification of the strong and weak nuclear forces, and electromagnetism.

Gravitation remains the odd "force" out. If it is "quantized" then its particle, the graviton, must have spin 2.

22

Supersymmetry is introduced, providing a possible link between fermions and bosons, separated by a spin gap of $1/2$.

The hypothetical graviton would have an even more hypothetical partner of spin $3/2$ or $5/2$. The simplest possibility is spin $3/2$, named the gravitino.

An elaborate theory, supergravity, was built on the graviton-gravitino supersymmetry.

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§7.1.19 1974 Stephen Hawking does one of the first successful calculations of quantum field theory in a curved spacetime, and arrives at the result that blacks holes can radiate.

In 1982 Alan Guth, studying grand unified theories in the early universe to explain the suppression of magnetic monopoles, proposes an extension of the big bang theory called inflationary cosmology.

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ADDITIONAL NOTES

§7.2 The Classical Theory of Light

§7.2.1 Maxwell's Laws ($\sim$1870)

1. Elec. fields are produced by elec. charges.

2. Mag. fields are produced by mag. poles.

3. Elec. fields also produced by changing mag. fields.

4. Mag. fields also produced by electric currents,

25 OR by changing elec. fields.

§7.2.2 Maxwell's Equations

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

26 $k = 9 \times 10^9$ N$\cdot$ m^2/C^2, and $\epsilon_0 = 1/4\pi k$

$\mu_0 = 4\pi \times 10^{-7}$ T$\cdot$m/A

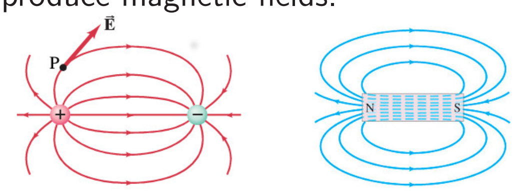
Three-and-a-half of Maxwell's Laws were a summary of the previous discoveries of others.

But his proposal that magnetic fields would be produced by changing electric fields was new.

Maxwell was partly motivated by the apparent symmetry between electricity and magnetism (although the absence of isolated magnetic poles spoils the

27 picture somewhat).

There's a deep truth hidden in Maxwell's Laws. One way to produce electric or magnetic fields is through sources: charges produce electric fields, poles produce magnetic fields.



28 (As in gravitation, masses produce gravitational effects.)

With sources, something concrete (a charge, a magnet, a mass) produces an abstraction, a field.

Why do we think this abstraction is "really there"?

Because it exerts forces, and thereby causes effects (such as motion) that we can measure.

For similar reasons we believe energy is "really there" even though we don't experience it directly.

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Maxwell's Laws say there's another way to produce electric and magnetic fields, not just via "concrete" sources.

By changing magnetic and electric fields, respectively.

And, as with fields caused by sources, we can equally well measure fields caused by fields.

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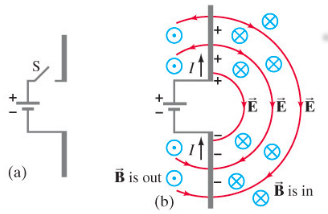
ADDITIONAL NOTES

One might think that a field that causes a secondary one should have a source, and one can view *that* source as the source of the secondary field.

For example, a scenario like this:



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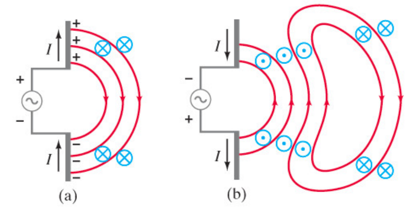
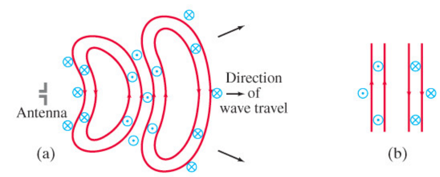
But, if we let DC $\longrightarrow$ AC:





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These oscillating EM fields persist in empty space.

Do they just hang out, or do they move?

They move, with a speed given by

$$\frac{1}{\sqrt{\epsilon_0\mu_0}}$$

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(3) Calculate $1/\sqrt{\epsilon_0\mu_0}$

$$\epsilon_0\mu_0 = \frac{1}{4\pi(9\times 10^9)} 4\pi \times 10^{-7}$$

$$= \frac{1}{9\times 10^{16}}$$

So $1/\sqrt{\epsilon_0\mu_0} = 3\times 10^8\,\mathrm{m/s}$.

(4) Do you recognize that value? Of course!!!!

It's our bff, c, the "speed of light."

34

Maxwell argued, correctly, that visible light is part of the "electromagnetic spectrum."

But checking it directly was difficult at the time because the known wavelengths of visible light were small ($\sim 500\,\mathrm{nm} = 5\times 10^{-7}\,\mathrm{m}$), hence the frequencies ($f = \underline{\;c/\lambda\;}$) were very large ($10^{15}$ Hz), and we were not able at the time to create such high frequency oscillations.

35

§7.2.3 Hertz's Experiments (1887)

Heinrich Hertz gave the first experimental confirmation of Maxwell's ideas by constructing an oscillating antenna that generated long-wavelength (radio-wave region) e-m waves.

His antenna had a frequency of 5×10^8 Hz, and the radiation it produced had a wavelength of $0.6\,\mathrm{m}$, giving the correct value for the speed, c.

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ADDITIONAL NOTES

Hertz showed that these electromagnetic waves could be reflected, refracted, focused, polarized, and made to interfere, just as visible light could.

Although we could not produce visible light by oscillating mechanisms, since Hertz's e-m waves had the same properties as visible light, including speed, the conclusion that visible light was also an electromagnetic wave was widely embraced.

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Now, Hertz's experiments did involve an initial source (the antenna).

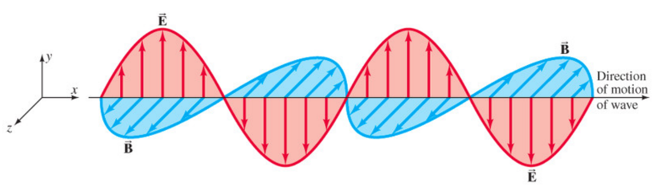
But, it turns out Maxwell's equations also allow self-sustaining electromagnetic waves that have no apparent source.

38

With $c = 1/\sqrt{\epsilon_0 \mu_0} = 1$ and $\vec{J} = 0$,

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{\partial t}$$



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§7.2.4 The Zeeman Effect (1896)

Pieter Zeeman discovered that a strong magnetic field changes the frequency of the light emitted by a glowing gas, an effect that follows from Maxwell's equations.

This was further proof that light was an electromagnetic wave.

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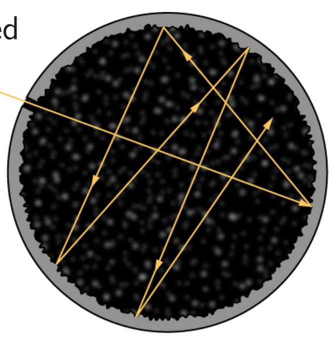
§7.3 Black Body Radiation

A blackbody is an idealized object that absorbs all the radiation that falls on it – hence "black."*

A hole in a cavity is a good approximation to a blackbody.



41 * Stay tuned

If *any* object (blackbody or not) is in thermal equilibrium with its surroundings it will both absorb energy and emit it.

One might think that this would be a complicated process, and would depend on the nature of the body, its shape, its composition, etc.

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ADDITIONAL NOTES

§7.3.1 Kirchoff's Law Gustav Kirchoff showed on thermodynamic grounds in 1859 that for *any* body in thermal equilibrium

$$e_f = J(f,T)A_f$$

where e_f is the power emitted per unit area per unit frequency, A_f is the absorption power (fraction of the incident power absorbed per unit area per unit frequency).

43

$J(f,T)$ is a *universal function* that depends only on frequency, f and temperature T, not on the nature of the body.

For a blackbody, $A_f = 1$.

44

This was consistent with previous observations that all objects, regardless of their chemical nature, size, or shape, become red at the same temperature, blue at the same higher temperature, etc.

By the mid-1800s it was also known that glowing solids emit continuous spectra rather than the bands or lines emitted by heated gases.

45

What does that mean?

If you break down the radiation emitted by a heated gas by wavelength (yeah spectroscopy!), you find that each gas emits radiation only at specific wavelengths. Heated hydrogen emits one set of precise wavelengths, heated Helium another.

46

But a solid object that's approximately blackbody emits a continuous set of wavelengths that depend only on temperature.

Not all the wavelengths were equally intense, though.

The color of heated objects that early observers had observed had to do with the peak intensity.

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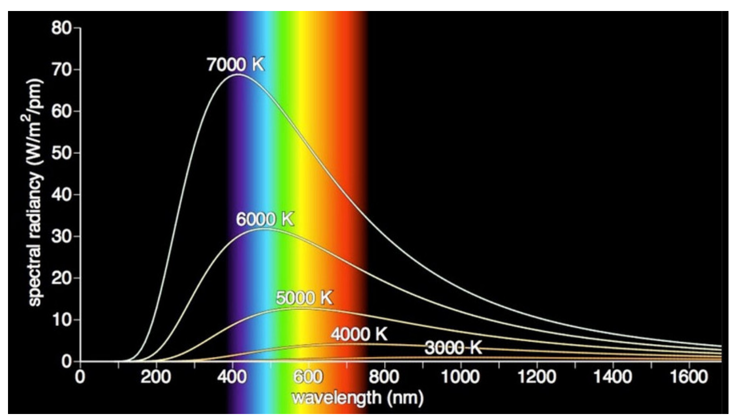

48 Power per unit area per unit wavelength vs wavelength

ADDITIONAL NOTES

§7.3.2 Stefan's Law In 1879 Josef Stefan found experimentally that the total power per unit area emitted at all frequencies by a hot solid, e_{total}, is given by

$$e_{\text{total}} = \int_0^\infty e_f df = \sigma T^4$$

where $\sigma = 5.67 \times 10^{-8}$ W·m^{-2}·K^{-4} is called the Stefan-Boltzmann constant. (Boltzmann derived the law from thermo. and Maxwell's equations.)

49

§7.3.3 Stefan & the sun Assume the sun is a blackbody with a radius $R_s = 7.0 \times 10^8$ m. The earth is a distance $R = 1.5 \times 10^{11}$ m from the sun. The power per unit area (at all frequencies) from the Sun is measured at the Earth to be 1400 W/m^2.

(5) What's the surface temperature of the sun?

You gotta have

$$e(R_s) \cdot 4\pi R_s^2 = e(R) \cdot 4\pi R^2$$

50

$$T = \left[\frac{e(R_s)}{\sigma}\right]^{1/4}$$

$$= \left[\frac{e(R) \cdot R^2}{R_s^2 \sigma}\right]$$

$$= \left[\frac{1400 \cdot (1.5 \times 10^{11})^2}{(7 \times 10^8)^2 \cdot (5.67 \times 10^{-8})}\right]$$

$= 5{,}800°$ K

51

§7.3.4 Wien's Displacement Law In 1893 Wien proposed a law that suggested

$$\lambda_{\text{max}} T = 2.898 \times 10^{-3} \text{ m·K}$$

where λ_{max} is the wavelength at peak intensity in the blackbody radiation spectrum.

(6) Does that seem plausible? Yeah, man!

52

(7) If the peak-intensity wavelength of the sun is 5×10^{-7} m, what's the temperature of the sun?

$$T = \frac{2.898 \times 10^{-3}}{5 \times 10^{-7}} \approx 5{,}800° \text{ K}$$

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§7.4 Planck's Law for Blackbody Radiation

$$e_f = J(f, T) A_f$$

e_f: power emitted per unit area per unit frequency

A_f: absorption power (fraction of incident power absorbed per unit area per unit frequency).

$J(f, T)$: "Universal function."

A_f is a fraction. So $J(f, T)$ is the *power* radiated *per unit area* per unit frequency by a blackbody.

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ADDITIONAL NOTES

55 Define $u(f,T)$ as the *energy per unit volume* per unit frequency of radiation. It can be shown from thermodynamics that

$$u(f,T) = \frac{4}{c} J(f,T)$$

Attempts to calculate $u(f,T)$ led to divergent results (i.e., to infinities) at short wavelengths. This was called the "ultraviolet catastrophe."

56 (**8**) Do the units of $u(f,T)$ and $J(f,T)$ work out?

Yes

Power is energy transferred per unit time, so energy is power multiplied by time.

The units of $J(f,T)$ need to be multiplied by time and divided by length to match the units of $u(f,T)$. This is what dividing by c does.

57 In 1900 Planck recalculated blackbody radiation. He made two assumptions:

1) The radiation is produced by vibrating (oscillating) submicroscopic resonators.

From Maxwell's theory, this radiation would have a frequency equal to that of the resonator (as Hertz had confirmed). The theory allows an oscillator to emit radiation of any energy.

58 Planck made the *new* proposal that

2) The total energy of *any* oscillator with frequency f is an integral multiple of hf, where

$$h = 6.626 \times 10^{-34}\ \text{J·s}$$

Further, radiation could only be emitted when an oscillator dropped its energy from one level to the next lowest one; i.e., radiation has energy hf.

59 (**9**) What, by Planck's hypothesis, is the energy in light of wavelength $300\,$nm?

$$E = hf = hc/\lambda$$

$$= \frac{(6.626 \times 10^{-34})(3 \times 10^{8})}{3 \times 10^{-7}}$$

$$= 6.626 \times 10^{-19}\ \text{J} = 4.14\ \text{eV}.$$

$(1\ \text{eV} = 1.6 \times 10^{-19}\ \text{J}.)$

60 Planck's proposal applies to all oscillators.

But, the small value of hf, even for relatively high frequency phenomena, such as visible light (in the previous, $f = 10^{15}$ Hz), is why we do not observe this "quantization of energy" in day-to-day life.

A pendulum swings at a frequency of ~ 1 Hz. It loses energy as it slows down in steps of hf, but the steps are too small ($\sim 10^{-34}$ J) to be detected.

ADDITIONAL NOTES

Based on his assumptions, Planck derived this formula (we'll call it eqn. P):

$$u(f,T) = \frac{8\pi h f^3}{c^3(e^{hf/kT}-1)} \qquad [P]$$

where k is a thermodynamic quantity called Boltzmann's constant:

$$k = 1.381 \times 10^{-23}\ \text{J}\cdot\text{K}^{-1}$$

61 We'll see this explains the blackbody spectrum.

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(10) Previous formulas had divergences at large frequencies (short wavelengths). What does eqn. P say as $f \to \infty$?

$$\lim_{f\to\infty} u(f,T) = \lim_{f\to\infty} \frac{8\pi h f^3}{c^3(e^{hf/kT}-1)} = 0,$$

because the $e^{hf/kT}$ in the denominator "dominates" f^3 in the numerator.

62

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(11) What does Plank's formula say as $f \to 0$?

Both numerator (N) and denominator (D) $\to 0$.

Use L'Hospital's rule (prime is derivative wrt f):

$$\lim_{f\to 0} u(f,T) = \lim_{f\to 0} \frac{N'}{D'} = \lim_{f\to 0} \frac{8\pi h(3f^2)}{c^3\big((h/kT)e^{hf/kT}\big)}$$

This limit is $\underline{\underline{\ 0\ }}$ because as $f \to 0$, $e^{hf/kT} \to 1$ and $f^2 \to 0$.

63

So Planck's formula has the energy function $u(f,T) \to 0$ both as $f \to \infty$ and as $f \to 0$.

This means the power function will do the same:

$$J(f,T) = \frac{c}{4}u(f,T) \to 0$$

as $f \to \infty$ and as $f \to 0$, i.e., as $\lambda \to 0$ or $\lambda \to \infty$, consistent with the experimental results shown on slide 48.

64

Now, frequencies, f, are always positive.

Examination of eqn. P on slide 61 shows that

$$u(f,T) > 0,\ \forall f.$$

So $J(f,T)$ must have at least one peak somewhere, again consistent with the graphs on slide 48.

65 We'll show that it has a *single* peak.

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(12) Find $J' \equiv dJ(f,T)/df$.

$$\frac{dJ(f,T)}{df} = \frac{2\pi h(3f^2)}{c^2\left(e^{\frac{fh}{kT}}-1\right)} - \frac{2\pi h f^3 e^{\frac{fh}{kT}}(h/kT)}{c^2\left(e^{\frac{fh}{kT}}-1\right)^2}$$

$$= \frac{6\pi h f^2}{c^2\left(e^{\frac{fh}{kT}}-1\right)} - \frac{2\pi h^2 f^3 e^{\frac{fh}{kT}}}{c^2\left(e^{\frac{fh}{kT}}-1\right)^2 kT}$$

66 using $J' = \left(\frac{N}{D}\right)' = N'D^{-1} + N(-D^{-2})D' = \frac{N'}{D} - \frac{ND'}{D^2}$.

ADDITIONAL NOTES

(13) Set $J' = 0$ (to figure out extrema).

$$\frac{6\pi h f^2}{c^2\left(e^{\frac{fh}{kT}}-1\right)} - \frac{2\pi h^2 f^3 e^{\frac{fh}{kT}}}{c^2\left(e^{\frac{fh}{kT}}-1\right)^2 kT} = 0$$

Multiplying by $\frac{c^2\left(e^{\frac{fh}{kT}}-1\right)^2}{(2h\pi f^2)}$ we get:

$$3\left(e^{\frac{fh}{kT}}-1\right) - \frac{hf}{kT}e^{\frac{fh}{kT}} = 0$$

67

(14) Rewrite the previous with $x = hf/kT$.

$$3(e^x - 1) - xe^x = (3-x)e^x - 3 = 0$$

Numerical methods give a single solution

$$x = \frac{hf}{kT} = 2.82$$

showing that there's a single peak to the graph.

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§7.5 Planck, Wien and Stefan

We've seen that Planck's formula yields the general features of the experimentally observed black-body radiation curve shown on slide 48.

We'll now check that it implies, more specifically, both Wien's Displacement Law (slide 52) and Stefan's Law (slide 49).

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§7.5.1 Wien's Law

There's a subtlety here: the graph on slide 48 plots

(power/unit area)/(unit λ) vs λ.

We're using a formula for

(power/unit area)/(unit *frequency*).

Since λ and f are "reciprocally related" Wein's displacement law changes to

70

$$f_{\mathsf{max}}/T = 5.9 \times 10^{10}\ \mathsf{Hz{\cdot}K^{-1}}$$

(15) Find f_{max}/T from the condition for the peak.

We had

$$\frac{hf_{\mathsf{max}}}{kT} = 2.82$$

So

$$\frac{f_{\mathsf{max}}}{T} = 2.82\frac{k}{h} = 2.82\frac{1.381\times 10^{-23}\ \mathsf{J{\cdot}K^{-1}}}{6.626\times 10^{-34}\ \mathsf{J{\cdot}s}}$$

71 Or, $f_{\mathsf{max}}/T = 5.9\times 10^{10}\ \mathsf{Hz{\cdot}K^{-1}}$

§7.5.2 Stefan's Law

(16) If you integrate $J(f,T) = (c/4)u(f,t)$ over f, switching to our new variable $x = \frac{hf}{kT}$, what is the dependence of the answer on T?

$$dx = \frac{h}{kT}df$$

72

ADDITIONAL NOTES

$$\int_0^\infty J(f,t)df = \frac{(c)}{4}\frac{8\pi[\![h]\!]}{(c^3)}\int_0^\infty \frac{x^3([kT]/[\![h]\!])^3}{e^x-1}\frac{[kT]}{[\![h]\!]}dx$$

$$= \frac{2\pi[(kT)^4]}{(c^2)[\![h^3]\!]}\left\{\int_0^\infty \frac{x^3}{e^x-1}dx\right\}$$

$$= \frac{2\pi(kT)^4}{c^2h^3}\left\{\frac{\pi^4}{15}\right\} = \frac{2\pi^5k^4}{15c^2h^3}T^4$$

$$= \sigma T^4\ \mathrm{W{\cdot}m^{-2}}$$

(where all the different brackets show how things group).

73

(17) Calculate σ from the previous.

$$\sigma = \frac{2\pi^5k^4}{15c^2h^3}$$

$$= \frac{2\pi^5\left(1.381\times10^{-23}\ \mathrm{J{\cdot}K^{-1}}\right)^4}{15\left(3\times10^8\ \mathrm{m/s}\right)^2\left(6.626\times10^{-34}\ \mathrm{J{\cdot}s}\right)^3}$$

$$= 5.67\times10^{-8}\ \mathrm{W{\cdot}m^{-2}{\cdot}K^{-4}}$$

74 the value of the Stefan-Boltzmann constant.

§7.6 The Photoelectric Effect

The effect refers to the production of electric effects by light falling on metal surfaces.

§7.6.1 <u>Experimental background</u>

The effect appears to have first been noticed by Hertz in 1887, ironically while he was doing experiments establishing the wave nature of electromagnetic radiation.

75

Subsequent work by J.J. Thomson in 1899 established that it was electrons that were emitted when ultraviolet light fell on metallic surfaces.

In 1902, Philip Lenard found that the maximum KE of the emitted electrons, K_{max}, does not depend on the intensity of the incident light, but on the frequency.

76

Classical Maxwellian theory predicts, on the other hand, that the energy in a light wave is continuously spread out over the wave and that this energy depends on the intensity of the light.

Further, it predicts that energy would be transmitted to the electrons continuously, and that there would be a time-lag before the electrons gained enough energy to escape.

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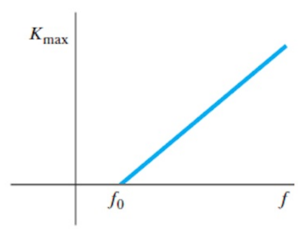
No such time-lag was observed. Further, no effect was observed below a certain frequency, after which K_{max} was found to increase linearly with frequency:



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ADDITIONAL NOTES

Einstein used Planck's "quantum hypothesis" to explain these puzzles. Planck had assumed that "quantization" applied only to emission and absorption of energy, not to EM waves.

Einstein argued that electromagnetic energy is not continuously spread out over the wave, but is concentrated in packets ("quanta") of energy hf.

79

If ϕ is the minimum energy with which an electron is bound to a particular metal, Einstein's proposal says

$$K_{\text{max}} = hf - \phi$$

consistent with observations.

(ϕ in eV for a few metals:
Aluminum: 4.08, Iron: 4.50, Platinum: 6.35.)

80

(18) Einstein's proposal is that there's a linear relationship between K_{max} and f. What is the slope of that line? h

This is a testable consequence, and it was confirmed by Millikan in 1916.

81

We now call Einstein's quanta photons.
(19) How many visible photons does a $100\,\text{W}$ lightbulb emit per second. (Assume $\lambda = 500\,\text{nm}$.)

A $100\,\text{W}$ lightbulb emits $100\,\text{J}$ of energy/second.

The number of photons per second will be the total energy per second, divided by the energy per photon.

82

Thus

$$N = \frac{E}{hf} = \frac{E\lambda}{hc}$$

$$= \frac{(100)(5 \times 10^{-7})}{(6.626 \times 10^{-34})(3 \times 10^{8})}$$

$$= 2.5 \times 10^{20}.$$

83

§7.7 The Compton Effect

Photons have zero rest-mass. But from Special Relativity, they have momentum p related to their energy E by $pc = E$.

Combining this with $E = hf$, we see that photons have momenta given by

$$p = \frac{hf}{c}.$$

84

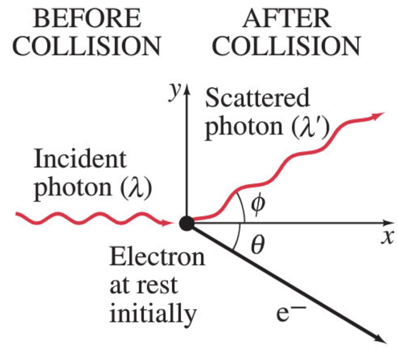
ADDITIONAL NOTES

Can this be tested?

It turns to that photons can "scatter" off electrons, moving at a different angle after and losing energy.

This is called the Compton Effect

85



$$\lambda' = \lambda + \frac{h}{m_e c}(1 - cos\,\phi).$$

86

(20) If a photon of wavelength 0.15nm is scattered at 90° off an electron, what is its new wavelength?

$1 - cos\,90^\circ = 1.$

$$\frac{h}{m_e c} = \frac{6.626 \times 10^{-34}}{(9.11 \times 10^{-31})(3 \times 10^8)} = 2.4{\times}10^{-12}\text{m}$$

87 So $\lambda' = 0.15 + .0024 = 0.1524\,$nm.

§7.8 Photon Interactions with Matter

1. The photoelectric effect: A photon may knock an electron out of an atom and in the process the photon disappears.

2. The photon may knock an atomic electron to a higher energy state in the atom. The photon disappears, and all its energy is given to the atom. Such an atom is then said to be in an excited state.

88

3. Compton effect: The photon can be scattered from an electron (or a nucleus) and in the process lose some energy. The photon is not slowed down. It still travels with speed c, but its frequency will be lower because it has lost some energy.

89

4. Pair production: A photon can actually create matter, such as the production of an electron and a positron (a positron has the same mass as an electron, but the opposite charge).

90

ADDITIONAL NOTES

ADDITIONAL NOTES