§3.1 Gravitational Force Near Earth

The gravitational force on an object of mass m near the earth is given to a very good approximation by $m\vec{g}$, where the direction of $\vec{g}$ is __________ and its magnitude is ________________.

This arises from a more fundamental force law.

1

(1) What law?

____

____

____

2

(2) With

$G = 6.67 \times 10^{-11}$ N·m^2/kg^2,

$M_{\text{E}} = 6 \times 10^{24}$ kg,

$r = R_{\text{E}} = 6.4 \times 10^{6}$ m,

calculate to four decimal places

$$a = \left[G \frac{M_{\text{E}}}{r^2} \right]$$

(i.e., mag. of grav. acc. of m at surface of earth)

3

4

(3) Calculate the same quantity 1 km above the surface of the earth.

$a =$

5

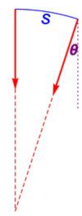
So, the magnitude of the acceleration of a mass, m, due to the earth's gravitational pull is constant to a very good approximation over vertical distances small compared to the radius of the earth.

What about the direction?

6

ADDITIONAL NOTES

Consider two points a distance s apart on the earth's surface Let the angle between the directions of $\vec{g}$ at the two points be θ, as shown.



____

____

7

(4) If θ is in radians, what's the connection between it, s and R_E?

(5) If $s = 1$ km, what's θ in radians and degrees?

Rad: ____________________________________.

Deg: _________________________________.

8

The differences in both magnitude and direction of $\vec{g}$ across ~ 1 km are negligible for many problems.

A particle dropping through a distance of 1 km will land $10^3 \times 1.6 \times 10^{-4} = 0.16$ m from the predicted spot. That's 16 cm, good enough for many purposes, such as artillery. If our region of interest had dimensions of 10^2 m, the accuracy would improve by a factor 10^{-2}, good enough for baseball.

9

§3.2 Projectile Motion

We'll use the assumption of constant $\vec{g}$ to study _________________.

A projectile is a point particle moving close enough to the surface of the earth, and over small enough horizontal distances, that our assumption of constant $\vec{g}$ is reasonable.

10 Call the mass of the particle m.

We'll take $t = 0$ as our initial time, and $\vec{v}_0 \equiv \vec{v}(0)$ as the initial velocity of the particle.

Decomposing $\vec{v}_0$ into horizontal and vertical components, we pick the y axis to be vertical (positive upward), and the x axis in the direction of the horizontal component of $\vec{v}_0$. If there is no horizontal component, we chose the x axis in an arbitrary horizontal direction.

11

We'll pick the initial position, $\vec{r}_0 = \vec{r}(0)$ to be some arbitrary point. That is,

$$\big(x(0), y(0), z(0)\big) \equiv (x_0, y_0, z_0).$$

The forces in the problem are that of gravity, $-m\vec{g}$ (constant by our assumption), acting in the downward y direction, and possible other forces $\vec{F}_{\text{other}}$.

12

ADDITIONAL NOTES

These other forces are usually frictional, because of resistance from the medium, often air.

From NSL, and our coordinate setup, etc.,

$$
\begin{aligned}
m\ddot{x} &= F_{\text{other}_x}, & v_x(0) &= v_{0_x}, & x(0) &= x_0 \\
m\ddot{y} &= -mg + F_{\text{other}_y}, & v_y(0) &= v_{0_y}, & y(0) &= y_0 \\
m\ddot{z} &= F_{\text{other}_z}, & v_z(0) &= 0, & z(0) &= z_0
\end{aligned}
$$

13

§3.3 Projectiles Without Resistance

The only force here is the force of gravity exerted by the earth on a particle of mass m; we assume $\vec{F}_{\text{other}} = \vec{0}$.

§3.3.1 <u>z motion</u>: It follows that $z(t) = 0\,\forall t$, so we have a 2-d, x-y problem.

To get the x and y motions, we use $v_x(t) \equiv \dot{x}(t)$ and $v_y \equiv \dot{y}(t)$, and integrate NSL twice.

14

§3.3.2 <u>x motion</u>:

$$m\ddot{x} = m\frac{dv_x(t)}{dt} = 0$$

(6) Get $x(t)$:

$$\dot{x}(t) = v_x(t) =$$

$$x(t) =$$

Since there are no forces in the x direction Newton's ________ law holds.

15

§3.3.3 <u>y motion</u>:

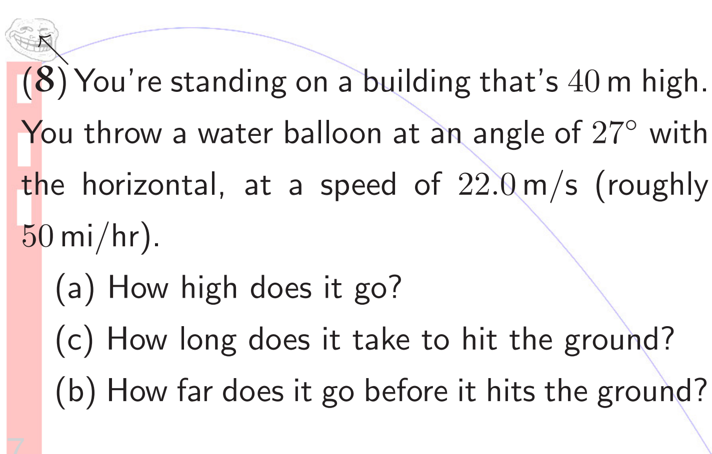
$$m\ddot{y} = m\frac{dv_y(t)}{dt} = -mg$$

(7) Get $y(t)$:

$$\dot{y}(t) = v_y(t) =$$

$$y(t) =$$

16



(8) You're standing on a building that's 40 m high. You throw a water balloon at an angle of $27°$ with the horizontal, at a speed of 22.0 m/s (roughly 50 mi/hr).

(a) How high does it go?

(c) How long does it take to hit the ground?

(b) How far does it go before it hits the ground?

17

Step 1: Resolve initial velocity into components:

$v_{x0} =$

$v_{y0} =$

Step 2: Get maximum height:

18

ADDITIONAL NOTES

Step 3: Get time to get to the ground.

19

(9) Which of these ts is relevant to the balloon hitting the ground after launch? ________

(10) What's with the negative answer?

____

____

____

Step 4: Get total horizontal distance.

20

(11) If you threw your balloon *horizontally* from the $40\,\mathrm{m}$ building at a speed of $27\,\mathrm{m/s}$, get a formula for how far it would drop vertically after $t\,\mathrm{s}$.

We have $v_{x0} =$ ________ and $v_{y0} =$ _______

So, $y =$

21

(12) In the previous situation, would it make a difference to the formula for y had you thrown the balloon at a horizontal speed of $2.7\,\mathrm{m/s}$? ______ $2{,}700{,}000\,\mathrm{m/s}$? ______ $0\,\mathrm{m/s}$? ________

____

____

____

22

§3.4 Projectiles With Resistance

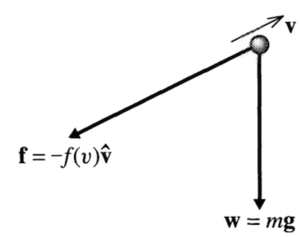
The resistance, as said above, is usually a frictional force, called drag, $\vec{f}$, exerted by the medium.

Drag is, in general, very complicated. In the simplest case, it opposes the velocity, $\vec{f} \propto -\vec{v}$, and that's all we'll look at.

Curveballs, airplane lift, etc., won't be discussed.

23

We assume the magnitude of the drag force depends only on the magnitude, v, of the velocity. Expanding $f(v)$ in powers of v, the lowest order terms are



$$f(v) = bv + cv^2 \equiv f_{\text{lin}} + f_{\text{quad}},$$

(No constant term 'cause ________________.)

24

ADDITIONAL NOTES

§3.4.1 The origin of f_{lin}

Arises from the viscous drag of the medium, and is proportional to the viscosity of the medium, and the linear dimensions of the particle:

$$f_{\text{lin}} \equiv bv = (3\pi\eta D)v,$$

where η is the viscosity of the medium, and D is a measure of the linear size of the particle.

25

Dimensions: $[D]$=L, so $[\eta]$=M·L^{-1}·S^{-1}.
In SI units, the units are kg·m^{-1}·s^{-1} ≡u.

Values (under "standard conditions"):

- Air: 1.7×10^{-5} u
- Water: 1.00 u
- Milk: 2.12 u
- Vinegar: ~ 13 u
- Honey: $\sim 6 \times 10^{3}$ u

26

§3.4.2 The origin of f_{quad}

OK, so f_{lin} arises from viscosity. What of f_{quad}?

27

Depending on the situation one, or the other, force is dominant – or both are equally important.

These are not easy problems, but we can look at a few simple examples.

28

§3.5 Linear resistance

$$m\ddot{\vec{\boldsymbol{r}}} = m\dot{\vec{\boldsymbol{v}}} = -m\vec{\boldsymbol{g}} - b\vec{\boldsymbol{v}}$$

Splitting into x and y components:

$$m\dot{v}_x = -bv_x$$
$$m\dot{v}_y = -mg - bv_y$$

29

§3.5.1 Horizontal motion

$$\dot{v}_x(t) = -(b/m)v_x(t)$$

(13) Solve $\dfrac{dv_x(t)}{dt} = -\dfrac{b}{m}v_x(t)$.

30

ADDITIONAL NOTES

(14) What's A in terms of the initial variables?

So

Remembering $v_x(t) = \dot{x}(t)$, what do we ask?

31

(15) What's $x(t)$?

$$x(t) =$$

$$=$$

(16) What's $x(0)$?

$$x(0) = x_0 =$$

32

Assume that $x_0 = 0$. Then,
(17) What's $x(t)$?

$$x(t) =$$

$$=$$

33

(18) What's $v_{x_\infty} = \lim_{t \to \infty} v_x(t)$?

In other words, under these assumptions, your speed slows not just to a crawl, but to 0.

34

(19) What's $x_\infty = \lim_{t \to \infty} x(t)$?

$$x_\infty =$$

Therefore, you can go "this far and no further."

35

36

ADDITIONAL NOTES