§14.1 Charge and Magnetism

Maxwell's differential equation

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

may be written in integral form as

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

where

- $\vec{J}$ is the current density,
- I is the current,
- Φ_E is the electric potential,
- $\vec{B}$ is the magnetic field,
- $\vec{E}$ is the electric field,
- $\epsilon_0 = (4\pi k)^{-1} = \left(4\pi(9 \times 10^9)\right)^{-1}$ C^2/N·m^2,

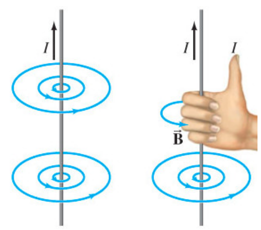
2 - $\mu_0 = 4\pi \times 10^{-7}$ T·m/A.

In situations where an electric current is the only
source of a magnetic field, the strength of the cur-
1 rent will determine the strength of the magnetic
field it produces.

Current is the rate of change of charge: $I = \frac{dQ}{dt}$.

The faster charges move, the larger the current,
and, hence, the larger the strength of the resulting
3 magnetic field.

Example: Magnetic Field of a Line Current

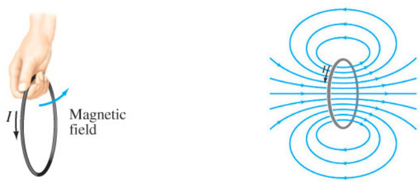


The magnitude of the field is

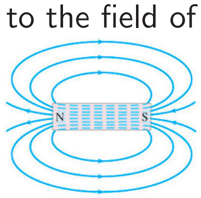
$$B = \frac{\mu_0}{2\pi} \frac{I}{r}$$

where r is the radial distance from the wire, and
you use the ________-________________ to figure out
4 the direction of $\vec{B}$.

Example: Magnetic Field of a Loop Current



This is very close to the field of a bar magnet:



5

Charges going round in circles mimic bar magnets.

(1) Where might we find such charges?

____

Like energy, the orbital angular momentum of electrons is quantized (i.e., can only take on specific values determined by quantum numbers).

The idea goes back to the Bohr atom, and is pre-
6 served in full-fledged quantum mechanics.

ADDITIONAL NOTES

Each orbital electron will mimic a magnet.

Angular momentum quantization will restrict the associated magnetic field strengths ("magnetic moments") to specific values.

7

§14.2 The Stern-Gerlach Experiment

In 1921, Otto Stern and Walter Gerlach set off to check angular momentum quantization, by looking at magnetic effects associated with the orbital electrons.

Discrete magnetic effects (opposed to continuous) would indicate underlying angular momentum quantization.

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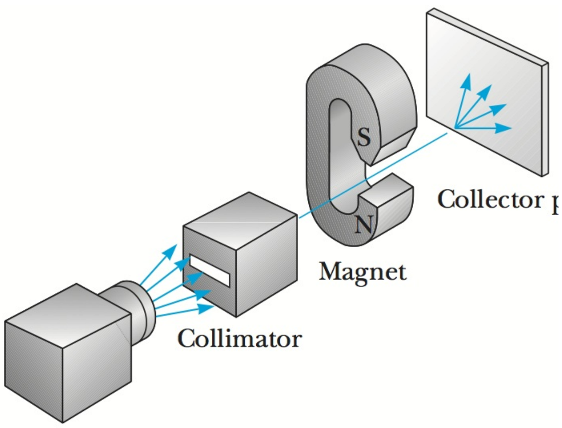
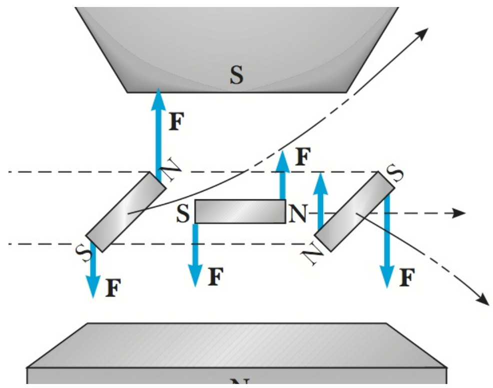


Setup: Silver atoms beamed through an inhomogenous magnetic field.

9



How magnets would behave under the circumstances.

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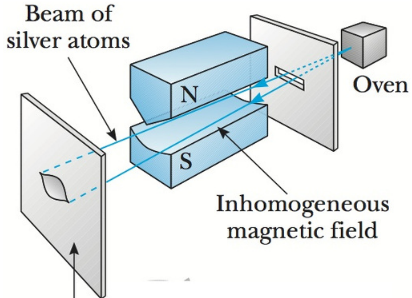
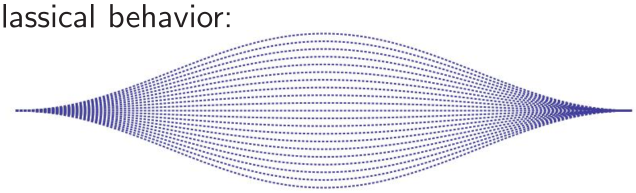


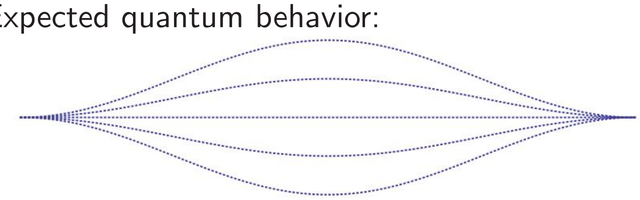
The deflection pattern on the screen.

11

Classical behavior:



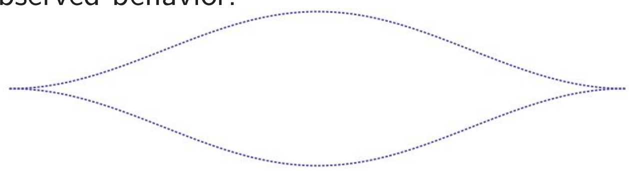
Expected quantum behavior:



12

ADDITIONAL NOTES

Observed behavior:



Finding just a pair of deflections was a shock. The experiment was repeated in 1927 by T.E. Phipps and J.B. Taylor with hydrogen atoms. They got
13 identical results: just two deflections.

This was explained in 1925 by Samuel Goudsmit and George Uhlenbeck (then graduate students) as evidence for an intrinsic angular momentum associated with particles.

They proposed the existence of a new quantum number, s, called the spin, with the number of components in the S-G experiment given by $2s+1$.

14

We now understand the S-G experiment as a result of the behavior of the outermost electron.

Since we associate the number of components in the experiment with $2s+1$ and the S-G experiment showed 2 components, it follows that the electron has a spin quantum number of ____________.

The angular momentum associated with spin has
15 magnitude $|\vec{S}| = \sqrt{s(s+1)}\hbar$ $(= (\sqrt{3}/2)\hbar$ for $e)$.

§14.3 Abstract(ish) QM

The states of a physical system (e.g., an atom) are represented by elements of a vector space.

The wavefunction is an example of a state vector.

The state vector tells us everything that's predictable about a system.

16 A state is conventionally denoted by $|\psi\rangle$.

The vector space (H) of quantum states has an inner product $(H \times H \to R)$, denoted by

$$\langle \phi \mid \psi \rangle$$

The vectors ϕ, ψ are called orthogonal $(\phi \perp \psi)$ if

$$\langle \phi \mid \psi \rangle = \underline{\quad}$$

In wave mechanics, the inner product is defined by

$$\langle \phi \mid \psi \rangle = \int \phi^* \psi \, d^3x.$$

17

(2) What are these in wave mechanics?

$\langle \phi \mid \psi \rangle^* =$

$\langle \phi \mid a_1\psi_1 + a_2\psi_2 \rangle =$

18 These are true in general.

ADDITIONAL NOTES

(3) What are these for wavefunctions?

$$\langle \psi \mid \psi \rangle =$$

$$\langle \vec{x} \rangle =$$

In the same way,

$$\langle \vec{p} \rangle =$$

19

A linear operator, Q, in a vector space, H, is a mapping $H \to H$: $Q\psi = \phi$.

Linearity requires that

$$Q(a_1\psi_1 + a_2\psi_2) =$$

The sum and product of operators, P and Q, are

$$(P + Q)\psi =$$

$$(PQ)\psi =$$

20

Suppose that P and Q are linear operators.

(4) Is $P + Q$ linear? ____________

$$(P + Q)(a_1\psi_1 + a_2\psi_2)$$

$$=$$

(5) Is PQ linear? ____________

21

If Q is a linear op., its adjoint, $Q^\dagger$, is defined by

$$\langle Q^\dagger \phi \mid \psi \rangle = \langle \phi \mid Q\psi \rangle, \quad \forall\, \psi,\, \phi.$$

A self-adjoint operator is one that obeys

$$Q^\dagger = Q.$$

An operator is self-adjoint iff $\langle \psi \mid Q\psi \rangle$ is real, $\forall\, \psi$.

Observables are represented by self-adjoint ops.

22

An eigenfunction of a linear operator, Q, is a vector, ψ, that obeys $Q\psi = q\psi$, where $\langle \psi \mid \psi \rangle \neq 0$, and q is a complex number.

(6) Show that q is real, if Q is self-adjoint.

23

If ψ_1, ψ_2 are eigenfunctions of a self adjoint Q with eigenvalues $q_1 \neq q_2$, then $\psi_1 \perp \psi_2$.

If $\langle \psi_1 \mid \psi_2 \rangle \neq 0$ we can cancel it to give ________ which is ______________. So ______________.

24

ADDITIONAL NOTES

§14.4 Mathematics v Physics

§14.4.1 Classical

Till the early 1900s, all mathematical descriptions of physical reality were approximate.

To describe the motion of the earth around the sun, for example, we approximate both as perfectly spherical, and (*apparently* even more outrageously) as "point particles."

25

(7) What's a point particle?

____

____

Nothing like this physically existed in our experience prior to ∼1900.

This is a mathematical abstraction that approximates, but doesn't exactly match, physical reality.

26

The dichotomy arises because we feel we've independent understandings of physics and math.

We feel we independently know

A) What a physical "object" is – the earth, a billiard ball, whatever, and

B) What a mathematical "object" is – a real number, a differential equation, whatever.

27

§14.4.2 Quantum

The situation here is radically different.

Nobody has a physical sense of what, say, an electron is because nobody has seen one, touched one, smelled one.

An electron is what the mathematics of quantum theory says it is.

28

(8) What does the math. of quantum theory say an electron is completely described by?

____

(9) What are its *intrinsic* attributes?

____

(10) What are its attributes in the outside world?

____

29

Why do position and momentum suffice?
Classically, because $F = ma$.

What does that have to do anything?
Because it's a second order DE.

Knowing the position and velocity (or momentum) of a particle at any instant will allow you, in principle, to know what a particle subsequently does in response to any force.

30

ADDITIONAL NOTES

Looking only at intrinsic attributes, there's a difference between the three that were listed above.

Mass and charge are fixed, no matter your pov.

Spin can be measured as $+1/2$ or $-1/2$ depending on your pov.

31

§14.5 Spin States

In the absence of forces, we can ignore position and momentum.

Mass and charge are fixed.

So, the quantum state of such an electron is determined by its spin: $\pm\frac{1}{2}$. We denote the two states (assumed normalized) as $|\uparrow\rangle$ and $|\downarrow\rangle$.

32

§14.5.1 Two single (one-electron) states

$|\uparrow\rangle$ and $|\downarrow\rangle$ form an orthonormal basis for the possible states of an arbitrary electron.

A pair of such states, $|A\rangle$ and $|B\rangle$ may be written as

$$|A\rangle =$$

$$|B\rangle =$$

33

From slides 18 and 19, we can get conditions on the coefficients. for $|A\rangle$ to be normalized (for example):

$$A^*_\uparrow A_\uparrow + A^*_\downarrow A_\downarrow = 1$$

One can form a (tensor) product of these to give two-electron states.

34

§14.5.2 Tensor product

Given two vector spaces the tensor product is a new vector space spanned by a combination of basis vectors of the original spaces.

It is denoted by $\otimes$.

35

§14.5.3 One double (two-electron) state

For two electrons we have ___ possible spin states.

(**11**) List them.

(**12**) What's the most general linear combination of these states?

36

ADDITIONAL NOTES

Some two electron states can be constructed as a tensor product of single electron states. Given

$$|A\rangle = A_{\uparrow}|\uparrow\rangle + A_{\downarrow}|\downarrow\rangle$$
$$|B\rangle = B_{\uparrow}|\uparrow\rangle + B_{\downarrow}|\downarrow\rangle$$

we can construct a state denoted $|A\rangle \otimes |B\rangle$:

37

Not all two-electron states are expressible as a product of two one-electron states.

Such states are called *entangled*. An example is

$$\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

This cannot be written as a product state.

38

§14.6 The EPR problem

Einstein bears responsibility for the photon (the quantum of light), and other early developments of quantum theory.

But once Born put forward the probabilistic interpretation of QM, his advocacy was replaced by opposition.

39

He and Niels Bohr engaged in (what's easily) the greatest debate in the history of human thought.

As a master of thought experiments, Einstein raised objection after principled objection to QM.

Bohr answered all, except for one.

In 1935, Einstein, in collaboration with B. Podolsky and N. Rosen produced the EPR problem.

40

Suppose that a two-spin system with zero net spin is split into components that are then widely separated.

The spin on one component, say the left one (L), is measure to be $1/2$.

We'll immediately know that the spin of the other component is __________.

41

What's the big deal?

For a classical system, we can know that one quantity is something, and thereby force some other quantity to be some other (or the same) value.

In a probabilistic theory such as QM, determining a variable at one place, seems to force a variable elsewhere to take on some value.

42

ADDITIONAL NOTES

ADDITIONAL NOTES