§14.4 Math v Physics

§14.4.1 Classical

Till the early 1900s, all mathematical descriptions of physical reality were approximate.

To describe the motion of the earth around the sun, for example, we approximate both as perfectly spherical, and (*apparently* even more outrageously) as "point particles."

(7) What's a point particle?

____

____

Nothing like this physically existed in our experience prior to ∼1900.

This is a mathematical abstraction that approximates, but doesn't exactly match, physical reality.

The dichotomy arises because we feel we've independent understandings of physics and math.

We feel we independently know

A) What a physical "object" is – the earth, a billiard ball, whatever, and

B) What a mathematical "object" is – a real number, a differential equation, whatever.

§14.4.2 Quantum

The situation here is radically different.

Nobody has a physical sense of what, say, an electron is because nobody has seen one, touched one, smelled one.

An electron is what the mathematics of quantum theory says it is.

(8) What does the math. of quantum theory say an electron is completely described by?

____

(9) What are its *intrinsic* attributes?

____

(10) What are its attributes in the outside world?

____

Why do position and momentum suffice?
Classically, because $F = ma$.

What does that have to do anything?
Because it's a second order DE.

Knowing the position and velocity (or momentum) of a particle at any instant will allow you, in principle, to know what a particle subsequently does in response to any force.

ADDITIONAL NOTES

Looking only at intrinsic attributes, there's a difference between the three that were listed above.

Mass and charge are fixed, no matter your pov.

Spin can be measured as $+1/2$ or $-1/2$ depending on your pov.

§14.5 Spin States

In the absence of forces, we can ignore position and momentum.

Mass and charge are fixed.

So, the quantum state of such an electron is determined by its spin: $\pm\frac{1}{2}$. We denote the two states (assumed normalized) as $|\uparrow\rangle$ and $|\downarrow\rangle$.

§14.5.1 Two single (one-electron) states

$|\uparrow\rangle$ and $|\downarrow\rangle$ form an orthonormal basis for the possible states of an arbitrary electron.

A pair of such states, $|A\rangle$ and $|B\rangle$ may be written as

$$|A\rangle =$$

$$|B\rangle =$$

From slides 18 and 19, we can get conditions on the coefficients. for $|A\rangle$ to be normalized (for example):

$$A^*_\uparrow A_\uparrow + A^*_\downarrow A_\downarrow = 1$$

One can form a (tensor) product of these to give two-electron states.

§14.5.2 Tensor product

Given two vector spaces the tensor product is a new vector space spanned by a combination of basis vectors of the original spaces.

It is denoted by $\otimes$.

§14.5.3 One double (two-electron) state

For two electrons we have ___ possible spin states. (**11**) List them.

(**12**) What's the most general linear combination of these states?

ADDITIONAL NOTES

Some two electron states can be constructed as a tensor product of single electron states. Given

$$|A\rangle = A_\uparrow|\uparrow\rangle + A_\downarrow|\downarrow\rangle$$
$$|B\rangle = B_\uparrow|\uparrow\rangle + B_\downarrow|\downarrow\rangle$$

we can construct a state denoted $|A\rangle \otimes |B\rangle$:

37

Not all two-electron states are expressible as a product of two one-electron states.

Such states are called *entangled*. An example is

$$\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

This cannot be written as a product state.

38

§14.6 The EPR problem

Einstein bears responsibility for the photon (the quantum of light), and other early developments of quantum theory.

But once Born put forward the probabilistic interpretation of QM, his advocacy was replaced by opposition.

39

He and Niels Bohr engaged in (what's easily) the greatest debate in the history of human thought.

As a master of thought experiments, Einstein raised objection after principled objection to QM.

Bohr answered all, except for one.

In 1935, Einstein, in collaboration with B. Podolsky and N. Rosen produced the EPR problem.

40

Suppose that a two-spin system with zero net spin is split into components that are then widely separated.

The spin on one component, say the left one (L), is measure to be ½.

We'll immediately know that the spin of the other component is __________.

41

What's the big deal?

For a classical system, we can know that one quantity is something, and thereby force some other quantity to be some other (or the same) value.

In a probabilistic theory such as QM, determining a variable at one place, seems to force a variable elsewhere to take on some value.

42

ADDITIONAL NOTES

ADDITIONAL NOTES