Arvind Borde / PHY 19, Week 13: Expectation & Uncertainty

§13.1 Introduction

The expectation value of a variable is the value we expect to measure, on average, of a variable.

It's not the value we expect at every measurement.

The $\mathcal{S}$ eq. is "deterministic" (knowledge of $\Psi(x,0)$ determines, mathematics willing, $\Psi(x,t)$, $\forall t$), but it's here that QM deviates from CM.

1

Ψ may be precisely determined, but physical quantities of interest, such as position and momentum, are given as probabilities, or expectation values.

This is analogous to tossing coins. If we toss two coins a large number of times, the expected value, in the sense of the mean value, of the number of heads is ________, although we do not expect to get that value on any given toss.

2

§13.2 Particle in a Box

The wave function is

$$\psi_n(x) = A\,sin\left(\frac{n\pi x}{L}\right) = \sqrt{\frac{2}{L}}\,sin\left(\frac{n\pi x}{L}\right)$$

where L is the box size.

(1) What's $\langle x \rangle$ for $n = 1$?

3

$$\langle x \rangle = \int_{-\infty}^{\infty} x\,|(\psi)x)|^2\,dx =$$

4

The first term integrates to __________.

The second needs int. by parts. We can view

$$x\,cos\left(\frac{2\pi x}{L}\right)$$

as

$$\left(\frac{x^2}{2}\right)' cos\left(\frac{2\pi x}{L}\right) \quad \text{or} \quad \frac{xL}{2\pi}\,sin'\left(\frac{2\pi x}{L}\right)$$

5 **(2)** Which is more useful?

The ____________. So

$$\int_0^L x\,cos\left(\frac{2\pi x}{L}\right)dx =$$

6 **(3)** So, what's $\langle x \rangle$? ______. Reasonable, no?

ADDITIONAL NOTES

(4) What is $\langle x^2 \rangle$?

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} \qquad dx =$$

7

The second requires ____________________,
giving ____________.

So

$$\langle x^2 \rangle =$$

(5) What's Δx, the uncertainty in x?

$$\Delta x = \sqrt{\qquad}$$

8

So we have,

$$\Delta x = \sqrt{\qquad\qquad} \approx$$

9

(6) What might you intuitively *expect* $\langle p \rangle$ to be for a particle in a box? ________.

(7) What might you *calculate* $\langle p \rangle$ to be?

$$\langle p \rangle =$$

10

(8) What does this work out to be?

____

11

Further, calculations of Δp lead to

This is an example of an uncertainty principle in quantum mechanics.

12

ADDITIONAL NOTES

§13.3 The Uncertainty Principle

Many pairs of physical variables, called conjugate variables, are related:

Position and momentum,
Time and energy, etc.

The connection between them is complex and deep.

13

For example, Emmy Noether proved in 1915 that in classical mechanics there's a connection between symmetry and conservation.

A system that's symmetric wrt shifts in position must conserve momentum.
A system that's symmetric wrt translations in time must conserve energy.

14

Etc.

The connection between these disparate quantities pops up in special relativity as well:

We combine space (3d) and time into a single 4d entity.

We also combine the 3d-vector of momentum with energy to create a 4d energy-momentum vector.

15

Werner Heisenberg proposed in 1927, based on what was emerging in QM, that all such conjugate variables must obey an uncertainty relationship:

The product of the uncertainties (as defined above) in the two variables is always greater than $\hbar/2$.

All experimental and mathematical evidence, so far, suggests that he was right.

16

ADDITIONAL NOTES

ADDITIONAL NOTES