**Arvind Borde: Phy 12** Homework 8

### Keep in mind

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \qquad (1)$$

$$m = \frac{h_i}{h_o} = -\frac{di}{d_o} \qquad (2)$$

$$d_o > 0, h_o > 0 \quad \text{(assumptions)} \qquad (3)$$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \qquad (4)$$

The last three apply to mirrors and single lenses; $d_o$ is the distance from the mirror/lens to the object, $d_i$ the distance to the image, and $f$ the focal length. The *signs* of the quantities follow the conventions spelled out in the text. Solving eqn. 4 for $d_i$:

$$d_i = \frac{d_o f}{d_o - f} \qquad (\hat{4})$$

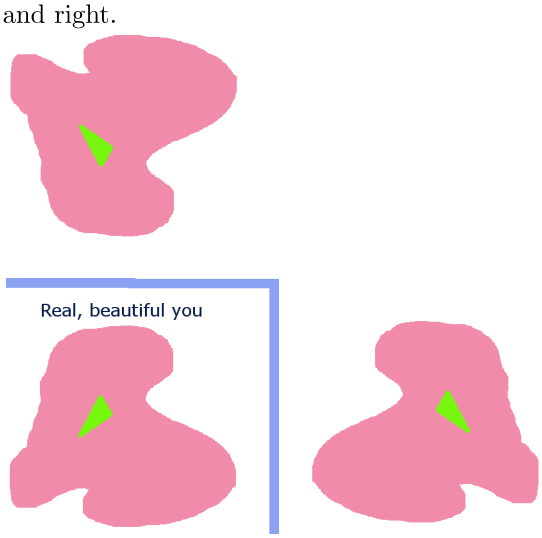
Dividing both sides by $d_o$, we get an alternate formula for $m$:

$$m = -\frac{di}{d_o} = -\frac{f}{d_o - f} = \frac{f}{f - d_o}. \qquad (\hat{2})$$

### Questions

**(2)** $f = \infty$ (parallel rays never focus); $m = 1$.

**(3)** Here you are, admiring yourself in two blue mirrors, one above you and one in front. The one in front reverses front and back and left and right, the one above reverses top and bottom and left and right.

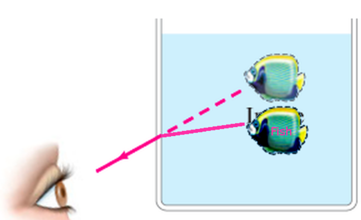



**(4)** $m < 0$, therefore $d_i > 0$, so real image.
∘ $h_i < 0$, so inverted.
∘ For a convex mirror, $f < 0$, so $d_i$ and $d_o$ cannot both be positive. Here they are, so the mirror is concave.
∘ Image is in front of mirror (same side as object).

**(5)** $d_i, d_o > 0 \Rightarrow m < 0 \Rightarrow h_i < 0$. Image is inverted.

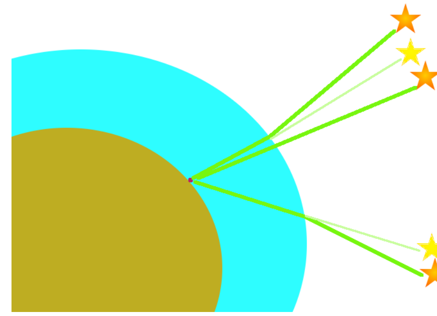
**(7)** Multiple reflections from the differently angled ripples.

**(12)** Here



**(16)** No. From eqn. 1, if $n_2 > n_1$, $\sin\theta_2 < \sin\theta_1$. Therefore, $\theta_2 < \theta_1$, and so $\theta_2 \not> 90°$.

**(18)** (a) Bend toward vertical because the light is going from a less dense medium (interstellar vacuum) to more dense (air).
(b) The positions shift slightly toward *your* vertical:

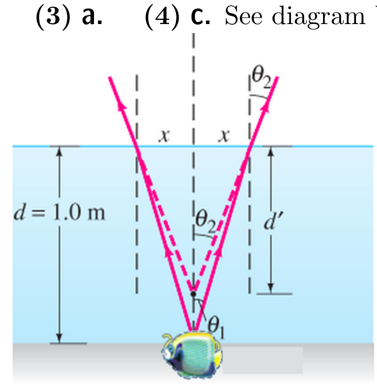


The thick lines are the actual light paths leading from the actual stars to your actual eye. The thinner lines lead to where the stars appear to be.

**(21)** No. Because real rays have to *converge* to a focus. Another reason: we need $d_i > 0$ for a real image made by a diverging lens (since the rays would need to focus on the opposite side of the lens from their original direction), but $f < 0$ for a diverging lens (convention), and $d_o > 0$. So $d_i \not> 0$.

### Misconceptuals

**(1) a.** **(2) c.** **(3) a.** **(4) c.** See diagram below.




**(5) e.** Reason: $n_2 > n_1$ (light bends toward normal in $1 \to 2$), $n_2 > n_3$ (light away from normal in $2 \to 3$). Also, $n_1 \sin\theta_1 = n_2 \sin\theta_2 = n_3 \sin\theta_3$, and clearly $\theta_3 > \theta_1 > \theta_2$. So we must have, $n_2 > n_1 > n_3$.

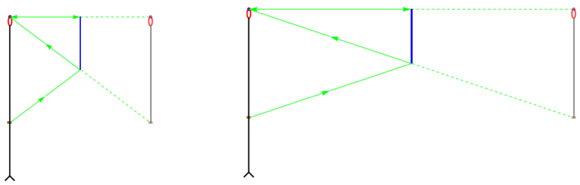
**(6) a.** Reverse the diagram for question 12 (imagine that light is not coming to your eye, but a laser beam is shooting from it).

**(7) b.** **(8) c.** **(9) e.** **(10) e.** **(12) c.** **(13) e.**

**(14) a.** From eqn. $\hat{4}$, If $d_o > f$, $d_i > 0$. If $d_o < f$, $d_i < 0$. So, as $d_o$ goes from outside $f$ to inside, $d_i$ goes from positive to negative, and $m$ goes from negative to positive (see eqn. 2). I.e., the image goes from inverted to upright. The "large" business arises from eqn. $\hat{2}$. For values of $d_o$ close to $f$ the denominator will be small and $m$ will be large.

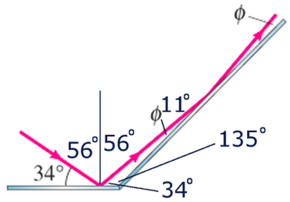
## Problems

**(1)** You're gazing at yourself in a mirror, thinking "Lookin' good from near; lookin' better from afar." In both cases, your eye (purple dot on top of your head) can see from the top of you till your belt buckle.



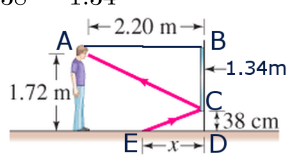
**(2)** 6.2 m. Explanation: Your image is 3.1 m behind the mirror, therefore 6.2 m from you. Alternatively, you have to focus on the distance that the light rays travel to get from the object (you) to the camera; in this case they travel 3.1 m to the mirror, then 3.1 m back.

**(3)** $\phi = 180° - (135° + 34°) = 11°$. See figure:




**(4)** In the figure below, $\angle ACB = \angle ECD$, so $\triangle ACB$ is similar to $\triangle ECD$. Therefore,

$$\frac{x}{0.38} = \frac{2.20}{1.34} \Rightarrow x = 0.62\,\text{m}.$$




**(8)** 37.6 cm.

**(9)** 10.5 cm.

**(10a)** 12 cm.
**(b)** From eqn. $\hat{4}$, $d_i = (38)(12)/(38 - 12) = 17.54\,\text{cm}$.
**(c)** Inverted, from eqn. 2.

**(13)** From eqn. 2, $1/2 = -d_i/3.4$. So, $d_i = -1.7\,\text{m}$ and

$$\frac{1}{f} = \frac{1}{3.4} + \frac{1}{-1.7} = -\frac{1}{3.4}\,\text{m}^{-1}.$$

So $f = -3.4\,\text{m}$ and $r = -6.8\,\text{m}$.

**(25)** $n = 3 \times 10^8 / 2.29 \times 10^8 = 1.31$.

**(26)** From the text, the values of $n$, therefore the values of $v = c/n$ are:
- Ethyl alcohol: $n = 1.36$, $v = 2.21 \times 10^8\,\text{m/s}$;
- Lucite: $n = 1.50$, $v = 2.00 \times 10^8\,\text{m/s}$;
- Crown glass: $n = 1.52$, $v = 1.97 \times 10^8\,\text{m/s}$.

**(27)** $v_{\text{water}} = c/n_{\text{water}} = 3 \times 10^8/1.33 = 2.25 \times 10^8\,\text{m/s}$.
So $v_{\text{substance}} = (0.82)(2.25 \times 10^8 = 1.85 \times 10^8\,\text{m/s}$.
$n_{\text{substance}} = 3 \times 10^8 / 1.85 \times 10^8 = 1.62$.

**(28)** $1 \sin 67° = 1.56 \sin\theta_r$. $\theta_r = \sin^{-1}(0.5901) = 36.2°$.

**(29)** $1.33 \sin 35.2° = 1 \sin\theta_r$. $\theta_r = \sin^{-1}(0.7665) = 50.1°$.

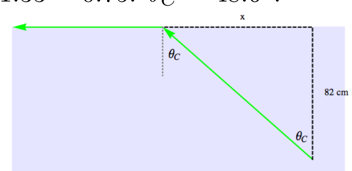
**(30)** $1.33 \sin\theta_i = 1 \sin 56.0°$. $\theta_i = \sin^{-1}(0.6233) = 38.6°$.

**(31)** $1 \sin\theta_i = 1.33 \sin 36.0°$. $\theta_i = \sin^{-1}(0.7818) = 51.4°$.
So, angle above horizon is $90° - 51.4° = 38.6°$.

**(35)** $\sin\theta_C = n_2/n_1$ where $n_1$ is the refractive index of the material the light is being totally internally reflected *from*. Here, $\sin\theta_C = 1.33/1.52 = 0.875$, So $\theta_C = 61.0°$.
The light must start in the medium with higher $n$, so crown glass.

**(36)** $\sin 47.2° = 1/n_1$. $n_1 = 1.36$.

**(37)** $\sin\theta_C = 1/1.33 = 0.75$. $\theta_C = 48.6°$.




From the diagram above $\tan\theta_C = x/82$. So $x = 82 \tan 48.6° = 93.0\,\text{cm}$.

**(41)** From eqn. 4,

$$\frac{1}{d_o} + \frac{1}{391} = \frac{1}{215} \quad \text{or} \quad \frac{1}{d_o} = \frac{1}{215} - \frac{1}{391} = \frac{391 - 215}{(215)(391)}.$$

So $d_o = 478\,\text{mm}$.

**(42a)** Converging. **(b)** $P = 1/f = 1/0.165 = 6.1$ diopters.

**(43a)** $P = 1/0.325 = 3.1$ diopters. $f > 0$, so converging.
**(b)** $f = 1/P == 1/(-6.75) = -0.15\,\text{m}$. $f < 0$, so diverging.

**(44)** From eqn. 4(after changing cm to m),

$$\frac{1}{f} = \frac{1}{1.55} + \frac{1}{0.483} = 2.72\,\text{m}^{-1}. \quad \text{So} \quad f = 0.37\,\text{m}.$$

Converging lens, because $f > 0$. Image is real.

**(45)** I'll work in mm. From eqn. $\hat{4}$,
**(a)** $d_o = 10^4\,\text{mm}$: $d_i = 106.1\,\text{mm}$.
**(b)** $d_o = 3 \times 10^3\,\text{mm}$: $d_i = 108.8\,\text{mm}$.
**(c)** $d_o = 10^3\,\text{mm}$: $d_i = 117.3\,\text{mm}$.
**(d)** From eqn. 4, the min value for $d_o$ comes from the max value for $d_i$ of 132 mm. Using that, the min value for $d_o = 513.3\,\text{mm}$, or 0.5133 m.

**(47a)** From eqn. $\hat{4}$,

$$d_i = \frac{(16)(28)}{16 - 28} = -37.3\,\text{cm}.$$

$d_i < 0$, image is behind the lens and upright.
**(b)** $m = -d_i/d_o = -(-37.3)/16 = 2.3$.