Arvind Borde / PHY 19, Week 1: Introduction

A crash course in the world

The whole world consists of two entities:
____________ and ________________.

Examples of matter are your chairs, your bodies, and the stars.

Examples of interactions are the ____________
1 ____________________________________

These four are the only known interactions, also called forces.

Unlike the fundamental forces, it may appear that matter is more complicated. Take our bodies.

(1) What are we mainly made of?
________________.

2 But there's other stuff: bones, flesh, hair, etc.

Similarly, if you look around the room, you'll see many different substances.

It's been known since the 1800s that the complexity of the material world is based on just a few basic things combining in different ways. These "basic things" are called ______________.

3

(2) Name some elements.
____

(3) Roughly how many elements are there?
____
____

(4) Elements come in basic "pieces." What are they called? ___________________________

4

But that is not the end of the story. Each atom has structure and is itself made up of three more basic things.

(5) What are the constituents of an atom called?
____

5

Protons and neutrons form the "nucleus" of the atom, and electrons swirl in a cloud around it.

Protons have positive electric charge, electrons an equal negative charge and neutrons are neutral. The electron cloud is "held in place" by the electric forces between them and the protons.

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ADDITIONAL NOTES

The simplest atom is that of hydrogen. It consists of a single proton and a single electron.

The nucleus is roughly 10^{-13}cm in radius and the electron cloud about 10^{-8}cm.
(6) That's factor of about 100,000. Why?

____

7

Can you subdivide further?

Not for electrons: they appear to have no internal structure.

But there's one step further for protons and neutrons: they have internal constituents called ____________.

8

At the micro level both matter and interactions are represented by particles, called ____________. (That's the plural. The singular is quantum.)

There are ________ and _______, distinguished by their ____. (Like a top.)

____

____

____

9

All the known quanta

Fermions (matter)

6 "quarks":
up, down, charm, strange, top, bottom
(combinations give the proton, neutron, etc.)
6 "leptons":
electron, muon, tau and their neutrinos

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Bosons (interactions)

Electromagnetism: Photon
Strong: 8 Gluons (hold the nucleus together)
Weak: W^+, W^-, Z^0 (radioactive decay)
Gravitation: Graviton??????

These 24-odd particles make up the world and all its interactions, along with a final particle called the Higgs boson.

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The weak and strong forces are short range. They drop to zero outside the nucleus. They play no direct role in the structure of the solar system or the Universe.

Electromagnetism is long range, but most large objects are electrically and magnetically neutral. So this force, too, is irrelevant over large distances.

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ADDITIONAL NOTES

That leaves gravity. It and it alone determines the large scale structure of the Universe. It explains why the moon goes around the earth, why planets move around the sun. Understanding gravity is intertwined with understanding the Universe.

Gravity depends on ______ and ________. They're the two fundamental things we need to understand in order to understand the Universe.

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____,
yet in some ways understand the least.

It's a magical force that doesn't, unlike electromagnetism or the nuclear forces, exist in the fabric of the Universe – it ____________________
___________.

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The most immediate experience we have of gravity is that things fall.

If the earth were at the center of the Universe, as was thought, one could attribute the tendency of things to fall as their natural tendency to go to the center of the Universe because of their weight. That was the view of Aristotle, and co. (∼300 BC).

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According to this view, the "heavens" were fixed (apart from the wandering planets) and objects fell because they were trying to get to the center of the Universe.

In the Aristotelian view heavier objects would fall more quickly.

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Galileo spoiled all that in the 1600s by dropping things.

Around the same time it became clear that the "heavens" were more complicated than had been thought: planets had moons that went around them, for example.

The earth was not the center of everything.

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In the late 1600s, Isaac Newton fixed everything – the shenanigans in the heavens and why things fall to earth by

Newton's Law of Universal Gravitation

$$F_{\text{grav}} = G\frac{m_1 m_2}{d^2},$$

$$\underbrace{G = 6.67 \times 10^{-11}\frac{N \cdot m^2}{kg^2}}_{\text{Gravitational constant}}$$

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ADDITIONAL NOTES

In English:

Every object (mass m_1) attracts every other object (mass m_2) by a force proportional to the product of their masses and inversely proportional to the square of the distance between them.

$$m_1 \bullet \underbrace{\rightarrow \text{attractive gravitational force} \leftarrow}_{distance = d} \circ m_2$$

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Why is this a universal law? Because __________ ______________________________________

It applies to you and the earth, to a piece of chalk and the earth, to a piece of chalk and another piece of chalk, to the earth and the moon, to the sun and Jupiter, . . .

The law uses "proportionality." What's that?

20

If $y = x$ then the quantity y equals x.

If $y = 5x$ then y is __________________ x. We write this as

$$y \propto x$$

This is true whenever $y = kx$ for any fixed k ("_________________________").

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In Newton's gravitational law

$$F_{\text{grav}} = G\frac{m_1 m_2}{d^2}$$

the quantity G is the proportionality constant.

_________________ – same value for any two objects:

$$G = 6.67 \times 10^{-11} \text{N} \cdot \text{m}^2/\text{kg}^2$$

(this is the value when you measure mass in kilograms and distance in meters).

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We also need ____________________. If

$$y = k \times \frac{1}{x} = \frac{k}{x}$$

where k is fixed, then y is said to be inversely proportional to x.

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Assume $k > 0$. If

(7) $y = kx$, as x goes up, y __________

(8) $y = kx$, as x goes down, y ____________

(9) $y = \frac{k}{x}$, as x goes up, y ____________

(10) $y = \frac{k}{x}$, as x goes down, y __________

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ADDITIONAL NOTES

Using Newton's law

$$F_{\text{grav}} = G\frac{m_1 m_2}{d^2}$$

(11) Does the gravitational force go up as the masses go up? ____________

(12) Does the gravitational force go up as the distance increases? ________________________

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We need to be more specific about *how much* the gravitational force goes up and down by.

The gravitational force on an object of mass m due to the earth (mass M_{E}) is

$$F = G\frac{mM_{\mathsf{E}}}{d^2}$$

where d is the distance to the *center of the earth*.

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In each case below, does the gravitational force on m go up or down, and by how much?

(13) m doubles: __________________________

(14) m triples: __________________________

(15) m halves: ______________________________ ___________

27

How the gravitational force depends on distance is *slightly* trickier:

If the distance between the two objects goes up, the force __________________________________ ___________.

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In each case below, does the gravitational force on m go up or down, and by how much?

(16) d doubles: __________________________

(17) d triples: __________________________

(18) d halves: ________________________

29

This means that if the distance between two objects doubles, one of their masses would have to go up by a factor of 4 in order to keep the force the same.

This is Newtonian gravity. The theory works spectacularly well, but not perfectly...

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ADDITIONAL NOTES

The Newtonian picture of the world changed significantly in 1905, when Eintein introduced the theory of relativity.

(19) What do you know of this theory?

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ON THE ELECTRODYNAMICS OF MOVING BODIES

By A. EINSTEIN

June 30, 1905

It is known that Maxwell's electrodynamics—as usually understood at the present time—when applied to moving bodies, leads to asymmetries which do not appear to be inherent in the phenomena. Take, for example, the reciprocal electrodynamic action of a magnet and a conductor. The observable phenomenon here depends only on the relative motion of the conductor and the

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In this 1905 paper, Einstein had *no references to previous work*, and he thanked nobody other than Michele Besso, an old friend:

to which, by the theory here advanced, the electron must move.

In conclusion I wish to say that in working at the problem here dealt with I have had the loyal assistance of my friend and colleague M. Besso, and that I am indebted to him for several valuable suggestions.

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Discussions with Michele Besso appear to have led Einstein to write to an ex-classmate, Marcel Grossman, in September 1901:

. . . *A considerably simpler method of investigating the relative motion of matter with respect to luminiferous ether. . . has just sprung to my mind. If only for once, relentless Fate gave me the necessary time and peace!. . .*

Doc. 122, Collected Papers of AE, Vol. 1.

After that, comments on the ether dwindle. . .

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Somewhere between 1901 and 1905 he had, apparently on his own, abandoned the idea that the ether was necessary. Relying only on constructs that he deemed necessary, he introduced a new approach.

What did Einstein present whole in 1905, after ten years of thought, as something entirely original?

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It's presented in paragraph 2 of his paper:

phenomena of electrodynamics as well as of mechanics possess no properties corresponding to the idea of absolute rest. They suggest rather that, as has already been shown to the first order of small quantities, the same laws of electrodynamics and optics will be valid for all frames of reference for which the equations of mechanics hold good.[1] We will raise this conjecture (the purport of which will hereafter be called the "Principle of Relativity") to the status of a postulate, and also introduce another postulate, which is only apparently irreconcilable with the former, namely, that light is always propagated in empty space with a definite velocity c which is independent of the state of motion of the emitting body. These two postulates suffice for the attainment of a simple and consistent theory of the electrodynamics of moving bodies based on Maxwell's

Developing these ideas requires us to understand how a "hatted" system, moving with respect to an unhatted, are mathematically related.

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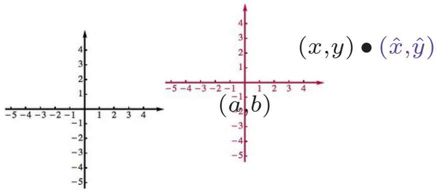
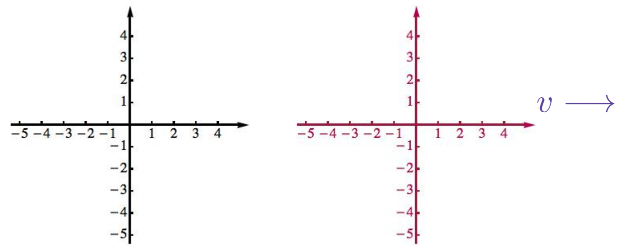
ADDITIONAL NOTES

(20) If the origin of the hatted coordinates is at (a,b) in the original (unhatted) coordinates, how are the hatted coordinates, $(\hat{x},\hat{y})$, of a point related to its unhatted coordinates, (x,y)?



37

Suppose the hatted coordinates initially coincide with the unhatted, but are now moving away from them at a fixed speed v in the positive x direction:



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At a time t,

(21) Looking at Q20 what's a here? ________

(22) How are $\hat{x}$ and x related? ________

(23) How are $\hat{y}$ and y related? ________

These relations between the hatted and unhatted coordinates are called ________

The assumption here is that ________

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We'll study the transformation of coordinates without this assumption, but guided by the two "postulates" that Einstein states at the start of his paper and repeats on page 4:

1. The laws by which the states of physical systems undergo change are not affected, whether these changes of state be referred to the one or the other of two systems of co-ordinates in uniform translatory motion.

2. Any ray of light moves in the "stationary" system of co-ordinates with the determined velocity c, whether the ray be emitted by a stationary or by a moving body. Hence

$$\text{velocity} = \frac{\text{light path}}{\text{time interval}}$$

where time interval is to be taken in the sense of the definition in § 1.

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Assume that the hatted coordinates initially coincide with the unhatted and are moving away from them at a fixed speed v in the positive x direction. The simplest transformation between the two sets of coordinates is a ________:

41 (α, β, γ, δ are fixed quantities that we'll determine.)

(24) Under the assumption of the previous slide, when $x = vt$, $\hat{x} =$ ________

(25) Plugging that into the $\hat{x}$ transformation equation, what do we get for δ?

____

____

(26) Is it OK to cancel t?

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ADDITIONAL NOTES

(27) Use this to kick δ out of the $\hat{x}$ equation:

____

(28) Now, from the p.o.v. of the hatted frame, the unhatted is moving away from *it* with velocity

____

(29) If the same laws apply (postulate 1), we have

$$x = \_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

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Using postulate 2 (speed of light, c, is the same in all coordinate systems), we have for a ray of light sent out in the $x/\hat{x}$ direction from the origin at the initial instant

$$x/t = c = \hat{x}/\hat{t}$$

(30) Solve these for t and $\hat{t}$.

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(31) Plugging these expressions for t and $\hat{t}$ into

$$\hat{x} = \gamma(x - vt)$$

$$x = \gamma(\hat{x} + v\hat{t}),$$

you get:

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(32) Multiply the two equations together and get a formula for γ.

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So, the space transformation equation becomes

$$\hat{x} = \gamma(x - vt) = \frac{(x - vt)}{\sqrt{1 - v^2/c^2}}. \qquad [\text{LT}x]$$

We can also figure out what α and β must be in the time transformation equation

$$\hat{t} = \alpha x + \beta t.$$

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We again use postulate 2 (speed of light, c, is the same in all coordinate systems)

$$\frac{x}{t} = c = \frac{\hat{x}}{\hat{t}}$$

and re-express it as $x = ct$ and $\hat{x} = c\hat{t}$. The space equation on the previous slide becomes

$$c\hat{t} = \gamma(ct - vx/c).$$

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ADDITIONAL NOTES

Dividing by c, we get

$$\hat{t} = \gamma(t - vx/c^2). \qquad [\text{LT}t]$$

(33) Comparing this with the time transformation equation, identify α and β.

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The equations

$$\hat{t} = \gamma(t - vx/c^2) \qquad [\text{LT}t]$$

$$\hat{x} = \gamma(x - vt) \qquad [\text{LT}x]$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

are called the Lorentz Transformations, in honor of Lorentz, who got there a year before Einstein. But Einstein got there independently and more deeply.

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the transformation equations which have been found become

$$\begin{aligned} \tau &= \beta(t - vx/c^2), \\ \xi &= \beta(x - vt), \\ \eta &= y, \\ \zeta &= z, \end{aligned}$$

where

$$\beta = 1/\sqrt{1 - v^2/c^2}.$$

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Understanding γ

(34) By looking at the structure of the formula,

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

and keeping things real, is there a maximum speed for the hatted system? ________________

(35) Are there minimum/maximum values for γ?

____

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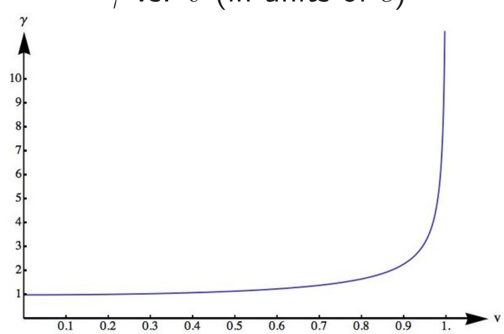
γ vs. v (in units of c)



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"Relativistic effects" are important only at high values of γ, which kick in as $v \to c$.

(36) Calculate γ for v equal to

(a) $0.25c$: ________________,

(b) $0.5c$: ________________,

(c) $0.75c$: ________________, and

(d) $0.9999c$: ________________.

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ADDITIONAL NOTES

The Lorentz Transformations

$$\hat{t} = \gamma(t - vx/c^2)$$

$$\hat{x} = \gamma(x - vt)$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

Our little algebraic excursion has important and odd consequences.

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Odd Consequence 1: Simultaneity

(37) Suppose two events occur at the same time (say $t = 0$) in the unhatted coordinates, but at different places: $x = +1$ for event 1 and $x = -1$ for event 2. When will they seem to occur in the hatted coordinates?

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(38) What's the big deal?

As Einstein observed

declare the clocks to be synchronous.

So we see that we cannot attach any *absolute* signification to the concept of simultaneity, but that two events which, viewed from a system of co-ordinates, are simultaneous, can no longer be looked upon as simultaneous events when envisaged from a system which is in motion relatively to that system.

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Odd Consequence 2: Time Dilation

Suppose you have a clock at a fixed place in the hatted coordinates, and you record two times with it $\hat{t}_1$ and $\hat{t}_2$. How long will the interval $t_2 - t_1$ be in the unhatted coordinates?

We have $$\hat{x}_2 = \gamma(x_2 - vt_2)$$

$$\hat{x}_1 = \gamma(x_1 - vt_1)$$

So $$\hat{x}_2 - \hat{x}_1 = \gamma((x_2 - x_1) - v(t_2 - t_1))$$

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Because the clock is at a fixed place in the hatted system, $\hat{x}_2 = \hat{x}_1$. So

$$0 = \gamma((x_2 - x_1) - v(t_2 - t_1))$$

or

$$(x_2 - x_1) = v(t_2 - t_1).$$

Put this into memory.

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From the time equations

$$\hat{t}_2 = \gamma(t_2 - vx_2/c^2)$$

$$\hat{t}_1 = \gamma(t_1 - vx_1/c^2)$$

(39) Find a formula for $\hat{t}_2 - \hat{t}_1$.

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ADDITIONAL NOTES

So

$$t_2 - t_1 = \gamma(\hat{t}_2 - \hat{t}_1)$$

This means that the time interval in the unhatted frame will be longer than in the hatted.

(40) Why? ______________________

____

____

____

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Gains from time dilation

Picking three modes of travel, say walking, driving and flying, let's assign each a plausible speed in km/sec. Using $c = 300,000$ km/sec, we'll calculate how much time you gain on a friend stationary on earth if you travel for an earth-year at that speed.

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$$t_2 - t_1 = \gamma(\hat{t}_2 - \hat{t}_1)$$

or

$$\Delta\hat{t} = \Delta t/\gamma$$

where you're the hatted traveler, and the unhatted frame is "stationary."

The amount of time you "gain" is

$$\text{Gain} = \Delta t - \Delta\hat{t} = \underline{\qquad\qquad}$$

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(41) To the nearest $1{,}000$, how many seconds are there in a non-leap year?

____

So we'll use __________________________

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Time gains at different speeds

Activity	Speed (km/s)	$1 - 1/\gamma$	Gain (sec)
Walking	6/3600	$1.5 \cdot 10^{-17}$	$4.9 \cdot 10^{-10}$
Driving	60/3600		
Flying	900/3600		

If flying time were 14 hrs (from ground pov), what would the time gain be in nanoseconds?

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Tests of time dilation

1) In 1971 Keating and Hafele flew four caesium atomic clocks around the world. The results of the experiment confirmed relativistic predictions within 10%. The experiment was repeated in 1996 on a trip from London to Washington and back, a 14 hour journey. The result, a 16.1 ns time gain from motion, was within 2 ns of the prediction.

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ADDITIONAL NOTES

2) Muon lifetime [Bailey,J. et al. Nature **268**, 301 (1977)]: Muons with "rest lifetime" of $2.198\,\mu$s were sped to high speed ($\gamma = 29.33$). The measured lifetimes at those speeds were found to be $64.368\,\mu$s.

(42) Is that consistent with relativity?

____

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Odd Consequence 3: Length Contraction

From $\hat{x}_2 = \gamma(x_2 - vt_2)$, $\quad \hat{x}_1 = \gamma(x_1 - vt_1)$ we see that

$$\hat{x}_2 - \hat{x}_1 = \gamma((x_2 - x_1) - v(t_2 - t_1))$$

If two position measurements are made at the same time in the unhatted frame ($t_1 = t_2$), we will have

$$\hat{x}_2 - \hat{x}_1 = \gamma(x_2 - x_1)$$

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So
$$x_2 - x_1 = (\hat{x}_2 - \hat{x}_1)/\gamma$$

The length of an object at rest in the hatted frame will be shorter in the unhatted frame.

(43) Why? ______________________________

____

____

____

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Paradoxes and Puzzles of Relativity

Relativity is an entirely consistent theory, but people often speak of "paradoxes" in it because it can sometimes lead to apparent self-contradictions.

They can always be resolved.

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ADDITIONAL NOTES