

Arvind Borde / PHY 19, Week 13: Expectation & Uncertainty

§13.1 Introduction

The expectation value of a variable is the value we expect to measure, on average, of a variable.

It's not the value we expect at every measurement.

The \mathcal{S} eq. is "deterministic" (knowledge of $\Psi(x, 0)$ determines, mathematics willing, $\Psi(x, t), \forall t$), but it's here that QM deviates from CM.

Ψ may be precisely determined, but physical quantities of interest, such as position and momentum, are given as probabilities, or expectation values.

This is analogous to tossing coins. If we toss two coins a large number of times, the expected value, in the sense of the mean value, of the number of heads is _____, although we do not expect to get that value on any given toss.

1

2

§13.2 Particle in a Box

The wave function is

$$\psi_n(x) = A \sin\left(\frac{n\pi x}{L}\right) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

where L is the box size.

(1) What's $\langle x \rangle$ for $n = 1$?

$$\langle x \rangle = \int_{-\infty}^{\infty} x |(\psi)x|^2 dx =$$

3

4

The first term integrates to _____.

The second needs int. by parts. We can view

$$x \cos\left(\frac{2\pi x}{L}\right)$$

as

$$\left(\frac{x^2}{2}\right)' \cos\left(\frac{2\pi x}{L}\right) \quad \text{or} \quad \frac{xL}{2\pi} \sin'\left(\frac{2\pi x}{L}\right)$$

5 (2) Which is more useful?

The _____. So

$$\int_0^L x \cos\left(\frac{2\pi x}{L}\right) dx =$$

6 (3) So, what's $\langle x \rangle$? _____. Reasonable, no?

ADDITIONAL NOTES

(4) What is $\langle x^2 \rangle$?

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} dx =$$

7

The second requires _____ giving _____.

So

$$\langle x^2 \rangle =$$

(5) What's Δx , the uncertainty in x ?

$$\Delta x = \sqrt{\quad}$$

8

So we have,

$$\Delta x = \sqrt{\quad} \approx$$

9

(6) What might you intuitively *expect* $\langle p \rangle$ to be for a particle in a box? _____.

(7) What might you *calculate* $\langle p \rangle$ to be?

$$\langle p \rangle =$$

10

(8) What does this work out to be?

11

Further, calculations of Δp lead to

This is an example of an uncertainty principle in quantum mechanics.

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ADDITIONAL NOTES
