Arvind Borde / PHY 19, Week 3: Momentum and Energy

§3.1 Inertial Frames

Class of frames which

a) Move uniformly wrt each other, and in which

b) Newton's First Law holds.

("No force, no acceleration.")

We assume our frames are inertial, but we'll see there are problems with the concept.

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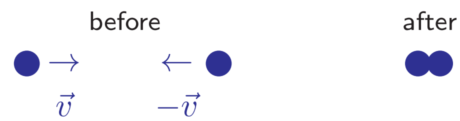
§3.2 Momentum

We need to redefine momentum as $\vec{p} = \gamma(v)m\vec{v}$ because otherwise ______________.

Example: Two balls of equal mass that collide and stick (inelastic collision) in a frame in which they have equal and opposite initial velocities:



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In this frame, momentum is conserved:
Initial: $\vec{p} = m\vec{v} + m(-\vec{v}) = 0$.
Final: $\vec{p} = 0$.

Now, consider a frame moving to the right with velocity $\vec{v}$.
(1) What is the initial momentum of the left ball in this frame? ______________

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With respect to the new frame, the old frame is moving in the negative x direction with speed v.
(2) Using relativistic addition of velocities, what is the initial velocity of the right ball in this frame?

(3) What is the total initial momentum?

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(4) What is the final velocity of the stuck-ball pair in the new frame? ___

(5) What's the final momentum here? ______

(6) Is momentum conserved in this frame? ___

In order to preserve the conservation of momentum, we need a new definition of momentum.

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§3.3 Newton's Second law

This law is expressed in introductory physics as ______________.

If m is assumed constant, and $\vec{p} = m\vec{v}$, we can re-express this as

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ADDITIONAL NOTES

In relativity, although these *steps* are invalid, the final result still holds: $\vec{F} = d\vec{p}/dt$.

One reason that this is the "correct" law is that it enforces the conservation of momentum.

If the total external force on a system is zero, then

$$\frac{d\vec{p}}{dt} = \frac{d(\gamma(v)m\vec{v})}{dt} = 0$$

in other words, $\vec{p}$ is constant (aka "conserved").

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(7) Calculate $d(\gamma(v))/dt$ in terms of dv/dt.

$$\frac{d(\gamma(v))}{dt} =$$

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(8) Confining ourselves to magnitudes $p = \gamma m v$. Calculate dp/dt.

$$\frac{dp}{dt} = \frac{d(\gamma m v)}{dt} =$$

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Therefore,

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§3.4 Energy

If a force moves an object from x_1 to x_2 on the x axis, it does work given by

$$W = \int_{x_1}^{x_2} F dx = \int_{x_1}^{x_2} \frac{dp}{dt} dx$$
$$= \int_{x_1}^{x_2} \gamma^3 m \frac{dv}{dt} dx$$

from the previous formula for dp/dt.

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But,

$$\frac{dv}{dt} dx = dv \frac{dx}{dt} = v dv$$

Assuming that the particle being moved by the force is at rest at x_1 and has velocity v at x_2,

$$W = \int_{x_1}^{x_2} \gamma^3 m \frac{dv}{dt} dx = m \int_0^v \gamma^3 v dv$$
$$= m \int_0^v \frac{v dv}{(1 - v^2/c^2)^{(3/2)}}$$

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ADDITIONAL NOTES

Using the substitution $w =$ ________________, we have $dw =$ ________________. So, we get

$$W =$$

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Since the work done by a force in setting a particle of rest mass m_0 in motion (from zero initial velocity) equals the KE of the particle, we get

$$E_{\text{kin}} = m_0 c^2(\gamma - 1)$$

which may also be rewritten as

$$m_0 c^2 \gamma = E_{\text{kin}} + m_0 c^2$$

where we think of $\gamma m_0 c^2$ as the total energy, and $m_0 c^2$ as the "mass-energy."

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(9) In units where $c = 1$ write the speed v as a function of E_{kin}/m in both the non-relativistic ($E_{\text{kin}} = \frac{1}{2}mv^2$) and relativistic cases.

Non-rel:

Rel:

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In both cases (Relativistic and Non-)

(10) what is v at $E_{\text{kin}} = 0$?

____

(11) how does v behave as $E_{\text{kin}} \to \infty$?

Non-rel:

Rel:

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(12) An electron has a (rest) mass of 9.11×10^{-31} kg. what is its "rest energy" (mc^2)?

Now, an electron Volt (eV) is the energy obtained by an electron when it is accelerated by a potential difference of 1 V.

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$$1\text{eV} = 1.60 \times 10^{-19}\,\text{J}$$

(13) What is the rest energy of an electron in eV?

This is a standard way now to express the masses of elementary particles, as eV, MeV, GeV, etc.

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ADDITIONAL NOTES

§3.5 Momentum and Energy

We can rewrite the expression for energy using momentum rather than velocity, using $E_{\text{tot}} = \gamma mc^2$ and $p = \gamma mv$.

(14) What is $E_{\text{tot}}^2 - p^2c^2$?

$$=$$

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So we have

$$E_{\text{tot}}^2 = p^2c^2 + (mc^2)^2$$

where m is the "rest mass."

This expression applies even to particles *with zero rest mass*, such as photons. For these we get

$$E = pc.$$

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Also, since the rest mass, m, and the speed of light, c, are the same in all reference frames,

$$E^2 - p^2c^2$$

is an invariant under Lorentz transformations.

Remember the invariance of the ____________.

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§3.6 Conservation of Mass-Energy

The separate conservation laws of mass and energy in pre-relativistic physics are combined onto a single, unified law of conservation of mass-energy (i.e., $E_{\text{tot}} = \gamma mc^2$) in special relativity:

"The sum of the mass-energy of a system of particles before interaction must equal the sum of the mass-energy of the system after interaction."

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Consider the sticky collision discussed earlier. Conservation of mass energy, in the first frame we looked at, says

$$\gamma(v)mc^2 + \gamma(-v)mc^2 = \gamma(0)Mc^2$$

where we are *not* assuming that $M = m + m$.

Since $\gamma(v) = \gamma(-v)$ and $\gamma(0) = 1$,

$$M = 2m\gamma(v).$$

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(15) Is $M = 2m$ or what (and why)?

____

(16) What is $(M - 2m)c^2$?

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ADDITIONAL NOTES

§3.7 Conservation of Relativistic Momentum

Consider the inelastic collision we looked at earlier.

In the first frame

Initial: $p = \gamma m v + \gamma m(-v) = 0$.

Final: $p = \gamma M \times 0 = 0$.

Relativistic momentum is conserved.

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In the second frame:

Initial momentum of left ball: $\gamma m \times 0 = 0$.

Initial momentum of right ball: $\gamma(v_{\text{R}}) m v_{\text{R}}$

Total initial momentum: $\gamma(v_{\text{R}}) m v_{\text{R}}$, where

$$v_{\text{R}} = \frac{-2v}{1 + v^2/c^2}$$

Final momentum: $\gamma(-v) M \times (-v) = -\gamma(v) M v$.

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(17) What is $1 - v_{\text{R}}^2/c^2$?

(18) What is $\gamma(v_{\text{R}})$?

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(19) What is the total initial momentum?

(20) What is the total final momentum?

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$\gamma(v)$ vs $\gamma(2v)$: high speeds

v	$0.49000c$	$0.49900c$	$0.49990c$	$0.49999c$
$\gamma(v)$	1.147	1.154	1.155	1.155
$\gamma(2v)$	5.025	15.819	50.002	158.115

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ADDITIONAL NOTES

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