§5.1 Review

(1) What do we do when we do GR?

____

(2) What is Einstein's equation?

where T_{ab} is the energy-momentum tensor of matter, and R and R_{ab} are curvatures.

1

(3) What does it mean to "solve Einstein's eq.?"

____

It's not possible to solve the equation without further assumptions. These are _______________

____

____

____

2

The theory is difficult computationally, and also conceptually because you are determining the structure (or geometry) of spacetime itself by finding the metric.

What does knowing the metric, g_{ab}, do for you?

3

Knowing the metric allows you to know

- _______________________________________
- ____________________________
- _______________________________________
  ____

4

Another twist:

Solving Einstein's equation

$$R_{ab} - \frac{1}{2} R g_{ab} = \frac{8\pi G}{c^4} T_{ab}$$

means ____________________________________

____

____

5

§5.2 Example 0: The vacuum, $T_{ab} = 0$

Einstein's equation becomes

$$R_{ab} - \frac{1}{2} R g_{ab} =$$

Solutions of this are called ______________.

An obvious vacuum solution is that ____________, making R_{ab} and R zero; i.e, spacetime is flat.

6

ADDITIONAL NOTES

This means that the metric can be reduced everywhere to the Minkowski metric

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

7

That's what you'd *naively* expect:

No matter

$\downarrow$

No gravitation

$\downarrow$

No curvature

Spacetime should be flat. Life is good (at least for now).

8

§5.3 Example 1: Cosmology

"*we are not able to make cosmological models without some admixture of ideology*"

– S.W.Hawking and G.F.R.Ellis
"The Large Scale Structure of Space-Time," 1973.

The ideology that we adhere to today is the anti-ideology of the anthropocentric view held in Western Europe for around 2,000 years: ____________

9

Less dramatically, we assume that the Universe is ____________ (all places the same) and ________ (all directions the same) on the largest scales.

10

§5.3.1 The form of the metric Under these restrictions, we assume there's "time coordinate" t and a set of "spatial coordinates," such that

a) $t =$ constant hypersurfaces, $\mathcal{H}$, do not intersect each other and are (going to be) "spacelike";

b) fixed values of the spatial coordinates define timelike lines in the "t direction."

11

The metric takes the form

$$ds^2 = dt^2 - a^2(t)d\sigma^2$$

where $d\sigma^2$ is a metric on a 3d space of constant curvature, k.

There are three possibilities: $k > 0$, $k = 0$, $k < 0$. They can be scaled to $k = 1$, $k = 0$, $k = -1$.

12

ADDITIONAL NOTES

Picking suggestive coordinates (t, r, θ, ϕ) fixes a form for the metric:

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right] \quad \begin{cases} k = -1 \\ k = 0 \\ k = +1 \end{cases}$$

where $d\Omega^2 = d\theta^2 + sin^2\theta d\phi^2$.

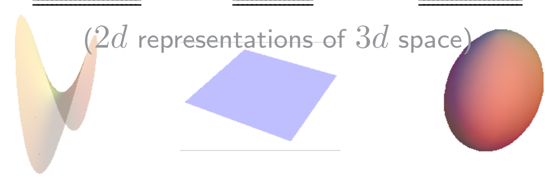
Cred: Friedman (Soviet Union), Lemaitre (France), Robertson and Walker (USA) in the 1920s–1940s.

13

The three possibilities, labeled by k, arise from the ________________ being

________ ________ ________

________ ________ ________



$\underbrace{\hspace{6em}}$ "open" universes (spatially infinite) $0 \leqslant r < \infty$

$\underbrace{\hspace{3em}}$ "closed" (finite) $0 \leqslant r < 2\pi$

14

$a(t)$ is called the ______________ of the Universe and is assumed positive. Its behavior follows from Einstein's equation and the nature of matter. Different behaviors arise in different circumstances.

The nature of matter is what the energy-momentum says it is. Our symmetry assumptions force matter to be uniformly sprinkled throughout the Universe.

15

We model this by assuming the Universe is filled entirely and uniformly by a fluid:

$$T_{ab} = (\rho + P)\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + P g_{ab}$$

where we identify ______________________________ __________ and ________________________.

16

The metric, g_{ab}, appears on the right hand side of Einstein's equation, too.

Our goal is to figure out the behavior of $a(t)$, ρ and P from Einstein's equation.

17

The steps are

1. Calculate the curvatures R_{ab} and R from the FLRW metric. The answer will involve $a(t)$, and its derivatives.
2. Set up Einstein's equation with the above form of T_{ab}:
$$R_{ab} - \frac{1}{2}R g_{ab} = \frac{8\pi G}{c^4} T_{ab}$$
3. Solve (as best you can).

18

ADDITIONAL NOTES

§5.3.2 Calculating $a(t)$

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)\right]$$

$$g_{ab} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\dfrac{a^2(t)}{1-kr^2} & 0 & 0 \\ 0 & 0 & -a^2(t)r^2 & 0 \\ 0 & 0 & 0 & -a^2(t)r^2\,sin^2\theta \end{pmatrix}$$

19

$$g^{ab} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\dfrac{1-kr^2}{a^2(t)} & 0 & 0 \\ 0 & 0 & -\dfrac{1}{a^2(t)r^2} & 0 \\ 0 & 0 & 0 & -\dfrac{1}{a^2(t)r^2\,sin^2\theta} \end{pmatrix}$$

20

Christoffel symbols:

$$\Gamma^m_{ij} = \frac{1}{2}g^{mk}\left(\frac{\partial g_{ik}}{\partial u^j} + \frac{\partial g_{kj}}{\partial u^i} - \frac{\partial g_{ij}}{\partial u^k}\right)$$

So

$$\Gamma^t_{ij} = \frac{1}{2}\left(\frac{\partial g_{it}}{\partial u^j} + \frac{\partial g_{tj}}{\partial u^i} - \frac{\partial g_{ij}}{\partial t}\right)$$

i.e. (no sum over t),

$$\Gamma^t_{tj} = \Gamma^t_{jt} = \quad .$$

21

(4) Calculate $\Gamma^t_{r\theta}$, $\Gamma^t_{r\phi}$, $\Gamma^t_{\theta\phi}$.

(5) Calculate Γ^t_{rr}.

$$\Gamma^t_{rr} =$$

…and so forth.

22

Here they all are (non-zeroes):

$\Gamma^t_{rr} = a\dot{a}/(1-kr^2)$ $\qquad \Gamma^r_{rr} = kr/(1-kr^2)$

$\Gamma^t_{\theta\theta} = a\dot{a}r^2$ $\qquad \Gamma^t_{\phi\phi} = a\dot{a}r^2\,sin^2\theta$

$\Gamma^r_{tr} = \Gamma^\theta_{t\theta} = \Gamma^\phi_{t\phi} = \dot{a}/a$

$\Gamma^r_{\theta\theta} = -r(1-kr^2)$ $\qquad \Gamma^r_{\phi\phi} = -r(1-kr^2)\,sin^2\theta$

$\Gamma^\theta_{r\theta} = \Gamma^\phi_{r\phi} = 1/r$

$\Gamma^\theta_{\phi\phi} = -\,sin\,\theta\,cos\,\theta$ $\qquad \Gamma^\phi_{\theta\phi} = cot\,\theta$

23

Then, we calculate the Ricci tensor:

$$R_{ij} \equiv R^m_{imj} \equiv \frac{\partial\Gamma^m_{ij}}{\partial u^m} - \frac{\partial\Gamma^m_{im}}{\partial u^j} + \Gamma^n_{ij}\Gamma^m_{nm} - \Gamma^n_{im}\Gamma^m_{nj}$$

$$R_{tt} = -3\ddot{a}/a$$

$$R_{rr} = (a\ddot{a} + 2\dot{a}^2 + 2k)/(1-kr^2)$$

$$R_{\theta\theta} = r^2(a\ddot{a} + 2\dot{a}^2 + 2k)$$

$$R_{\phi\phi} = r^2(a\ddot{a} + 2\dot{a}^2 + 2k)\,sin^2\theta$$

24

ADDITIONAL NOTES

and

$$R = g^{ij} R_{ij} = -6\left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right]$$

then set

$$R_{ij} - \frac{1}{2} R g_{ij} = \frac{8\pi G}{c^4} T_{ij},$$

with T_{ij} for a perfect fluid.

25

The result of steps (1) and (2) is:

$$(\diamondsuit)$$

$$(\heartsuit)$$

where $k = -1$, $k = 0$, and $k = +1$ gives us the three cases we've mentioned.

26

(6) What does eqn. $\diamondsuit$ say ρ has to be if $a(t) = \mathbf{C}$ (a constant)?

27

In practice, both ρ and P are positive for normal forms of matter.

(7) In this case, what does eqn. $\heartsuit$ say whether $a(t)$ can be constant?

28

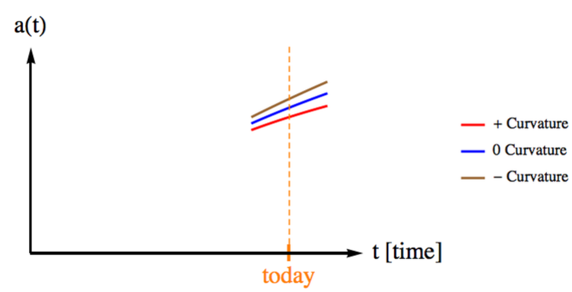
We can get more information by making further assumptions about ρ and P, usually a relationship between the two ("equation of state").

One scenario: $P = \rho/3$ ("radiation dominated")

$$a(t) = \begin{cases} \sqrt{2t + t^2} & (-) \\ 4^{1/4}\sqrt{t} & (0) \\ \sqrt{2t - t^2} & (+) \end{cases}$$

Plotting this around "today":

29

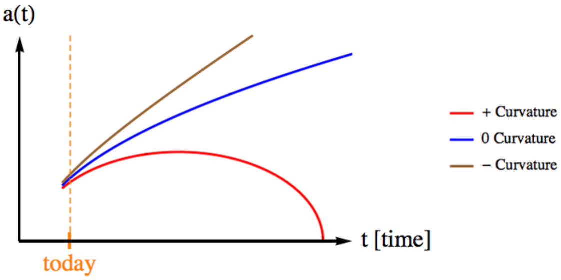


$\dot{a} > 0$: The Universe is expanding.

30

ADDITIONAL NOTES

Projecting into the future:

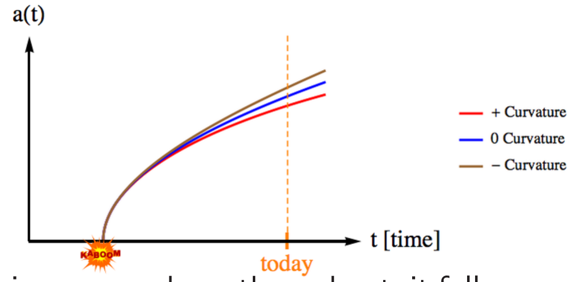


$\ddot{a} < 0$: The expansion is decelerating.

The graph is concave down.

31

Projecting into the past:



If the graph is concave down throughout, it follows that $a(t)$ was zero in the past.

(The Universe had a beginning.)

32

This consequence holds up in a wide variety of scenarios where "gravity is attractive." This happens when $\rho > 0$ and $P > 0$, and it forces the expansion of the Universe to decelerate. I.e., the equation

$$\frac{3\ddot{a}(t)}{a(t)} = -\frac{4\pi G}{c^4}(\rho + 3P)$$

forces $\ddot{a} < 0$. Observations indicate that the Universe *is* expanding, we are led to the inevitable conclusion: **There is always a beginning**.

33

(8) Now, rewrite the ($\diamondsuit$) equation with all the $a(t)$ terms on the left.

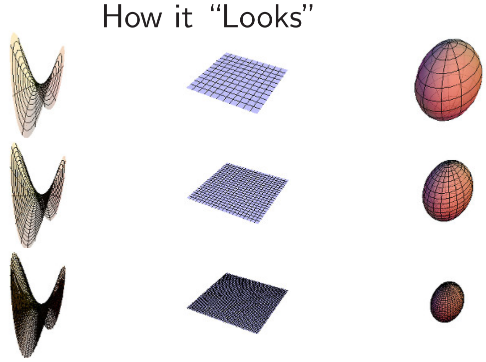
(9) If $a(t) \to 0$ what can you say about the left-hand side, and therefore ρ on the right?

____

____

34

How it "Looks"



BACK IN TIME ↓

35

As $t \to 0$, $a(t) \to$ _____, and $\rho \to$ __________.

____

____

____

Infinities are problematic (____________________).
Still, the Universe was in a very hot, dense state in the past from which it exploded and expanded:

36 ________________________

ADDITIONAL NOTES

You'd think Einstein would be happy at this major consequence, but he had sensed as far back as 1917 that these issues, especially that of singularities, might be a problem.

In fact, these issues arise in Newtonian gravity, as well, as Newton mentioned in letters he wrote in 1692-93 ("Four Letters from Sir Isaac Newton to Doctor Bentley").

37

To the Reverend Dr. RICHARD BENTLEY, *at the Biſhop of* Worceſter's *Houſe in* Park-ſtreet, Weſtminſter.

SIR,

WHEN I wrote my Treatiſe about our Syſtem, I had an Eye upon ſuch Principles as might work with conſidering Men, for the Belief of a Deity, and nothing can rejoice me more than to find it uſeful for that Purpoſe.

December 10, 1692

38

As to your firſt Query, it ſeems to me that if the Matter of our Sun and Planets, and all the Matter of the Univerſe, were evenly ſcattered throughout all the Heavens, and every Particle had an innate Gravity towards all the reſt, and the whole Space, throughout which this Matter was ſcattered, was but finite; the Matter on the outſide of this Space would by its Gravity tend towards all the Matter on the inſide, and by conſequence fall down into the middle of the whole Space, and there compoſe one great ſpherical

December 10, 1692

39

Maſs. But if the Matter was evenly diſpoſed throughout an infinite Space, it could never convene into one Maſs, but ſome of it would convene into one Maſs and ſome into another, ſo as to make an infinite Number of great Maſſes, ſcattered at great Diſtances from one to another throughout all that infinite Space.

December 10, 1692

Newton then goes on to say how this might explain why the Universe contains what it does, *as known at the time* (stars and planets) ...

40

thus might the Sun and fixt Stars be formed, ſuppoſing the Matter were of a lucid Nature. But how the Matter ſhould divide itſelf into two ſorts, and that Part of it, which is fit to compoſe a ſhining Body, ſhould fall down into one Maſs and make a Sun, and the reſt, which is fit to compoſe an opaque Body, ſhould coaleſce, not into one great Body, like the ſhining Matter, but into many little ones .... I do not think explicable by meer natural Cauſes,

December 10, 1692

41

Bentley raised objections and Newton replied:

I Agree with you, that if Matter evenly diffuſed through a finite Space, not ſpherical, ſhould fall into a ſolid Maſs, this Maſs would affect the Figure of the whole Space, provided it were not ſoft, ... the Protuberances might ſometimes ſink a little by their Weight, and thereby the Maſs might, by Degrees, approach a ſpherical Figure.

January 17, 1693

42

ADDITIONAL NOTES

harder it is to ſuppoſe all the Particles in an infinite Space ſhould be ſo accurately poiſed one among another, as to ſtand ſtill in a perfect Equilibrium. For I reckon this as hard as to make not one Needle only, but an infinite number of them (ſo many as there are Particles in an infinite Space) ſtand accurately poiſed upon their Points.

January 17, 1693

43

Yet I grant it poſſible, at leaſt by a divine Power; and if they were once to be placed, I agree with you that they would continue in that Poſture without Motion for ever, unleſs put into new Motion by the ſame Power. When therefore I ſaid, that Matter evenly ſpread through all Space, would convene by its Gravity into one or more great Maſſes, I underſtand it of Matter not reſting in an accurate Poiſe.

January 17, 1693

44

But you argue, in the next Paragraph of your Letter, that every Particle of Matter in an infinite Space, has an infinite Quantity of Matter on all Sides, and by conſequence an infinite Attraction every way, and therefore muſt reſt in Equilibrio, becauſe all Infinites are equal. Yet you ſuſpect a Paralogiſm in this Argument; and I conceive the Paralogiſm lies in the Poſition, that all Infinites are equal.

January 17, 1693

45

After a long discussion of infinity, Newton ends:

I fear what I have ſaid of Infinites, will ſeem obſcure to you; but it is enough if you underſtand, that Infinites when conſidered abſolutely without any Reſtriction or Limitation, are neither equal nor unequal, nor have any certain Proportion one to another, and therefore the Principle that all Infinites are equal, is a precarious one.

January 17, 1693

Newton ties himself into knots, with good reason.

46

The same issue arises in General Relativity. A year after his fundamental paper introducing that theory, Einstein wrote a paper on cosmology:

COSMOLOGICAL CONSIDERATIONS ON THE GENERAL THEORY OF RELATIVITY

BY

A. EINSTEIN

Translated from "Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie," Sitzungsberichte der Preussischen Akad. d. Wissenschaften, 1917.

47

He saw that difficulties with the Newtonian theory were serious

If we apply Boltzmann's law of distribution for gas molecules to the stars, by comparing the stellar system with a gas in thermal equilibrium, we find that the Newtonian stellar system cannot exist at all. For there is a finite ratio

and that General Relativity would have the same problem. A static Universe would be unstable.

48

ADDITIONAL NOTES

Gravity is a destabilizing phenomenon because it is (apparently) ________________

Newton and Einstein realized that the attractive nature of gravity made systems that are controlled by ____________ inherently prone to collapsing on themselves. Yet, much of the Universe is stable. We don't see things rushing to each other, overwhelmed by a fatal gravitational attraction.

49

Against gravitational collapse, what supports

(10) the atomic nucleus? ________________

(11) molecules? ________________

(12) your nose? ________________

50

But, what supports and gives stability to

(13) the solar system? ____________

(14) galaxies? ____________

No electromagnetic forces at work here.

51

The motion that gives stability to the solar system and to galaxies is ____________

There is no evidence for large-scale rotation of the Universe. But expansion could have been used by Einstein to stabilize the Universe.

52

Instead, he changed his theory

> **§ 2. The Boundary Conditions According to the General Theory of Relativity**
>
> In the present paragraph I shall conduct the reader over the road that I have myself travelled, rather a rough and winding road, because otherwise I cannot hope that he will take much interest in the result at the end of the journey. The conclusion I shall arrive at is that the field equations of gravitation which I have championed hitherto still need a slight modification, so that on the basis of the general theory of relativity those fundamental difficulties may be avoided which have been set forth in § 1 as confronting the Newtonian theory. This modification corresponds perfectly to the transi-

53

admitting that

> will not here be discussed. In order to arrive at this consistent view, we admittedly had to introduce an extension of the field equations of gravitation which is not justified by our actual knowledge of gravitation. It is to be emphasized,

This is a glimpse of how some of the remainder of Einstein's career would play itself out.

54

ADDITIONAL NOTES

Einstein proposed altering his equation to

$$R_{ab} - \frac{1}{2} R g_{ab} + \boxed{\Lambda g_{ab}} = \frac{8\pi G}{c^4} T_{ab}$$

Λ is called the ________________ and is meant to give a small effective large scale repulsion that would balance the instability caused by the inherently attractive nature of gravity, and so give a static Universe.

55

Einstein found the idea of a static Universe attractive because, among other reasons, it would eliminate the question of the initial singularity (the origin of the Universe).

The Universe would not begin, end, or change.

It would simply be.

56

The cosmological solutions now become

$$\frac{3\dot{a}^2(t)}{a^2(t)} = \frac{8\pi G}{c^4}\rho - \frac{3k}{a^2(t)} \qquad (\diamondsuit)$$

$$\frac{3\ddot{a}(t)}{a(t)} = -\frac{4\pi G}{c^4}(\rho + 3P) + \Lambda \qquad (\heartsuit^{\Lambda})$$

Einstein showed that if $k = 1$ and $P = 0$ there is a particular ("critical") value of Λ that allows a static solution with $a(t) =$ constant.

57

We've seen that if $a(t) = \mathbf{C} =$ constant,

$$\rho = \frac{3kc^4}{8\pi G \mathbf{C}^2}$$

Solving this for $a(t) = \mathbf{C}$:

$$\mathbf{C} = \sqrt{\frac{3kc^4}{8\pi G\rho}}$$

58

(15) Einstein had assumed that $k = 1$. Did he need that assumption? ________

____

____

59

(16) Use ($\heartsuit^{\Lambda}$) to get the critical value of Λ, in terms of ρ, that gives Einstein's static Universe.

60

ADDITIONAL NOTES

Therefore, we can write

$$\frac{3\ddot{a}(t)}{a(t)} = -\Lambda_{\text{crit}} + \Lambda$$

(17) What would happen if Λ were slightly bigger than the critical value and what would happen if it were slightly smaller?

61

If $\Lambda > \Lambda_{\text{crit}}$, we have $\ddot{a}(t) > 0$. If the Universe is expanding it accelerates; if it is contracting it decelerates. The effective repulsion that Λ gives dominates the attractive nature of gravity.

If $\Lambda < \Lambda_{\text{crit}}$, we have $\ddot{a}(t) < 0$. If the Universe is expanding it decelerates; if it is contracting it accelerates. The effective repulsion that Λ gives is dominated by the attractive nature of gravity.

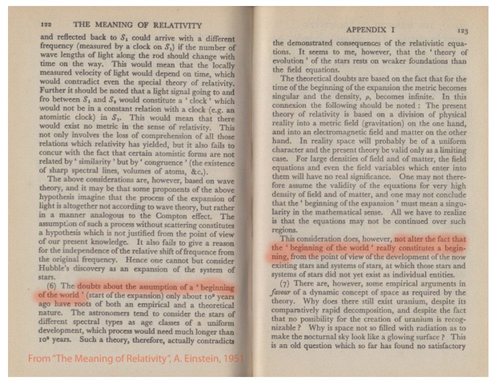
62

We have a single value of Λ that gives a static Universe, with very different behavior on either side.

This was criticized right away as too delicate a "fine tuning." Further, as the years went on, it was observed that the Universe is, in fact, expanding.

Eventually Einstein came around, dropped the cosmological constant, accepted a dynamic Universe, and agreed that the Universe had a beginning.

63



122 THE MEANING OF RELATIVITY

and reflected back to S_1 could arrive with a different frequency (measured by a clock on S_1) if the number of wave lengths of light along the rod should change with time on the way. This would mean that the locally measured velocity of light would depend on time, which would contradict even the special theory of relativity. Further it should be noted that a light signal going to and fro between S_1 and S_2 would constitute a 'clock' which would not be in a constant relation with a clock (e.g. an atomistic clock) in S_1. This would mean that there would exist no metric in the sense of relativity. This not only involves the loss of comprehension of all those relations which relativity has yielded, but it also fails to concur with the fact that certain atomistic forms are not related by 'similarity' but by 'congruence' (the existence of sharp spectral lines, volumes of atoms, &c.).

The above considerations are, however, based on wave theory, and it may be that some proponents of the above hypothesis imagine that the process of the expansion of light is altogether not according to wave theory, but rather in a manner analogous to the Compton effect. The assumption of such a process without scattering constitutes a hypothesis which is not justified from the point of view of our present knowledge. It also fails to give a reason for the independence of the relative shift of frequence from the original frequency. Hence one cannot but consider Hubble's discovery as an expansion of the system of stars.

(6) The doubts about the assumption of a 'beginning of the world' (start of the expansion) only about 10^9 years ago have roots of both an empirical and a theoretical nature. The astronomers tend to consider the stars of different spectral types as age classes of a uniform development, which process would need much longer than 10^9 years. Such a theory, therefore, actually contradicts

APPENDIX I 123

the demonstrated consequences of the relativistic equations. It seems to me, however, that the 'theory of evolution' of the stars rests on weaker foundations than the field equations.

The theoretical doubts are based on the fact that for the time of the beginning of the expansion the metric becomes singular and the density, ρ, becomes infinite. In this connexion the following should be noted: The present theory of relativity is based on a division of physical reality into a metric field (gravitation) on the one hand, and into an electromagnetic field and matter on the other hand. In reality space will probably be of a uniform character and the present theory be valid only as a limiting case. For large densities of field and of matter, the field equations and even the field variables which enter into them will have no real significance. One may not therefore assume the validity of the equations for very high density of field and of matter, and one may not conclude that the 'beginning of the expansion' must mean a singularity in the mathematical sense. All we have to realize is that the equations may not be continued over such regions.

This consideration does, however, not alter the fact that the 'beginning of the world' really constitutes a beginning, from the point of view of the development of the now existing stars and systems of stars, at which those stars and systems of stars did not yet exist as individual entities.

(7) There are, however, some empirical arguments in *favour* of a dynamic concept of space as required by the theory. Why does there still exist uranium, despite its comparatively rapid decomposition, and despite the fact that no possibility for the creation of uranium is recognizable? Why is space not so filled with radiation as to make the nocturnal sky look like a glowing surface? This is an old question which so far has found no satisfactory

From "The Meaning of Relativity", A. Einstein, 1951

But, was he right in saying that he was wrong?

64

§5.4 The Expansion of the Universe

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)\right]$$

We've said that $\dot{a}(t) > 0$ is equivalent to saying that the Universe is expanding, but how does this connect to what we measure?

Note:

We call $H_0 \equiv \dot{a}(t)/a(t)$ Hubble's "constant."

65

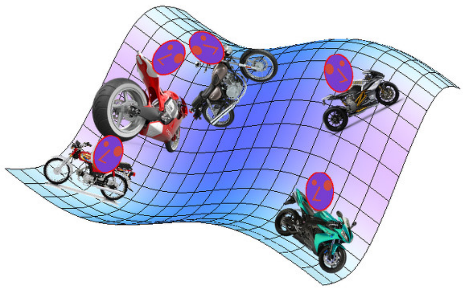
Consider observers separated by fixed coordinate distances r. The physical distance between them is $R(t) = a(t)r$.

Even if they have fixed coordinates, the distance between them changes because $a(t)$ does.

66

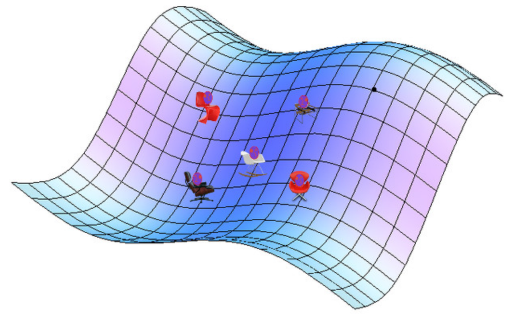
ADDITIONAL NOTES

They're not moving away from everything else by actively running away:



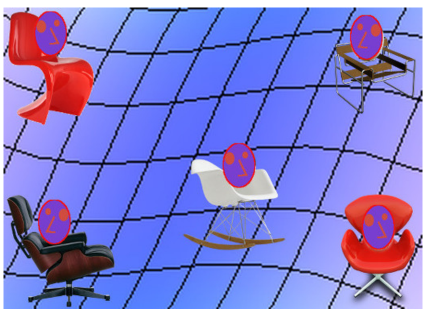
67

Instead, they're sitting in place



68

and enjoying at no charge the benefits of the expansion of the Universe:



69

The "hot big bang theory" makes testable predictions.

It predicts that the Universe should have begin mainly with Hydrogen (75%) and some Helium (25%).

(**18**) Why these two elements?

70

Highlights from History

Time (s)	What happened?	Temp ($^\circ$K)
?	Matter forms	?
$2.0 \cdot 10^{-12}$	Electromagnetism	$1.2 \cdot 10^{18}$
$2.0 \cdot 10^{-6}$	quarks combine	$1.7 \cdot 10^{12}$
1.0	neutrinos released	$1.2 \cdot 10^{10}$
$1.8 \cdot 10^{2}$	Hydrogen forms	$1.2 \cdot 10^{9}$
$1.2 \cdot 10^{13}$	photons released	$2.9 \cdot 10^{3}$
$\vdots$	$\vdots$	$\vdots$
$4.4 \cdot 10^{17}$	Today	2.8

71

Are we making all this up? Maybe. But,...
We can estimate the temperature and density as we plot back in time.

$\downarrow$

We have theories of particle physics that tell us how particles behave at high energies.

$\downarrow$

We can test this in particle accelerators on earth, where we have achieved effective temperatures of

72 $(1.4 \cdot 10^{16})^\circ$ K.

ADDITIONAL NOTES

Is there anything more definite?

Well, there are the photons released about $370{,}000$ years after the big bang, if it occurred. They would still be in the Universe.

It can be calculated that they would be at a little under $3°$K now, and the expansion of the Universe will have stretched their wavelengths to around 1 cm, the microwave region. Can we find them?

73

Hunted by Dicke and Peebles at Princeton in the 1960s, but accidentally discovered as noise at Bell Labs by Penzias and Wilson.

74

§5.5 The Critical Density

Remember: $$\frac{3\dot{a}^2(t)}{a^2(t)} = \frac{8\pi G}{c^4}\rho - \frac{3k}{a^2(t)} \qquad (\diamondsuit)$$

(19) Solve for ρ (and use $H_0 = \dot{a}(t)/a(t)$).

$\rho =$

75

(20) Find a relation (equation or inequality) between ρ and $\frac{c^4}{8\pi G}(3H_0^2)$ when $k = -1,\ 0,\ 1.$

$k = -1:$

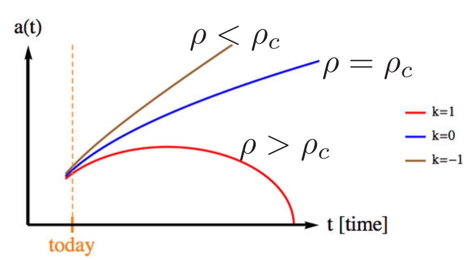
$k = 0:$

$k = 1:$

76

We call $\rho = \dfrac{c^4}{8\pi G}(3H_0^2)$ the critical density of the Universe, ρ_c. It's the dividing density between an open and a closed Universe, and between continual expansion and recollapse.

77



78

ADDITIONAL NOTES

That's what you'd expect on physical grounds. It takes matter to curve spacetime. If there's a low level of matter, you don't expect enough curvature to "close the Universe" or cause recollapse:

"Matter tells geometry how to curve."

79

§5.6 Big Bang Problems

By the late 1970s, the big bang theory was considered on solid footing.

The origin of the uniformity of the Universe was unknown, but it was indisputable: the uniformity of CMBR was solid proof.

The density of the Universe was unknown, but it appeared to be close to ρ_c.

80

Then, at a conference in 1979 to celebrate the 100th anniversary of the birth of Albert Einstein, Dicke and Peebles (Drs. Microwave Background), discussed puzzling aspects of the big bang theory:

9. The big bang cosmology – enigmas and nostrums†

R. H. DICKE AND P. J. E. PEEBLES

81

They said

> The big bang cosmology that developed out of Einstein's ideas and Hubble's observations has stood the test of time and observation, but even the staunchest advocate would admit that it is at best only a reasonable first approximation that certainly does not tell the whole story. There are in particular some curious and enigmatic features of this cosmology that lead us to think that an important piece of the picture may be missing. It is useful to review and reconsider these curiosities from time to time because they certainly have something to teach us. But what is it?

82

then went on with a quick account of what was understood about the Universe in 1917:

> It is not clear how familiar Einstein was with the observational situation in astronomy, or how much attention he paid to it. There was at the time speculation that the spiral nebulae are island universes like the Milky Way galaxy, but there were also some good arguments that these objects must be only minor satellites. It was considered well established, from star counts, that the Milky Way star system is finite and bounded, shaped roughly like a flattened spheroid. The spiral nebulae seemed to be concentrated at the poles of this star system, which would suggest they are related to it. Also, by 1916 van Maanen had found the first tentative evidence of proper motions in some of the larger spirals (van Maanen, 1916). If valid, and if the internal velocities in these systems are less than the velocity of light, it would make them quite close and much smaller than the Milky Way.

83

Nebulae were hazy patches that had been catalogued because they got in the way of finding the really interesting objects: comets.

The nature of these nebulae was debated. Some astronomers said they were in the Milky Way, others argued that they were outside it.

One argument that they were within the MW was the observations of novae in some of them.

84

ADDITIONAL NOTES

If a single star outside the MW could be observed brightening, the event would have to be "on a scale of magnitude such as the imagination recoils from contemplating."

"A Popular History of Astronomy during the Nineteenth Century," Agnes M. Clerke, page 438.

85

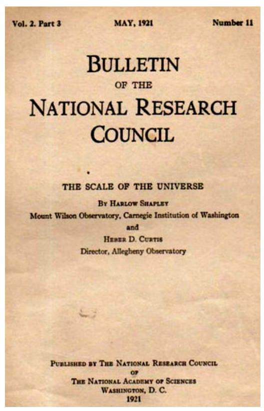
The question relates to the size of the Universe: Is the MW the entire Universe, or does the Universe extend outside the MW?

The Milky Was was already known to be vast.

86

The question was debated on 26 April 1920 by two leading astronomers.

The event became known as the Shapley-Curtis "Great Debate."



Vol. 2. Part 3 MAY, 1921 Number 11

BULLETIN OF THE NATIONAL RESEARCH COUNCIL

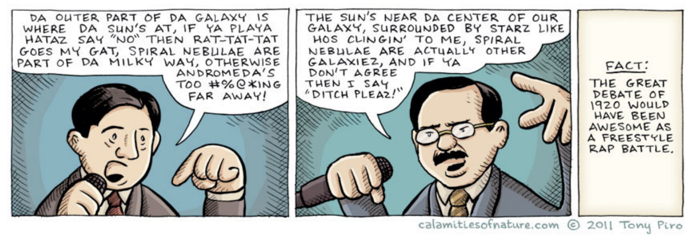
THE SCALE OF THE UNIVERSE

By Harlow Shapley
Mount Wilson Observatory, Carnegie Institution of Washington
and
Heber D. Curtis
Director, Allegheny Observatory

Published by The National Research Council of The National Academy of Sciences
Washington, D. C.
1921

87

Here's the debate in your language:



The "within MW" view was thought to have won.

88

Although Einstein had been trying to build a larger cosmology, the indications were that the Milky Way was the entire Universe. But:

> Over the next two decades it became clear that these indications are misleading, the former because interstellar dust in the plane of the Milky Way obscures the galaxies and the latter because of observational problems. Einstein's vision was remarkably good. In 1924 Hubble showed, by the identification of variable stars of known intrinsic luminosity, that the spiral nebulae are well outside the Milky Way and at least comparable to it in size (Hubble, 1924). Hubble's surveys of the galaxy distribution, begun in 1926 (Hubble, 1926) and continuing through the 1930s, gave the first direct evidence of large-scale homogeneity and isotropy. This has been confirmed by recent observations of the precise isotropy of the radiation background – X-ray, microwave,

89

The observations that the Universe was much bigger than we'd originally thought had also provided evidence of large scale homogeneity and isotropy. This is remarkable:

> The concept of large-scale homogeneity has been with us so long that cosmologists tend to take it as a commonplace, but it is remarkable simply because it stands in such contrast to our experience that things have structure – from the properties of subatomic particles on up to the organization of galaxies in great clouds. Milne (1935) was the first to

90

ADDITIONAL NOTES

It's not only remarkable, large scale homogeneity is a problem:

The distant galaxies observed in well-separated parts of the sky are so far apart from each other that there is not time enough since the big bang for a signal to have traveled from one to the other. Observers on Earth can see and compare them, being about half-way in between, and in line with homogeneity it is found that the galaxies are quite similar. By comparing radiation background intensities across the sky it is also found that the temperature and expansion rate are precisely synchronized across the visible universe. Even though the separate parts of the visible universe are not visible to each other they are evolving in very precise unison.

91

We believe the Universe is $\sim$ 14 billion years old.

A patch of the Universe 12 billion light years from us in one direction cannot have communicated with another patch 12 billion light years from us in the opposite direction.

Yet they look virtually identical. What made them that way, if there was no communication between them?

92

Is this really a problem?

Are structural relations between widely separated parts of the universe a problem? In the past these parts were much closer together. But close proximity in earlier times does not eliminate the problem. Assum-

This problem, the problem of inexplicable uniformity, is known as the

____

93

Whether you think it a problem depends on how content you are with coincidence.

OK, we have a problem. Actually, we have two.

Dicke and Peebles, go right on to say:

The relationships of widely separated parts of the universe are not the only problem. There is a remarkable balance of mass density and expansion rate. In general relativity theory with $\Lambda = 0$ the two are

94

related by the equation

$$H^2 = \left(\frac{1}{a}\frac{da}{dt}\right)^2 = \tfrac{8}{3}\pi G\rho(t) - \frac{c^2}{R^2 a^2}, \tag{9.1}$$

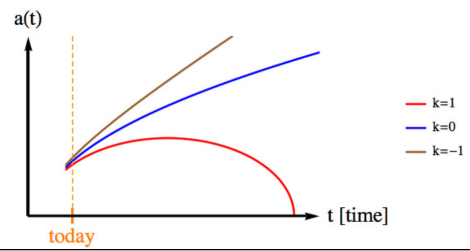
where $a(t)$ is the expansion parameter, R is a constant, and $|R|a(t)$ is the magnitude of the space curvature (measured in a hypersurface of roughly constant galaxy proper number density, at fixed cosmic time t). The present relative value of the two terms on the right side of this equation is poorly known, because the mean mass density, ρ, is so uncertain, but it is unlikely that the first term is less than 3 per cent of the magnitude of the second. Since ρ varies as a^{-3} (or more rapidly if pressure is important) the mass term on the right-hand side dominates the curvature term when a is less than about 3 per cent of its present value. Tracing the expansion back in time, one finds that at $t \sim 1$ s, when much of the helium is thought to have been produced, the mass term is some 14 orders of magnitude larger than the curvature term. This means the expansion rate has been tuned to agree with the mass density to an accuracy better than 1 part in 10^{14}. In the limit $t \to 0$ this balance

95

Remember, $k = 0$ gives the critical density, the dividing density between

a closed Universe ($k = 1, \rho > \rho_c$) and

an open Universe ($k = -1, \rho < \rho_c$).



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ADDITIONAL NOTES

In Slide 75, from ($\diamondsuit$) we'd obtained:

$$\rho = \frac{c^4}{8\pi G}\left(3H_0^2 + \frac{3k}{a^2(t)}\right) = \frac{c^4}{8\pi G}(3H_0^2) + \frac{c^4}{8\pi G}\frac{3k}{a^2(t)}.$$

Writing the right-hand side in terms of ρ_c:

$$\rho = \rho_c + \frac{3kc^4}{8\pi G}\frac{1}{a^2(t)}$$

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Solving for ρ_c:

$$\rho_c = \rho - \frac{3kc^4}{8\pi G}\frac{1}{a^2(t)}$$

(**21**) What is $\rho_c/\rho - 1$?

$$\frac{\rho_c}{\rho} - 1 = -\frac{3kc^4}{8\pi G}\frac{1}{\rho a^2(t)}$$

98

The quantity ρ/ρ_c is called the density parameter and denoted by Ω . So we have

$$(\Omega^{-1} - 1)\rho a^2(t) = -\frac{3kc^4}{8\pi G} = \text{ constant.}$$

Writing $\rho a^2(t)$ as $\rho a^3(t)/a(t)$, and arguing that $\rho a^3(t)$ is roughly fixed, $\rho a^2(t)$ goes down (rapidly) as the Universe expands.

Therefore, $(\Omega^{-1} - 1)$ must go up rapidly.

99

Why is that a problem?

The observed density of the Universe is estimated to be between a tenth and twice the critical density of about 10^{-30} g/cc: i.e., $0.1 < \Omega < 2$.

(**22**) What are the bounds on $(\Omega^{-1} - 1)$?

100

For a rapidly growing quantity to be this small today (after 14 billion years of growth), the density parameter $\Omega = \rho/\rho_c$ would have had to have been between 0.99999999999999 and 1.00000000000001 1 sec after the big bang.

the expansion rate has been tuned to agree with the mass density to an accuracy better than 1 part in 10^{14}. In the limit $t \to 0$ this balance

101

Even small deviations from this cause the Universe to behave very differently from what we see.

A slightly greater value for ρ would make the Universe to be closed, and recollapse rapidly. A slightly smaller value would give too rapid an expansion.

Why the Universe should be so precisely tuned to the critical (i.e, flat) value of the density is called the_______________.

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ADDITIONAL NOTES

§5.7 Inflation

Dicke and Peebles presented the horizon and flatness problems at the Einstein centenary conference in 1979, but had already been giving lectures about them at other venues.

One lecture was by Dicke at Cornell on November 13, 1978, on the flatness problem. It was attended by a young particle physicist, Alan Guth.

103

Guth got his Ph.D. from MIT in 1972 and had hopped around in temporary positions since:

- Princeton, 1971–74
- Columbia, 1974–76
- Cornell, 1976–79
- SLAC, 1979–1980

In 1978 Guth felt cosmology wasn't interesting. He went to Dicke's lecture because Penzias & Wilson had just received a Nobel Prize.

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Guth was working on "Grand Unified Theory" at the time – the theory that unifies the strong nuclear, weak nuclear and electromagnetic forces at high energies (temperatures).

In 1970, we'd had a good understanding of just electromagnetism (and gravity).

By 1976, we had a unified theory of three of the four fundamental forces.

105

Guth was investigating the existence of magnetic monopoles in GUT: Could they have existed in the early Universe and, if they did, why might we not see them today?

On the night of December 6, 1979, Guth felt he had found the reason why we see no monopoles today, through a mechanism called supercooling.

Then he remembered Dicke's talk (a year ago)...

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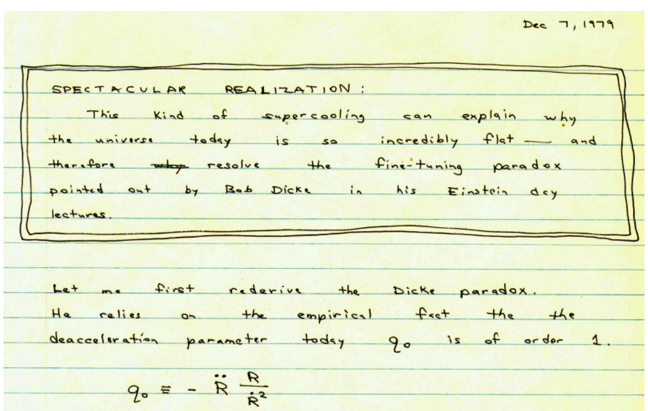

Dec 7, 1979

SPECTACULAR REALIZATION:

This kind of supercooling can explain why the universe today is so incredibly flat — and therefore ~~why~~ resolve the fine-tuning paradox pointed out by Bob Dicke in his Einstein day lectures.

Let me first rederive the Dicke paradox. He relies on the empirical fact the the deacceleration parameter today q_0 is of order 1.

$$q_0 \equiv - \ddot{R} \frac{R}{\dot{R}^2}$$

107

Later that same month, Guth was told over lunch about the horizon problem of cosmology (he didn't know of it at the time). He went home and figured out, that afternoon itself, that his "supercooling" mechanism would solve this problem too.

It was, as he said later, as if he'd found the master key to the Universe: door after door opened with that single key.

108

ADDITIONAL NOTES

Guth's career after December 1979:

- January 23, 1980: Announces his ideas in a seminar at SLAC
- January 24, 1980, before lunch: gets invitations to present his ideas from three different universities.
- January 24, 1980, after lunch: Is invited to spend three further years at SLAC; hears that U. Penn, and UC Davis are considering offering permanent professorships.
- January 28, 1980: U. Penn offers the job.
- February-March, 1980: Lectures at ten universities, including Harvard, Princeton, Columbia and Cornell.

109

- On his return has offers of professorships from Minnesota, Rutgers, Harvard, Princeton, Maryland, UC Davis and UC Santa Barbara.
- But he wants MIT – and shortly after, he gets it. He's been there ever since.

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What was this big idea of Alan Guth's?

Inflationary universe: A possible solution to the horizon and flatness problems

Alan H. Guth*

Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

(Received 11 August 1980)

The standard model of hot big-bang cosmology requires initial conditions which are problematic in two ways: (1) The early universe is assumed to be highly homogeneous, in spite of the fact that separated regions were causally disconnected (horizon problem); and (2) the initial value of the Hubble constant must be fine tuned to extraordinary accuracy to produce a universe as flat (i.e., near critical mass density) as the one we see today (flatness problem). These problems would disappear if, in its early history, the universe supercooled to temperatures 28 or more orders of magnitude below the critical temperature for some phase transition. A huge expansion factor would then result from a period of exponential growth, and the entropy of the universe would be multiplied by a huge factor when the latent heat is released. Such a scenario is completely natural in the context of grand unified models of elementary-particle interactions. In such models, the supercooling is also relevant to the problem of monopole suppression. Unfortunately, the scenario seems to lead to some unacceptable consequences, so modifications must be sought.

111

Guth was able to show that the problems of big bang cosmology could be solved if you had an exponential scale factor that doubled $a(t)$ every 10^{-37} seconds for a hundred doublings.

This would stretch the Universe a million trillion trillion times in ten trillionth-trillionth-trillionth of a second.

112

It solves the Horizon Problem by making everything was close enough in the past in order to have communicated.

It solves the Flatness Problem by blowing the Universe up so much, it looks flat.

113

Cosmic inflation was quickly embraced.

In 1982, Stephen Hawking organized a conference around the very early Universe.

Major advances occurred here in figuring how inflation predicts structure formation: quantum fluctuations give rise to tiny irregularities that act as seeds for larger structures (stars, galaxies, etc.).

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ADDITIONAL NOTES

Directly looking for fluctations in CMBR...

BICEP project, South Pole

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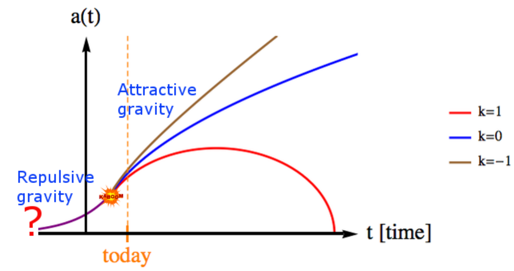


116

Revisiting the Initial Singularity

The conclusion that there's an initial singularity is an important one, because it says ________________ ________________. That's one of the most important results of standard modern cosmology.

Many people have resisted this: ________________ ____ ____

117

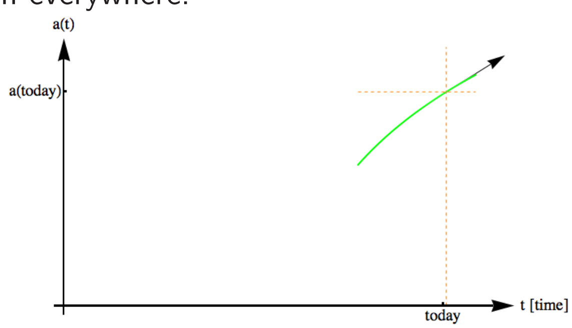
How does Standard Cosmology Imply a Beginning?

Observations support the fact that the Universe is expanding. Therefore, the graph of the scale factor, $a(t)$, is increasing at $t =$ today and, *if gravity is always attractive*, is concave down everywhere.

We can show based only on these two conditions that $a(t) = 0$ at a finite time in the past.

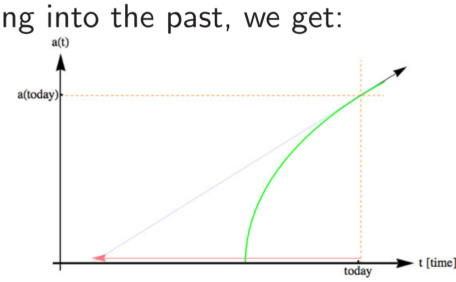
118

The graph of $a(t)$ is increasing today and concave down everywhere:



119

Projecting into the past, we get:



The graph of $a(t)$ can't wriggle out from under the straight line that's the past-projection of $\dot{a}$(today) without going concave up.

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ADDITIONAL NOTES

So, $a(t)$ must be zero a finite time in the past in an expanding Universe in which gravity is attractive. As you approach that time, the matter density, ______. That's how *calculations* indicate that the Universe had a beginning.

But ______________________________.

What happens in more realistic situations?

121

Even without symmetry assumptions, the theorems of Hawking and Penrose show that there is an initial singularity (in the form of ______ ______) as long as some energy condition holds: weak or strong. (______________________ ______________________.) That was a very important conclusion. But, it needs to be revisited in inflation for technical and philosophical reasons.

122

1. Technical reasons:

In inflation, the

1a: ______________________________.

1b: ______________________________.

Theorems that say there was an initial singularity that depend on these conditions no longer apply.

2. Philosophical reason: ______________

123

2. Philsophical Reason: Inflation is Forever

Because the mechanism of inflation involves quantum processes, it ______________________ ______________________. (That would lead to another version of the horizon problem.)

Inflation will end at different times at different places.

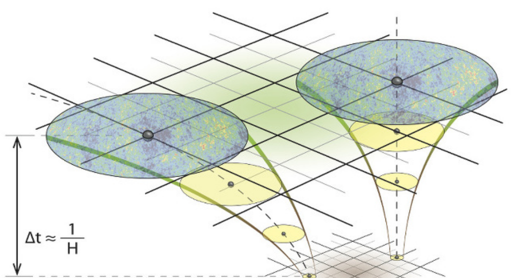
124

Each post-inflationary region will expand at the "normal" cosmological rate, but these regions will be separated by a background that is expanding so fast, they will not, in general, merge.

A "bubble" ("pocket Universe") such as ours comes from a tiny post-inflationary patch of spacetime in a hot, dense, explosive state. The explosion ("bang"?) would lead to our observed Universe.

125

But, we'd be part of a multiverse:



P.D. Will, "Eternal Inflation and the Multiverse", 2012
https://esc.fnwi.uva.nl/thesis/centraal/files/f252559910.jpg

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ADDITIONAL NOTES

When inflation ends at a particular place cannot be predicted with certainty. The quantum nature of inflation means we can only assign probabilities.

____

____

Inflation is, in general, ____________: it continues somewhere forever into the future

127

It's natural to ask if inflation is also past-eternal: does it continue forever into the past?

The explosion from which a post-inflation bubble such as ours evolves would not itself be a singularity, and there would not automatically be a singularity prior to it, just a state of eternal inflation.

In this scenario, ______________________________

____

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Non-singular Inflationary Cosmology

A short history

In the early 1980s attempts were made to construct inflationary, non-singular cosmologies. They were not successful.

Occasional attempts were made to study if this lack of success was *necessary*.

129

1987: "Geodesic focusing, energy conditions and singularities," A. Borde, Class. Quant. Grav., 4, 343-356 (1987).

"situations where there are repeated violations of the energy conditions are shown to still lead to the focusing of geodesics, . . . The existence of singularities in situations where the energy conditions are violated, as in inflationary cosmological models, is also discussed."

130

1992: "Did the Universe have a beginning?"' A. Vilenkin, Phys. Rev. D 46, 2355 (1992).

"It is argued that 'eternal inflation' must have a beginning in time. Conditions are formulated for a spacetime to describe an eternally inflating universe without a beginning, and it is shown that these conditions cannot be satisfied. A rigorous proof is given for a two-dimensional spacetime, and a plausibility argument for four dimensions."

131

A Tale of Two Afternoons

Afternoon 1: September 1993

132

ADDITIONAL NOTES

1993–1997

- "Eternal inflation and the initial singularity," AB & AV, Phys. Rev. Lett., 72, 3305 (1994).
- "The impossibility of steady-state inflation," AB & AV, Eighth Yukawa Symposium, Japan (1994).
- "Open and closed universes, initial singularities and inflation," AB, Phys. Rev., D50, 3692 (1994).
- "Inflation and initial singularities," AB, Seventh M. Grossmann Meeting, Stanford (1996).
- "Singularities in inflationary cosmology," AB & AV, Sixth Quant. Grav. Seminar, Moscow (1996).
- "Violations of the weak energy condition in inflating spacetimes," AB & AV, Phys. Rev., D56, 717 (1997).

133

A Tale of Two Afternoons

Afternoon 2: August 2001

134

The BGV Theorem

Inflationary Spacetimes Are Incomplete in Past Directions

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(Received 5 October 2001; revised manuscript received 24 January 2003; published 15 April 2003)

Many inflating spacetimes are likely to violate the weak energy condition, a key assumption of singularity theorems. Here we offer a simple kinematical argument, requiring no energy condition, that a cosmological model which is inflating—or just expanding sufficiently fast—must be incomplete in null and timelike past directions. Specifically, we obtain a bound on the integral of the Hubble parameter over a past-directed timelike or null geodesic. Thus inflationary models require physics other than inflation to describe the past boundary of the inflating region of spacetime.

135

I. Introduction.—Inflationary cosmological models [1–3] are generically eternal to the future [4,5]. In these models, the Universe consists of postinflationary, thermalized regions coexisting with still-inflating ones. In comoving coordinates the thermalized regions grow in time and are joined by new thermalized regions, so the *comoving volume* of the inflating regions vanishes as $t \to \infty$. Nonetheless, the inflating regions expand so fast that their *physical volume* grows exponentially with time. As a result, there is never a time when the Universe is completely thermalized. In such spacetimes, it is natural to ask if the Universe could also be past-eternal. If it could, eternal inflation would provide a viable model of the Universe with no initial singularity. The Universe would never come into existence. It would simply exist.

136

After reviewing the background

More general theorems showing that inflationary spacetimes are geodesically incomplete to the past were then proved [8]. One of the key assumptions made in these theorems is that the energy-momentum tensor obeys the weak energy condition. Although this condition is satisfied by all known forms of classical matter, subsequent work has shown that it is likely to be violated by quantum effects in inflationary models [9,10]. Such violations must occur whenever quantum fluctuations result in an increase of the Hubble parameter H—i.e., when $dH/dt > 0$—provided that the spacetime and the fluctuation can be approximated as locally flat. Such upward fluctuations in H are essential for the future-eternal nature of chaotic inflation. Thus, the weak energy condition is generally violated in an eternally inflating universe. These violations appear to open the door again to the possibility that inflation, by itself, can eliminate the need for an initial singularity. Here we argue that this is not the case. In fact,

We state our main claim.

137

The theorem requires one condition, the ________ ________________:

considering spacetimes for which the past region obeys an *averaged expansion condition*, by which we mean that the average expansion rate in the past is greater than zero:

$$H_{av} > 0. \qquad (1)$$

With a suitable definition of H and the region over which the average is to be taken, we show that the averaged expansion condition implies past-incompleteness.

138

ADDITIONAL NOTES

metric. We define the Hubble parameter as [14]

$$H \equiv \frac{\Delta u_r}{\Delta r} = \frac{-v_\mu (Du^\mu/d\tau)}{\gamma^2 - \kappa}. \qquad (8)$$

Since $\mathcal{O}$ is a geodesic, we have $(Dv^\mu/d\tau) = 0$, and therefore

$$H = \frac{-d\gamma/d\tau}{\gamma^2 - \kappa} = \frac{d\tau}{d\tau} F(\gamma(\tau)), \qquad (9)$$

where

$$F(\gamma) = \begin{cases} \gamma^{-1} & \text{null observer } (\kappa = 0), \\ \frac{1}{2}\ln\frac{\gamma+1}{\gamma-1} & \text{timelike observer } (\kappa = 1). \end{cases} \qquad (10)$$

139

As in Sec. II, we now integrate H along $\mathcal{O}$ from some initial τ_i to some chosen τ_f:

$$\int_{\tau_i}^{\tau_f} H d\tau = F(\gamma_f) - F(\gamma_i) \leq F(\gamma_f). \qquad (11)$$

In the null case $F(\gamma_f) = \gamma_f^{-1}$, which is equal to the value of $d\tau/dt$ at t_f, normalized in Sec. II to unity.

Equation (11) therefore reproduces exactly the results of Eqs. (4) and (6), but in a much more general context. Again we see that if $H_{\mathrm{av}} > 0$ along any null or noncomoving timelike geodesic, then the geodesic is necessarily past-incomplete.

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IV. Discussion.—Our argument shows that null and timelike geodesics are, in general, past-incomplete in inflationary models, whether or not energy conditions hold, provided only that the averaged expansion condition $H_{\mathrm{av}} > 0$ holds along these past-directed geodesics. This is a stronger conclusion than the one arrived at in previous work [8] in that we have shown under reasonable assumptions that almost all causal geodesics, when extended to the past of an arbitrary point, reach the boundary of the inflating region of spacetime in a finite proper time (finite affine length, in the null case).

What can lie beyond this boundary? Several possibil-

141

Whatever the possibilities for the boundary, it is clear that unless the averaged expansion condition can somehow be avoided for all past-directed geodesics, inflation alone is not sufficient to provide a complete description of the Universe, and some new physics is necessary in order to determine the correct conditions at the boundary [16]. This is the chief result of our Letter. The result depends on just one assumption: the Hubble parameter H has a positive value when averaged over the affine parameter of a past-directed null or noncomoving timelike geodesic.

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Spreading the Message

Alan Guth spoke at Hawking's 60th birthday in 2002, partly on the fluctuations that led to structure formation, partly on the BGV theorem.

In the historical part of his talk, Guth mentioned how in the early days of inflation, he was excited that Stephen Hawking had organized a conference on the early Universe and invited him.

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On the BGV theorem, Guth asked

". . . can inflation by itself be the complete theory of cosmic origins? Can inflation be eternal into the past as well as the future, allowing a model which on very large scales is steady state, eliminating the need for a beginning? The answer I believe is no . . . Borde, Vilenkin, and I have proven a rigorous theorem, . . . [that] shows that the simplest type of inflationary models still require a beginning, even though they are eternal into the future. The difficulty is that we have no way of discussing the class of all possible inflationary models, so we cannot say that our theorem applies to all cases."

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ADDITIONAL NOTES

And so word got out, but it sometimes got to unexpected places . . .

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§5.8 Milne's Universe

Assume that $P = 0$ in the FLRW equations

$$\frac{3\dot{a}^2(t)}{a^2(t)} = \frac{8\pi G}{c^4}\rho - \frac{3k}{a^2(t)}$$

$$\frac{3\ddot{a}(t)}{a(t)} = -\frac{4\pi G}{c^4}(\rho + 3P)$$

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What are the possibilities as $\rho \to 0$?
Setting $P = 0$:

$$\frac{3\dot{a}^2(t)}{a^2(t)} = \frac{8\pi G}{c^4}\rho - \frac{3k}{a^2(t)}$$

$$\frac{3\ddot{a}(t)}{a(t)} = -\frac{4\pi G}{c^4}\rho$$

What happens in these equations as $\rho \to 0$?

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$$\dot{a}^2(t) \approx -k \quad \text{and} \quad \ddot{a}(t) \approx 0$$

with $k = 0$ or -1.

The second equation simply confirms the first, and has no further content. So we're working with

as our two cases.

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Case 1: $k = 0$, $\therefore\ \dot{a}(t) \approx 0$.

If we scale the FLRW r coordinate to $\hat{r} = ar$, we get $d\hat{r} = a dr$ and the metric becomes

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Case 2: $k = -1$, $\therefore\ \dot{a}(t) \approx \pm 1$.

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ADDITIONAL NOTES

We've rediscovered "Milne's Universe."

(Wikipedia sagely observes "the assumption of zero energy content limits its use as a realistic description of the universe.")

But, it's thrilling.

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§5.9 Spacelikeness and Timelikeness

For a vector $\vec{V}$, we've defined

$$\vec{V} \text{ is } \begin{cases} \text{timelike} & \text{if } g_{ab}V^aV^b > 0, \\ \text{null} & \text{if } g_{ab}V^aV^b = 0, \\ \text{spacelike} & \text{if } g_{ab}V^aV^b < 0, \end{cases}$$

where $g_{ab}V^aV^b$ is $g(\vec{V}, \vec{V})$ if you're working abstractly, or $g_{ij}V^iV^j$ if thinking concretely in components. In the same spirit, we'll use V^a for $\vec{V}$.

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A (smooth) curve represents the path of:

a) a material test particle (such as a neutron), if its tangent is everywhere timelike.

b) a massless test particle (such as a photon), if its tangent is everywhere null.

If no (other) forces are at play, these paths are geodesic.

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Let $f : \mathcal{M} \to \mathbb{R}$ be a scalar function on an nd spacetime, $\mathcal{M}$. Define a (local) hypersurface, $\mathcal{H}$, as the $(n-1)$d submanifold given by $f =$ constant.

The normal to $\mathcal{H}$ is obtained from the gradient of f, calculated in coordinates $\{x^i\}$ as $N_i = \{\partial_i f\}$, where $\partial_i f \equiv \partial f/\partial x^i$.

We define $N^i \equiv g^{ij}N_j$.

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$$\mathcal{H} \text{ is called } \begin{cases} \text{spacelike} & \text{if } N^i \text{ is timelike,} \\ \text{null} & \text{if } N^i \text{ is null,} \\ \text{timelike} & \text{if } N^i \text{ is spacelike} \end{cases}$$

For example, in 4d Minkowski spacetime with coordinates (t, x, y, z), t is a scalar function whose level hypersurfaces are spacelike ($N^i = (1, 0, 0, 0)$).

A spacelike hypersurface is "an instant of time," or, as Kurt Gödel put it, "a layer of 'now'."

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Gödel came up with his characterization in an attempt to answer whether "reality" can always be split into layers of 'now.'

The answer he came up with was, in general, no.

In cosmology we start with the simplifying assumption that it can.

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ADDITIONAL NOTES