§13.1 Charge and Magnetism

Maxwell's differential equation

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

may be written in integral form as

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

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where

- $\vec{J}$ is the current density,
- I is the current,
- Φ_E is the electric potential,
- $\vec{B}$ is the magnetic field,
- $\vec{E}$ is the electric field,
- $\epsilon_0 = (4\pi k)^{-1} = \left(4\pi(9 \times 10^9)\right)^{-1}$ C^2/N·m^2,
- 2 $\mu_0 = 4\pi \times 10^{-7}$ T·m/A.

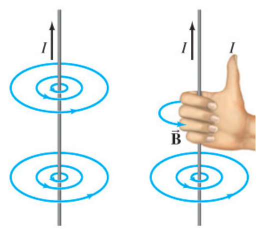
In situations where an electric current is the only source of a magnetic field, the strength of the current will determine the strength of the magnetic field it produces.

Current is the rate of change of charge: $I = \frac{dQ}{dt}$.

The faster charges move, the larger the current, and, hence, the larger the strength of the resulting

3 magnetic field.

Example: Magnetic Field of a Line Current



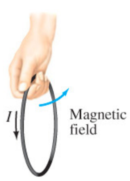
The magnitude of the field is

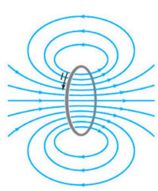
$$B = \frac{\mu_0}{2\pi} \frac{I}{r}$$

where r is the radial distance from the wire, and you use the ________________ to figure out

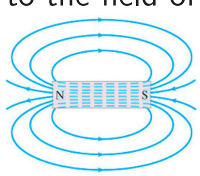
4 the direction of $\vec{B}$.

Example: Magnetic Field of a Loop Current





This is very close to the field of a bar magnet:



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Charges going round in circles mimic bar magnets.

(1) Where might we find such charges?

____

Like energy, the orbital angular momentum of electrons is quantized (i.e., can only take on specific values determined by quantum numbers).

The idea goes back to the Bohr atom, and is pre-

6 served in full-fledged quantum mechanics.

ADDITIONAL NOTES

Each orbital electron will mimic a magnet.

Angular momentum quantization will restrict the associated magnetic field strengths ("magnetic moments") to specific values.

7

§13.2 The Stern-Gerlach Experiment

In 1921, Otto Stern and Walter Gerlach set off to check angular momentum quantization, by looking at magnetic effects associated with the orbital electrons.

Discrete magnetic effects (opposed to continuous) would indicate underlying angular momentum quantization.

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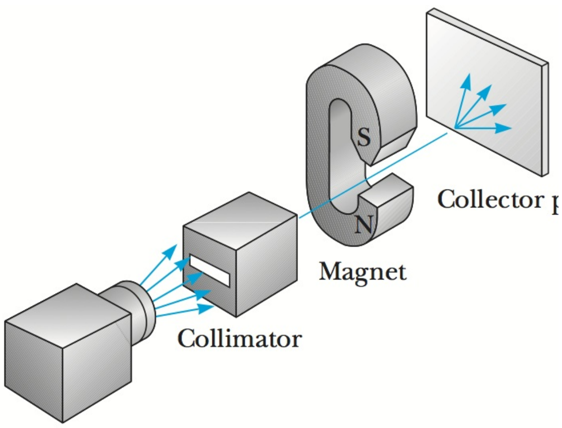

Setup: Silver atoms beamed through an inhomogenous magnetic field.

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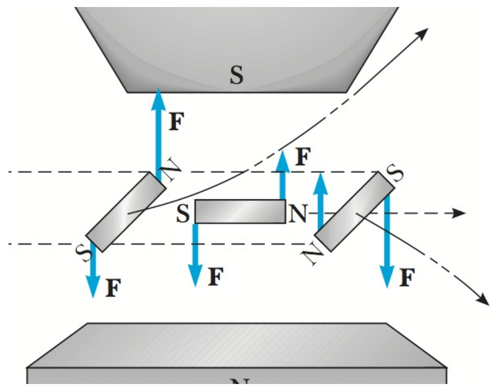

How magnets would behave under the circumstances.

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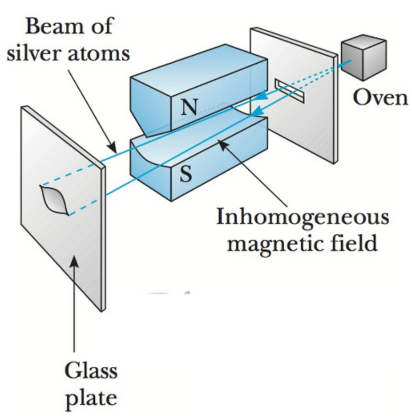

The deflection pattern on the screen.

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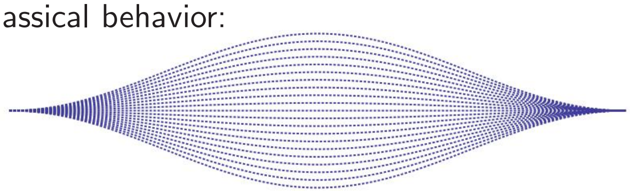
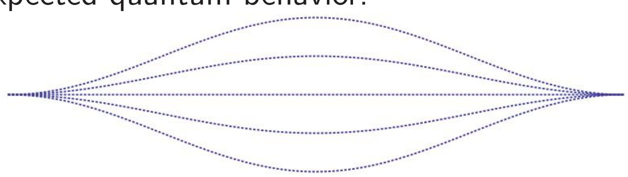
Classical behavior:



Expected quantum behavior:



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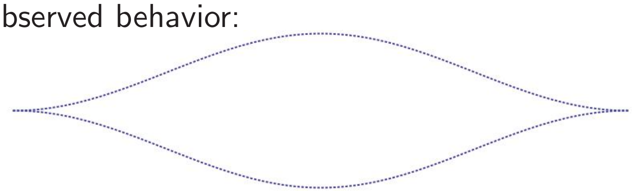
ADDITIONAL NOTES

Observed behavior:



Finding just a pair of deflections was a shock. The experiment was repeated in 1927 by T.E. Phipps and J.B. Taylor with hydrogen atoms. They got
13 identical results: just two deflections.

This was explained in 1925 by Samuel Goudsmit and George Uhlenbeck (then graduate students) as evidence for an intrinsic angular momentum associated with particles.

They proposed the existence of a new quantum number, s, called the spin, with the number of components in the S-G experiment given by $2s+1$.

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We now understand the S-G experiment as a result of the behavior of the outermost electron.

Since we associate the number of components in the experiment with $2s+1$ and the S-G experiment showed 2 components, it follows that the electron has a spin quantum number of ________.

The angular momentum associated with spin has
15 magnitude $|\vec{S}| = \sqrt{s(s+1)}\hbar \; (= (\sqrt{3}/2)\hbar$ for e).

§13.3 Abstract(ish) QM

The states of a physical system (e.g., an atom) are represented by elements of a vector space.

The wavefunction is an example of a state vector.

The state vector tells us everything that's predictable about a system.

A state is conventionally denoted by $|\psi\rangle$.

16

The vector space (H) of quantum states has an inner product ($H \times H \to R$), denoted by

$$\langle \phi \mid \psi \rangle$$

The vectors ϕ, ψ are called orthogonal ($\phi \perp \psi$) if

$$\langle \phi \mid \psi \rangle = \_\_$$

In wave mechanics, the inner product is defined by

$$\langle \phi \mid \psi \rangle = \int \phi^* \psi \, d^3x.$$

17

($\mathbf{2}$) What are these in wave mechanics?

$\langle \phi \mid \psi \rangle^* =$

$\langle \phi \mid a_1\psi_1 + a_2\psi_2 \rangle =$

18 These are true in general.

ADDITIONAL NOTES

(3) What are these for wavefunctions?

$$\langle \psi \mid \psi \rangle =$$

$$\langle \vec{x} \rangle =$$

In the same way,

$$\langle \vec{p} \rangle =$$

19

A linear operator, Q, in a vector space, H, is a mapping $H \to H$: $Q\psi = \phi$.

Linearity requires that

$$Q(a_1\psi_1 + a_2\psi_2) =$$

The sum and product of operators, P and Q, are

$$(P+Q)\psi =$$

$$(PQ)\psi =$$

20

Suppose that P and Q are linear operators.

(4) Is $P + Q$ linear? __________

$$(P+Q)(a_1\psi_1 + a_2\psi_2)$$

$$=$$

(5) Is PQ linear? __________

21

If Q is a linear op., its adjoint, $Q^\dagger$, is defined by

$$\langle Q^\dagger \phi \mid \psi \rangle = \langle \phi \mid Q\psi \rangle, \quad \forall\, \psi,\, \phi.$$

A self-adjoint operator is one that obeys

$$Q^\dagger = Q.$$

An operator is self-adjoint iff $\langle \psi \mid Q\psi \rangle$ is real, $\forall\, \psi$.

Observables are represented by self-adjoint ops.

22

An eigenfunction of a linear operator, Q, is a vector, ψ, that obeys $Q\psi = q\psi$, where $\langle \psi \mid \psi \rangle \neq 0$, and q is a complex number.

(6) Show that q is real, if Q is self-adjoint.

23

If ψ_1, ψ_2 are eigenfunctions of a self adjoint Q with eigenvalues $q_1 \neq q_2$, then $\psi_1 \perp \psi_2$.

If $\langle \psi_1 \mid \psi_2 \rangle \neq 0$ we can cancel it to give ______ which is ____________. So __________.

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ADDITIONAL NOTES