Dimensional Analysis "Lab"

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You're used to the idea that you must express physical quantities using the correct units: forces in newtons, magnetic fields in tesla, and so forth (in the SI system). While it's convenient for almost every quantity to have its own unit, these units can all be reduced to four fundamental ones: the units for mass, length, time, and charge, represented abstractly by M, L, T, and Q, respectively.[†‡] So we'll say that the unit of mass, m, is M (where M would be in kg if we are using the SI system, or g if we are using cgs).

Expressing the units of a quantity abstractly in terms of these four fundamental ones is called getting the **dimensions** of the quantity. If X is a physical quantity, we'll use the notation that $[X]$ is the dimension of X in terms of M, L, T, Q. If there are numerical factors in the formula for a quantity, such as π, $1/2$, etc., we simply omit them from the dimensions, since they are dimensionless. As an example, the formula for kinetic energy is $\frac{1}{2}mv^2$, and $v = d/t$. So, since $[d] = \mathsf{L}$ and $[t] = \mathsf{T}$, we'd write

$$[KE] = \mathsf{M}\,\mathsf{L}^2\mathsf{T}^{-2}$$

because

$$[v^2] = [d^2/t^2] = \mathsf{L}^2/\mathsf{T}^2 = \mathsf{L}^2\mathsf{T}^{-2}.$$

You may need to look up in the textbook the definitions of work, momentum, etc., as you go through the list below. At several points we'll use the abbreviation PE for potential energy. Other abbreviations should be obvious. The list builds step by step. For example, once you get the dimensions of acceleration, a, you can use that to get the dimensions of F (using $F = ma$). Once you know the dimensions of F you can use that in all formulas that use F, such as the one for work ($W = F \cdot d$).

OK, let's have at it:

Mechanics

Velocity: $[v] =$

Acceleration: $[a] =$

Force: $[F] =$

A Newton (N), being a unit of force, has the same dimensions.

Kinetic energy: $[KE] =$

Grav. PE: $[PE_{\text{grav}}] =$

Work: $[W] =$

Momentum: $[p] =$

Electricity

Electric field: $[E] =$

Coulomb's const.: $[k] =$

Permittivity: $[\epsilon_0] =$

Electric PE: $[PE_{\text{elec}}] =$

Current: $[I] =$

Voltage/emf: $[V] =$

Resistance: $[R] =$

Magnetism

Magnetic field: $[B] =$

Permeability: $[\mu_0] =$

Electromagnetism

Speed of EM rad.: $[1/\sqrt{\epsilon_0\mu_0}] =$

NOTES:

† Technically, current is used instead of charge as a basic quantity, with charge defined as $I \cdot t$, but it's more intuitive to use charge as fundamental.

‡ There are three further basic units to express temperature, luminosity and "amount" (moles), but they are not relevant to E&M.