§4.1 Beyond Special Relativity

Einstein used Minkowski's idea of spacetime to further develop the theory of relativity.

When developing Special Relativity, Einstein had several parallel investigators – Lorentz, Poincaré, and others.

When developing its extension, General Relativity,
1 Einstein initially acted alone.

After many false starts and finishes, Einstein hit upon his final formulation in 1915.

He summarized his journey and where it had led him in his landmark paper of 1916:

2

The Foundation of the General Theory of Relativity

by A. Einstein

The theory which is presented in the following pages conceivably constitutes the farthest-reaching generalization of a theory which, today, is generally called the "theory of relativity"; I will call the latter one—in order to distinguish it from the first named—the "special theory of relativity," which I assume to be known. The generalization of the theory of relativity has been facilitated considerably by Minkowski, a mathematician who was the first one to recognize the formal equivalence of space coordinates and the time coordinate, and utilized this in the construction of the theory. The mathematical tools that are necessary for general relativity were readily available in the "absolute differential calculus," which is based upon the research on non-Euclidean manifolds by Gauss, Riemann, and Christoffel, and which has been systematized by Ricci and Levi-Civita and has already been applied to problems of theoretical physics. In section B of the present paper I developed all the necessary mathematical tools—which cannot be assumed to be known to every physicist—and I tried to do it in as simple and transparent a manner as possible, so that a special study of the mathematical literature is not required for the understanding of the present paper. Finally, I want to acknowledge gratefully my friend, the mathematician Grossmann, whose help not only saved me the effort of studying the pertinent mathematical literature, but who also helped me in my search for the field equations of gravitation.

3

§ 1. Observations on the Special Theory of Relativity

THE special theory of relativity is based on the following postulate, which is also satisfied by the mechanics of Galileo and Newton.

If a system of co-ordinates K is chosen so that, in relation to it, physical laws hold good in their simplest form, the *same* laws also hold good in relation to any other system of co-ordinates K′ moving in uniform translation relatively to K. This postulate we call the "special principle of relativity." The word "special" is meant to intimate that the principle is restricted to the case when K′ has a motion of uniform translation relatively to K, but that the equivalence of K′ and K does not extend to the case of non-uniform motion of K′ relatively to K.

4

Einstein argued that it's *necessary* to extend special relativity because it's inherently defective.

This defect was present in Galilean relativity (Newtonian physics), as well, but had gone unresolved and often unnoticed for hundreds of years.

§ 2. The Need for an Extension of the Postulate of Relativity

In classical mechanics, and no less in the special theory of relativity, there is an inherent epistemological defect which

5

Physics till then had relied on the concept of "inertial frames," a class of frames which

a) Move uniformly wrt each other, and in which

b) Newton's First Law holds.

("No force, no acceleration.")

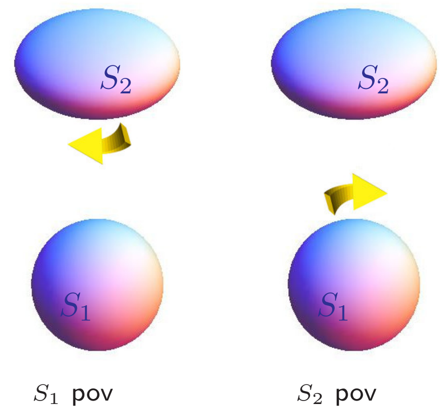
But how do you know there are "no forces."

6

ADDITIONAL NOTES

Two fluid bodies of the same size and nature hover freely in space at so great a distance from each other and from all other masses that only those gravitational forces need be taken into account which arise from the interaction of different parts of the same body. Let the distance between the two bodies be invariable, and in neither of the bodies let there be any relative movements of the parts with respect to one another. But let either mass, as judged by an observer at rest relatively to the other mass, rotate with constant angular velocity about the line joining the masses. This is a verifiable relative motion of the two bodies. Now let us imagine that each of the bodies has been surveyed by means of measuring instruments at rest relatively to itself, and let the surface of S_1 prove to be a sphere, and that of S_2 an ellipsoid of revolution. Thereupon we put the question—What is the reason for this difference in the two bodies? No answer can

7



8

"Newtonian mechanics does not give a satisfactory answer to this question. It pronounces as follows:—The laws of mechanics apply to the space R_1, in respect to which the body S_1 is at rest, but not to the space R_2, in respect to which the body S_2 is at rest. But the privileged space R_1 of Galileo, thus introduced, is a merely *factitious* cause, and not a thing that can be observed. It is therefore clear that Newton's

Einstein argues that Newtonian physicists would say frame 1, in which S_1 is at rest, is inertial, and frame 2 is not, but that decision would only be made *after* observing what's happening.

9

After some further discussion of the asymmetry in the appearance of two fluid spheres in relative rotation, Einstein concludes

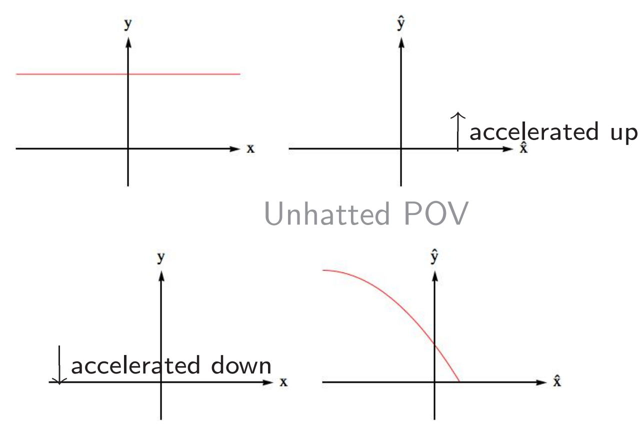
factitious cause R_1. Of all imaginable spaces R_1, R_2, etc., in any kind of motion relatively to one another, there is none which we may look upon as privileged *a priori* without reviving the above-mentioned epistemological objection. *The laws of physics must be of such a nature that they apply to systems of reference in any kind of motion.* Along this road we arrive at an extension of the postulate of relativity.

10

Einstein goes on with a further "thought experiment" (the Einstein elevator):

knowledge, there is a well-known physical fact which favours an extension of the theory of relativity. Let K be a Galilean system of reference, i.e. a system relatively to which (at least in the four-dimensional region under consideration) a mass, sufficiently distant from other masses, is moving with uniform motion in a straight line. Let K′ be a second system of reference which is moving relatively to K in *uniformly accelerated* translation. Then, relatively to K′, a mass sufficiently distant from other masses would have an accelerated motion such that its acceleration and direction of acceleration are independent of the material composition and physical state of the mass.

11



12

ADDITIONAL NOTES

Uniform downward acceleration, such as what you see in the hatted frame, is exactly what you expect, though, in a uniform gravitational field.

The answer is in the negative; for the above-mentioned relation of freely movable masses to K′ may be interpreted equally well in the following way. The system of reference K′ is unaccelerated, but the space-time territory in question is under the sway of a gravitational field, which generates the accelerated motion of the bodies relatively to K′.

13

This led Einstein to an important conclusion:

It will be seen from these reflexions that in pursuing the general theory of relativity we shall be led to a theory of gravitation, since we are able to "produce" a gravitational field merely by changing the system of co-ordinates. It will

Having argued that it would be more satisfactory that all reference frames be treated as equal, Einstein further argues that such a theory must incorporate gravitation.

14

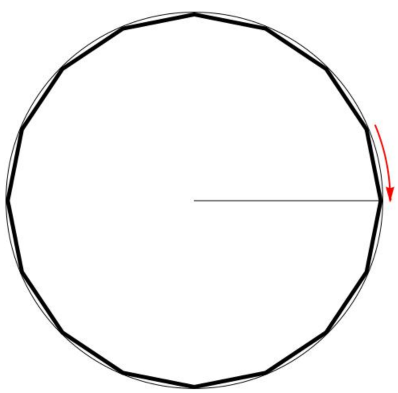
Einstein goes on to a second important argument, in order to make the case that a general theory of relativity would need to incorporate gravitation, *and non-Euclidean geometry.*

He imagines a system K and a system K' rotating with respect to it. He further imagines measuring the circumferences and diameters of circles in each system:

15

time be regarded as a circle in the X′, Y′ plane of K′. We suppose that the circumference and diameter of this circle have been measured with a unit measure infinitely small compared with the radius, and that we have the quotient of the two results. If this experiment were performed with a measuring-rod at rest relatively to the Galilean system K, the quotient would be π. With a measuring-rod at rest relatively to K′, the quotient would be greater than π. This is readily understood if we envisage the whole process of measuring from the "stationary" system K, and take into consideration that the measuring-rod applied to the periphery undergoes a Lorentzian contraction, while the one applied along the radius does not. Hence Euclidean geometry does not apply to K′. The notion of co-ordinates defined above, which pre-

16



Length contraction in direction of rotation; none in radial direction.

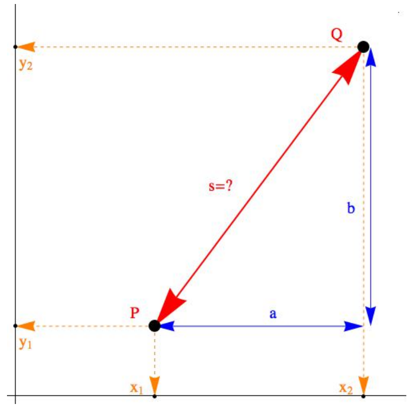
17

§4.2 Geometry and the Metric

(1) What's the Pythagorean theorem?

18

ADDITIONAL NOTES



19

(2) What's that in terms of the distance between the two points (x_1, y_1) and (x_2, y_2)?

20

We'll need the notation that "dx" means a (*very* small) difference in the variable x. The squared distance formula for flat space is then just the sum of squares of coordinate differences:

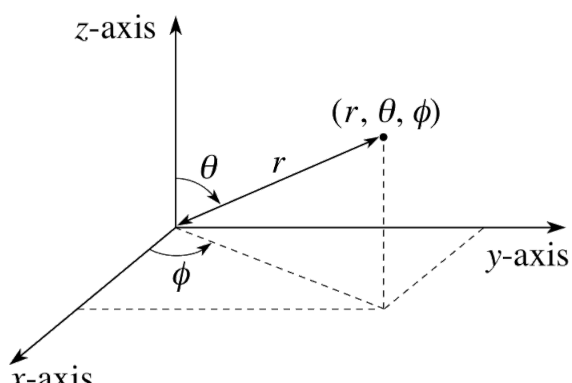
This is the basic distance formula of 2d Euclidean geometry.

21

Every geometry, Euclidean (flat) or Non- (curved), has a characteristic distance formula expressible via squares of coordinate differences.

If you're discussing the geometry of a sphere you use a distance formula that defines *that* geometry. The standard coordinates here are basically latitude (θ) and longitude (ϕ).

22



Spherical coordinates

23

The squared distance formula on a sphere is

$$ds^2 = r^2 d\theta^2 + r^2\, sin^2\, \theta d\phi^2$$

This ______ allows us to discuss the geometry of a sphere from properties of the sphere itself, not its embedding in a larger space.

Non-euclidean geometry is best studied through ideas of ______________________________.

24

ADDITIONAL NOTES

Reminder: The metric is a special matrix that allows you to find "distances." For example, distances on

$3d$ space in Cartesian coordinates:

$$ds^2 = dx^2 + dy^2 + dz^2$$

2d sphere of radius r:

$$ds^2 = r^2 d\theta^2 + r^2 \, sin^2 \, \theta d\phi^2$$

25

These formulas are examples of a general purpose formula for an n-dimensional distance

$$\begin{aligned} ds^2 &= \sum_{i,j}^{n} g_{ij} dx^i dx^j \\ &= g_{11} dx^1 dx^2 + g_{12} dx^1 dx^2 \ldots \end{aligned}$$

where dx^i is the differential (small difference) in the ith coordinate, *not a power*.

26

The expression $\sum_{i,j}^{n} g_{ij} dx^i dx^j$ can be calculated using matrix multiplication, *provided you write it out in a specific form*:

$$(dx^1, dx^2, ...) \begin{pmatrix} g_{11} & g_{12} & \cdots \\ g_{21} & g_{22} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} dx^1 \\ dx^2 \\ \vdots \end{pmatrix}$$

27

The expression $\sum_{i,j}^{n} g_{ij} dx^i dx^j$ occurs so often that the summation is often taken as implied, and it's written simply as

$$g_{ij} dx^i dx^j$$

(the "Einstein summation convention.")

28

3d Pythagorean metric

$$\begin{aligned} ds^2 &= (dx^1, dx^2, dx^3) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} dx^1 \\ dx^2 \\ dx^3 \end{pmatrix} \\ &= (dx^1)^2 + (dx^2)^2 + (dx^3)^2. \end{aligned}$$

29

Minkowski metric:

$$\begin{aligned} ds^2 &= (cdt, dx, dy, dz) \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} cdt \\ dx \\ dy \\ dz \end{pmatrix} \\ &= c^2 dt^2 - dx^2 - dy^2 - dz^2. \end{aligned}$$

The matrix for the Minkowski metric above will be denoted by η_{ij}.

30

ADDITIONAL NOTES

The Pythagorean and Minkowski metrics are constant. Metrics such as these are called _________, because the curvatures that can be computed from them are zero.

Metrics are coordinate-dependent. Switching from cartesian to polar coordinates in 2d will give metrics that look different. A metric that can be reduced to Minkowski form is called “Lorentzian.”

31

§4.3 Differential Geometry

(3) What's differential geometry?

(4) Why calculus in geometry?

32

Brief, sketchy history of geometry

- First(0ld), “Pythagorean theorem”:
  - ∼500–1800 BCE: Examples. Egypt/Mesopotamia, Plimpton Tablets.
  - ∼800 BCE: Statement. India, *Shubha Shastra*, Baudhayan.
  - ∼600 BCE: Proof. China, *Zhou Bi Suan Jing*.
  - ∼500 BCE: Statement. Pythagoras.
  - ∼300 BCE: Proof. Greece. *Elements*, Euclid.

33

- Next (Newish):
  - 1827: *General Investigation of Curved Surfaces*, Gauss. 2d surfaces in 3d space.
  - 1854: *On the Hypotheses which lie at the bases of Geometry*, Riemann.
  - 1887–1912: Ricci, Tensor Calculus.
  - 1913–1916: General Relativity

34

§4.4 General Relativity: A Summary

Theory developed between 1905 and 1916, primarily by Albert Einstein.

First version (1905), called Special Relativity. Einstein worked for a decade on extending it, till he succeeded in 1915 (published in 1916) with the General Theory. General Relativity has four main ingredients:

35

1. _______________
2. ______________________
3. ___________________________________
   ____
   ____
   ____
4. ___________________________________
   ____

36

ADDITIONAL NOTES

It's not all words:

Einstein's theory, expressed via equations,

$$G_{ab} \equiv R_{ab} - \frac{1}{2} g_{ab} R = \frac{8\pi G}{c^4} T_{ab}$$

$\downarrow$ $\downarrow$

Spacetime Geometry Matter

Ricci Curvature, R_{ab} Energy-Momentum

Curvature Scalar, R

37 Metric, g_{ab}

$$2R_{ab} = \partial_c\left[g^{cd}\left(\partial_a g_{bd} + \partial_b g_{ad} - \partial_d g_{ab}\right)\right] - \partial_a\left[g^{cd}\left(\partial_b g_{cd} + \partial_c g_{bd} - \partial_d g_{bc}\right)\right] - \left[g^{cd}\left(\partial_a g_{ed} + \partial_e g_{ad} - \partial_d g_{ae}\right)\right] \times \left[g^{ed}\left(\partial_c g_{bd} + \partial_b g_{cd} - \partial_d g_{cb}\right)\right] + \left[g^{cd}\left(\partial_a g_{bd} + \partial_b g_{ad} - \partial_d g_{ab}\right)\right] \times \left[g^{ed}\left(\partial_e g_{cd} + \partial_c g_{ed} - \partial_d g_{ec}\right)\right]$$

Ricci curvature:

$$R_{ij} = R^m_{imj}$$

Reimann-Christoffel curvature tensor:

$$R^m_{ikj} \equiv \frac{\partial \Gamma^m_{ij}}{\partial u^k} - \frac{\partial \Gamma^m_{ik}}{\partial u^j} + \Gamma^n_{ij}\Gamma^m_{nk} - \Gamma^n_{ik}\Gamma^m_{nj}$$

Christoffel symbols (second kind):

$$\Gamma^m_{ij} = \frac{1}{2} g^{mk}\left(\frac{\partial g_{ik}}{\partial u^j} + \frac{\partial g_{kj}}{\partial u^l} - \frac{\partial g_{ij}}{\partial u^k}\right)$$

38 where repeated indices are summed over.

§4.5 The Principle of Equivalence

The reduction of g_{ij}, in an appropriate basis, to η_{ij} fulfills an important physical requirement: Just as acceleration can mimic gravitation, an appropriate reference frame can "cancel gravity locally."

That either happens – "cancellation" or "creation" of gravity(like) effects – is because of the principle

39 of equivalence.

In Newtonian physics: forces cause accelerations.

How much acceleration?

Given by Newton's second law of motion, $F = ma$, where m represents the mass, a measure of the resistance to the force, the inertia. For the same force, the greater the mass, the smaller the acceleration.

40

Forces are given by "force laws."

Example 1: Electrostatic Forces:

Coulomb's Law

$$F_{\text{elec}} = k\frac{q_1 q_2}{d^2},$$

$$k \approx 9 \times 10^9 N \cdot m^2/C^2$$

Coulomb constant

(Charge measured in Coulombs.)

41

Coulomb's law tells you that the electrical force between two objects with charges q_1 and q_2 is attractive or repulsive and points in the direction of the straight line between them.

$$q_1(+)\bullet\underbrace{\rightarrow\text{attractive electrostatic force}\leftarrow}_{distance = d}\circ q_2(-)$$

$$q_1(+)\bullet\underbrace{\leftarrow\text{repulsive electrostatic force}\rightarrow}_{distance = d}\circ q_2(+)$$

42

ADDITIONAL NOTES

It says that the force on a charge q "due to" an "external charge" Q, $k\dfrac{qQ}{d^2}$, is proportional to q.

If q doubles, the force on it doubles.

We call the electrical influence of external charges, etc., the electric field.

43

Now, a Helium nucleus has twice the charge of a Hydrogen nucleus and roughly four times its mass.

(5) Facing the same electric field, which of the two will experience a greater electric force and by how much?
The Helium.
It will experience twice the force, because $F = qE$ and $q_{\text{He}} = 2q_{\text{H}}$.

44

(6) Which will accelerate more in this situation (and why)?
The Hydrogen nucleus.
The Helium nucleus feels twice the force, but has four times the inertia, and resists the force four times as strongly.

45

Example 2: Gravitational Forces:

Newton's Law of Universal Gravitation

$$F_{\text{grav}} = G\frac{m_1 m_2}{d^2},$$

$$G = 6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}$$

Gravitational constant

46

This law tells you that the gravitational force between two objects with masses m_1 and m_2 is always attractive and points in the direction of the straight line between them.

$$m_1 \underbrace{\bullet\!\longrightarrow \text{attractive gravitational force} \longleftarrow\!\circ}_{distance\,=\,d} m_2$$

47

(7) Subject to the same gravitational field which of the two nuclei, Hydrogen or Helium, will experience a greater gravitational force and by how much?
The Helium.
It will experience four times the force, because $m_{\text{He}} = 4m_{\text{H}}$.

48

ADDITIONAL NOTES

(8) Which will accelerate more?
They will accelerate the same, because the Helium nucleus, although it feels four times the force, also has four times the inertia.

49

Electrostatic and grav. forces are similar, but:
In electrostatics, charge creates force, and mass (inertia) resists it.
In gravitation, mass creates force, and mass (inertia) resists it.

The principle of equivalence says that the mass that creates gravitational forces (gravitational mass) equals the mass that resists forces (inertial mass).

50

§4.6 Motion

Postulate: In the absence of other forces, small and "unmassive" enough particles ("test particles") follow "geodesic paths" of g_{ij}.

A geodesic is the straightest possible line on a curved background (like a great circle on a sphere).

51

A curve $\{u^i(t)\}$ is a geodesic with parameter t if its tangent $V^i(t) = du^i/dt$ obeys

$$\frac{dV^i}{dt} + \Gamma^i_{kj} V^k V^j = 0.$$

The equation will preserve this form under linear reparametrizations, but not nonlinear ones. The class of parametrizations under which the geodesic equation has this form is called "affine."

52

If we knew the metric, we could calculate geodesics and thus predict the paths that test bodies follow – rays of light, planets, etc.

Einstein's equation is a differential equation for obtaining g_{ij} by linking curvature (which would represent gravity) with matter ("mass"), as an extension of Newtonian theory.

53 But what is "matter"?

§4.7 Matter

Special relativity, and mass-energy equivalence, necessitate extending our notions of mass and energy into a unified "energy-momentum tensor."

Example (4d): an idealized fluid characterized by an energy-density ρ, a pressure P, and a "flow" vector field V^i. In the rest frame of the fluid,

54

ADDITIONAL NOTES

$$V^i = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

and

$$T_{ij} = (\rho + P) \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + P\eta_{ij}$$

55

The rule of thumb that relativists follow, following Einstein, is to take special-relativistic expressions that have η_{ij} in them and replace η_{ij} with g_{ij}:

$$T_{ij} = (\rho + P) \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + Pg_{ij}$$

56

Then, if in Newtonian theory,

Gravity $\sim$ Mass,

a new, extended, geometrical theory should have

Spacetime curvature $\sim$ E-m tensor.

A more specific suggestion came from Poisson's equation for the gravitational field of a continuous mass distribution of mass-density ρ:

$$\nabla^2 \phi = 4\pi\rho.$$

57

The quantity ϕ here is the gravitational potential, the gravitational potential energy per unit mass. It was known that the first derivative of ϕ gave the gravitational force, and the second derivative gave the differential, or "tidal" force.

A study of geodesics in curved geometry suggested that it was the curvature that represented "tidal" effects in geodesic behavior.

58

By the time Einstein was working through these ideas, it was well-established that the Reimann-Christoffel tensor represented curvature.

But it couldn't simply be equated to the energy-momentum tensor.

59

A tensor introduced by Ricci, the Ricci curvature,

$$R_{ij} \equiv R^k ikj \equiv \sum_k R^k ikj$$

came to the rescue. Einstein initially tried to set the Ricci curvature proportional to T_{ij}, but it turned out to violate mass-energy/momentum conservation.

60

ADDITIONAL NOTES

He realized in 1915 that the following had, at least, no obvious problems:

$$G_{ij} = R_{ij} - \frac{1}{2} R g_{ij} = \frac{8\pi G}{c^4} T_{ij}$$

We call this Einstein's equation, and G_{ij} the Einstein tensor.

61

This looks a bit like Poisson's equation:

$$\nabla^2 \phi = 4\pi\rho.$$

But there's a significant difference, and not just in complexity.

In Poisson's equation, the r.h.s. is assumed known, and we have to find ϕ.

62

In Einstein's equation, g_{ij} is the unknown, but it appears *on both the left- and right-hand sides*, as (for example) in the case of T_{ij} for a perfect fluid.

Conceptually a matter "distribution" is meaningless until we know what "where" is and "when" is, and we only know that *after* we know g_{ij}.

63

Summary

g_{ij}	$\sim$	gravitational potential
Γ^k_{ij}	$\sim$	gravitational force
R^m_{ijk}	$\sim$	tidal gravitational force

Einstein's Equation (with $G = 1$ and $c = 1$)

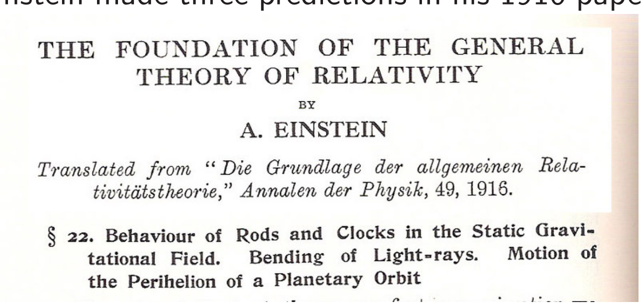
$$G_{ij} = 8\pi T_{ij}$$

determines the metric. What could be simpler?

64

§4.8 Tests of GR

Einstein made three predictions in his 1916 paper:


THE FOUNDATION OF THE GENERAL THEORY OF RELATIVITY

BY

A. EINSTEIN

Translated from "Die Grundlage der allgemeinen Relativitätstheorie," Annalen der Physik, 49, 1916.

§ 22. Behaviour of Rods and Clocks in the Static Gravitational Field. Bending of Light-rays. Motion of the Perihelion of a Planetary Orbit

We'll discuss them in reverse order.

65

Einstein Test 3

Motion of the perihelion of a planet

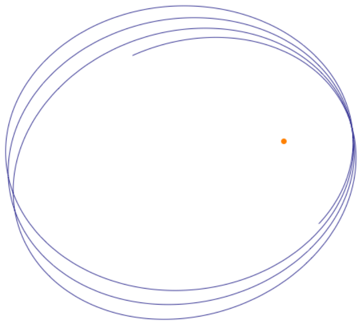
The ________________ of a planetary orbit is ____

A planet (Mercury, e.g.) goes around the sun on an elliptical path. But, the path does not close: the perihelion is not at the same point every year.

This is called ____________________

66

ADDITIONAL NOTES



A precessing ellipse

67

Till Einstein, we could explain most of the precession, except for a small amount:

$0.012°$ – every hundred years!

Einstein's proposal was that the matter of the sun warps surrounding spacetime geometry. Mercury moves on a straight line on this curved background.

Sounds weird, but you get exactly the extra $0.012°$ that you need.

68

Einstein Test 2

The bending of light

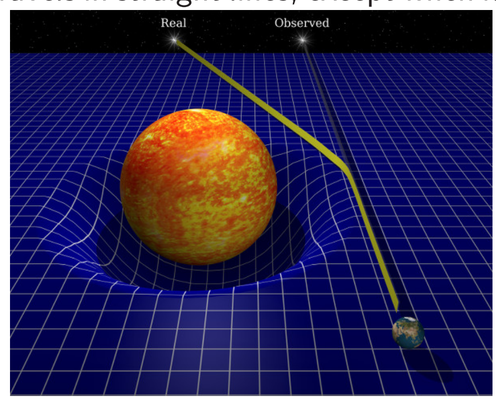
Light bends around objects like the sun.

Really? Does Gravity affect light?

Really (even though in Newtonian gravity, it's just mass that's involved).

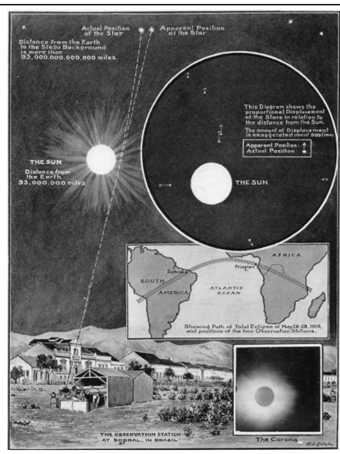
69

Light travels in straight lines, except when it bends:



70

The English astronomer Arthur Stanley Eddington, and others, proposed a test of Einstein's prediction of the bending of light, to be done during a solar eclipse in Brazil on May 29, 1919.



71

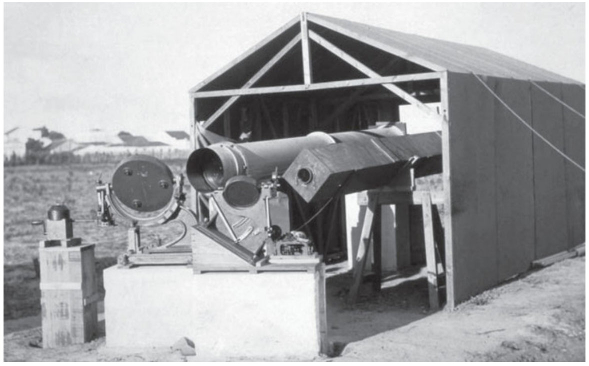
(9) Why solar eclipse? ______________________

____

72

ADDITIONAL NOTES

An expedition was organized:



73

They took photos with two telescopes in cloudy conditions of about a dozen stars near the sun during the eclipse, then of the same stars at night two months later.

(10) Why did they return two months later?

74

The team returned to England to compare and analyze the photographic plates.

They asked for a special joint meeting of the Royal Astronomical Society and the Royal Society of London for November 6, 1919, to make an announcement.

75

The prediction from Einstein's theory was that the angular positions of stars near the sun would shift by $1.75''$. The Eddington expedition results were

Telescope 1: $(1.98 \pm 0.12)''$

Telescope 2: $(1.61 \pm 0.30)''$

Given the small number of stars looked at, these are not completely convincing results.

76

Reaction at the meeting was mixed.

One person present called it

"the most important result obtained in connection with the theory of gravitation since Newton's day."

But another pointed to a portrait of Newton hanging in the room and urged caution:

"We owe it to that great man to proceed very carefully in modifying or retouching his Law of Gravitation."

77

Einstein had been kept informed as the data was analyzed. He had always been confident.

On September 27, nearly 6 weeks before the official announcement, he wrote to his mom:

"...joyous news today. ...the English expeditions have actually measured the deflection of starlight from the Sun."

78

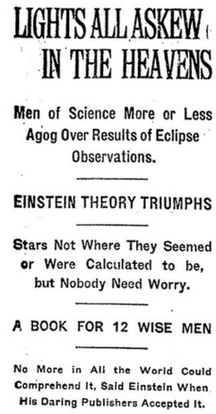
ADDITIONAL NOTES

The Press shared Einstein's enthusiasm. The London Times of November 7, 1919, one day later, carried a long article about the Royal Society meeting, headlined

REVOLUTION IN SCIENCE
NEW THEORY OF THE UNIVERSE

Three days later The New York Times got into it. . .

79


LIGHTS ALL ASKEW IN THE HEAVENS

Men of Science More or Less Agog Over Results of Eclipse Observations.

EINSTEIN THEORY TRIUMPHS

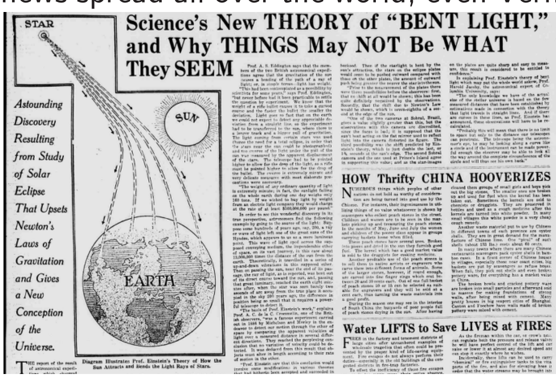
Stars Not Where They Seemed or Were Calculated to be, but Nobody Need Worry.

A BOOK FOR 12 WISE MEN

No More in All the World Could Comprehend It, Said Einstein When His Daring Publishers Accepted It.

80

The news spread all over the world, even Vermont:


Science's New THEORY of "BENT LIGHT," and Why THINGS May NOT Be WHAT They SEEM

Astounding Discovery Resulting from Study of Solar Eclipse That Upsets Newton's Laws of Gravitation and Gives a New Conception of the Universe.

HOW Thrifty CHINA HOOVERIZES

Water LIFTS to Save LIVES at FIRES

81

In 1921 Einstein visited New York:

82

Einstein Test 1

The behavior of rods and clocks

Space *and* time are warped.

The behavior of time is particularly interesting.

Clocks tick slightly slower on the surface of the earth than on the top of tall buildings or in planes.

Motion also affects the "flow" of time.

83

Tests of Time Alterations

a) In 1971 Keating and Hafele flew four caesium atomic clocks around the world on commercial aircraft, first traveling from east to west, then from west to east. The results of the experiment confirmed the relativistic predictions within 10%. The experiment was repeated in 1996 on a trip from London to Washington and back, a 14 hour journey. The result was within 2 ns of the prediction.

84

ADDITIONAL NOTES

b) Muon lifetime [Bailey,J. et al. Nature **268**, 301 (1977)]: Muons with "rest lifetime" of $2.198\,\mu$s were sped to high speed $(.999c)$. The measured lifetimes at those speeds were found to be $64.368\,\mu$s, consistent with relativity.

85

c) paper in *Science*, 2010:

Optical Clocks and Relativity

C.W.Chou, D.B.Hume, T.Rosenband, D.J.Wineland

Science, 24 Sep 2010, Vol.329, Issue 5999, pp.1630-1633

"Observers in relative motion or at different gravitational potentials measure disparate clock rates. . . . We observed time dilation from relative speeds of less than 10 meters per second by comparing two optical atomic clocks connected by a 75-meter length of optical fiber. We can now also detect time dilation due to a change in height near Earth's surface of less than 1 meter."

86

d) GPS


Los Angeles Air Force Base

HOME NEWS PHOTOS ART LIBRARY UNITS QUESTIONS JOIN THE AIR FORCE

Library > Fact Sheets > Global Positioning Systems Directorate

GLOBAL POSITIONING SYSTEMS DIRECTORATE

Posted 4/8/2015 Printable Fact Sheet

The Global Positioning Systems Directorate is a joint service effort directed by the United States Air Force and managed at the Space and Missile Systems Center, Air Force Space Command, Los Angeles Air Force Base, Calif. The directorate is the Department of Defense acquisition office for developing and producing Global Positioning System (GPS) satellites, ground systems and military user equipment.

GPS is a space-based dual use radio navigation system nominally consisting of a minimum of 24-satellite constellation that provides positioning, velocity and timing to military and civilian users worldwide. GPS satellites, in one of six medium earth orbits, circle the earth every 12 hours transmitting continuous ranging signals. In addition to the satellites, the system consists of a worldwide satellite control network and GPS receiver units that acquire the satellite's signals and compute navigation solutions to provide positioning, velocity and timing to the user.

GPS provides the following:

24-hour, worldwide service
Highly accurate, three-dimensional location information
Precision velocity and timing services
Accessibility to an unlimited number of global military, civilian, and commercial users

Photos

GLOBAL POSITIONING SYSTEMS DIRECTORATE NAVSTAR Any Time, Any Place Right Time, Right Place

Global Positioning Systems Directorate

Download HiRes

87

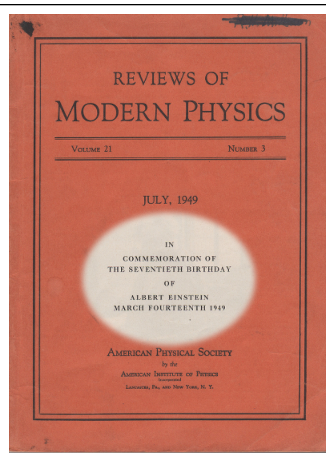
From: www.losangeles.af.mil/shared/media/document/AFD-100302-043.doc

3.3.1.1 Frequency Plan. . . . The SV carrier frequency and clock rates – as they would appear to an observer located in the SV – are offset to compensate for relativistic effects. The clock rates are offset by $\Delta f/f = -4.4647 \times 10^{-10}$, equivalent to a change in the I5 and Q5-code chipping rate of 10.23 MHz offset by a $f = 4.5674 \times 10^{-3}$Hz.

88

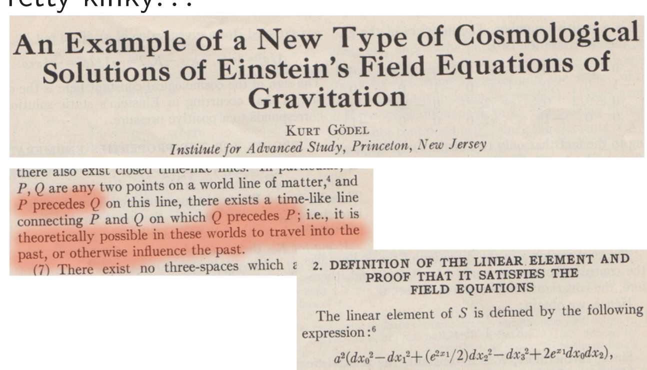
So time can be twisted in relativity.

How kinky can time get?



89

Pretty kinky. . .


An Example of a New Type of Cosmological Solutions of Einstein's Field Equations of Gravitation

Kurt Gödel

Institute for Advanced Study, Princeton, New Jersey

there also exist closed time-like lines. In particular, if P, Q are any two points on a world line of matter,[4] and P precedes Q on this line, there exists a time-like line connecting P and Q on which Q precedes P; i.e., it is theoretically possible in these worlds to travel into the past, or otherwise influence the past.

(7) There exist no three-spaces which

2. DEFINITION OF THE LINEAR ELEMENT AND PROOF THAT IT SATISFIES THE FIELD EQUATIONS

The linear element of S is defined by the following expression:[6]

$$a^2(dx_0^2 - dx_1^2 + (e^{2x_1}/2)dx_2^2 - dx_3^2 + 2e^{x_1}dx_0dx_2),$$

90

ADDITIONAL NOTES

ADDITIONAL NOTES