Arvind Borde / PHY 17, Week 7: Energy II

§7.1 Motion in 1-d: Linear

This is motion along a straight line. We can choose our x axis along the motion. So $\vec{\boldsymbol{F}}$ is simply F_x, and whether it is positive or negative tells us the direction. The work in going from a to b is

$$W_{ab} = \int_a^b F_x(x)\,dx.$$

1 We'll just say $F(x)$ here for $F_x(x)$.

For such a force to be conservative it must be a function of x alone, and the work integral must be path-independent. In this particular case, the first condition implies the second.

This follows from

$$\int_p^q F(x)\,dx = -\int_q^p F(x)\,dx.$$

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Let's assume that in

$$W_{ab} = \int_a^b F(x)\,dx.$$

the point b is to the right of a.

Different paths from a to b differ only in how much they go back and forth. But for every back there must be a forth *on the same path*, and their contributions to the work will cancel.

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So, as long as the force depends only on position, we can define

$$U(x) = -\int_{x_0}^x F(x)\,dx.$$

In 1-d, $\vec{\nabla}U$ reduces to dU/dx, so we have

$$F(x) = -\frac{dU}{dx}.$$

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§7.1.1 <u>A spring</u>

By Hooke's law, a mass hanging on a spring feels a force given by

$$F = -kx,$$

where x is the extension/compression from the spring equilibrium position, $x = 0$, and k, the spring constant, is a measure its stiffness.

5 We'll pick $x_0 = 0$.

(1) From $U = -\int F dx$ what's U?

$$U(x) =$$

(2) Consistent with $F = -(dU/dx)$? _____

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ADDITIONAL NOTES

§7.1.2 <u>The graph of $U(x)$, and what it tells us</u>

(3) If $f(x)$ is any function, what's the geometrical interpretation of $f'(x) = df(x)/dx$? ____________ ____

In regions where the graph goes up, ________, and where the graph goes down, ________.

That's true of the graph of $U(x)$, as well.

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Remembering that $F = -\dfrac{dU}{dx} = -U'$, we see that

- $F > 0$, i.e., points right, when $U' < 0$, i.e., the graph of U goes down.
- $F < 0$, i.e., points left, when $U' > 0$, i.e., the graph of U goes up.
- $F = 0$ when $U' = 0$, i.e., the graph is level.

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In the following graphs a particle is shown "on the x-axis." The graph of $U(x)$ is plotted overhead, with a ghost of the particle shown on it.

As you answer the next questions, imagine simultaneously what you'd expect if a real object were placed where the ghost particle is, but now on a physical hill with the shape of the graph of $U(x)$.

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(4) At each location of the particle,

- Does the graph of U go up ($U' > 0$) or down ($U' < 0$)?
- Does the force, F, point left or right?

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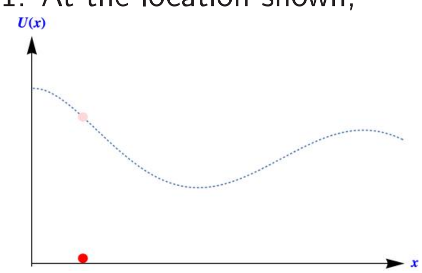
Scenario 1: At the location shown,

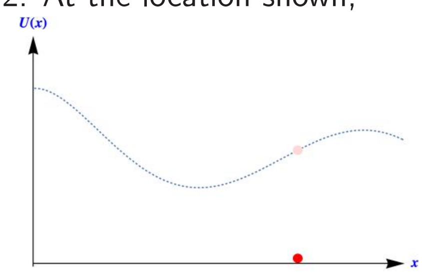


Graph of U is ______________

Force is ______________

11 If physical hill, ghost rolls ______________

Scenario 2: At the location shown,



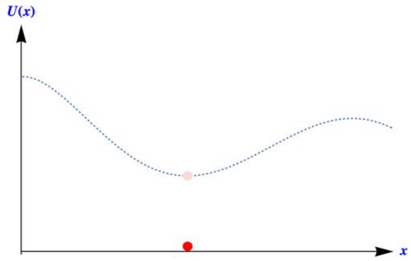
Graph of U is ____________

Force is _____________

12 If physical hill, ghost rolls ______________

ADDITIONAL NOTES

Scenario 3: At the location shown,



Graph of U is ________

Force is ______

13 If physical hill, ghost object would roll _________

The analogy with "rolling down a hill" is extremely useful, not just literally, but also in extended situations where we plot potential (hello PE) versus state (hello x).

For example, on the following graph where the particle starts tells you how it behaves.

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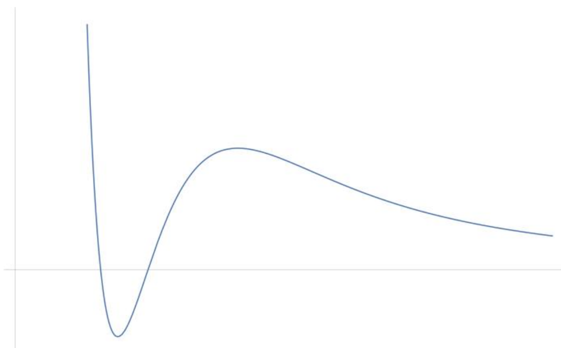

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§7.2 Motion in 1-d: Curvilinear

Motion along a line, even if curved, can be handled by the same methods.

In fact, any system that depends on a single parameter can be viewed as 1-dimensional.

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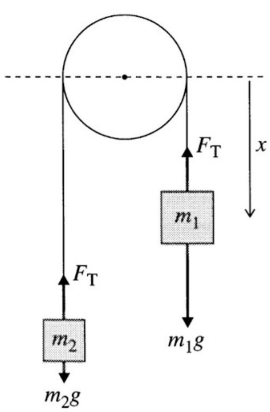
For example, x is all that matters here:



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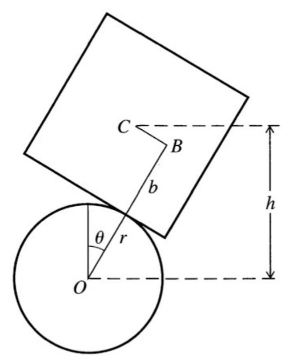
Or, θ, when a block balances on a cylinder:



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ADDITIONAL NOTES

§7.3 Central Forces

A problem that's also *essentially* 1-d is a central force problem between a particle, and a fixed force center separated by a distance r.

(5) Why?

____

____

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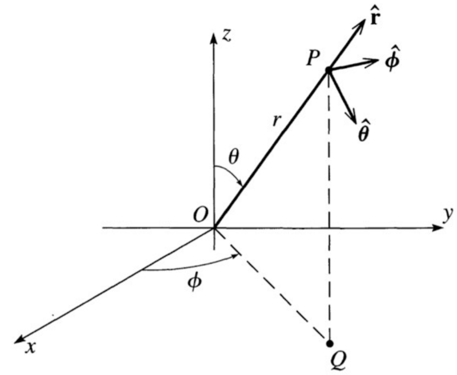
These are best studied with spherical coords:



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The transformations from (r, θ, ϕ) to (x, y, z) are

$$x = \underline{\hspace{6cm}}$$

$$y = \underline{\hspace{6cm}}$$

$$z = \underline{\hspace{4.5cm}}$$

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The gradient of a function f in spherical coordinates is slightly more tedious to calculate, but this is it

$$\vec{\nabla} f = \frac{\partial f}{\partial r}\hat{\boldsymbol{x}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\,sin\,\theta}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}}$$

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When the force depends only on r, so must U. Therefore,

$$\vec{\boldsymbol{F}} = \vec{\nabla} U = -\frac{df}{dr}\hat{\boldsymbol{x}}.$$

It follows that we have basically a 1-d problem.

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ADDITIONAL NOTES