Arvind Borde / PHY 11, Week 8: Rotational Motion

When we discussed centripetal acceleration, we looked at objects moving in a circle at fixed speed.

We'll study rotational motion from a more general point of view here.

In general, rotational motion is related to linear: the wheels on a car go ________________ and the car moves forward ______________.

1

The magnitudes of *kinematical* rotational quantities are related to those of linear ones through multiplication by the radius:

Displacement: $d = r\theta$

Speed/velocity: $v = r\omega = r\dfrac{\Delta\theta}{\Delta t}$

Acceleration: $a_T = r\alpha = r\dfrac{\Delta\omega}{\Delta t}$

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In each case the same word is used in the rotational world with a "rotational" or "angular" tagged onto the linear word.

For example:
ω: "rotational velocity" or "angular velocity."

If a new word is needed, it is explicitly stated.

3

When the equations $d = r\theta$, $v = r\omega$ hold, where v is the total linear velocity of an object, we call it "rotation without slipping."

____
____

Radians are unitless (they are ratios of lengths). Thus d and r above will have the same units, and, since v is in m/s (SI), w is in ____________.

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Conversions (based on $360^\circ \sim 2\pi$ – full circle):

Degrees to radians: $[\theta^\circ] \times \dfrac{\pi}{180^\circ}$

Radians to degrees: $[\theta_{\text{rad}}] \times \dfrac{180^\circ}{\pi}$

(1) Convert 15° to radians. ________________

(2) Convert $15\,$rad to degrees. _______________

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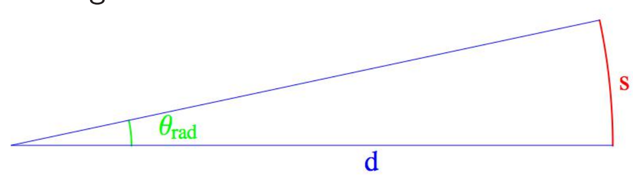
There's a relationship between the radian measure of an angle and distance:



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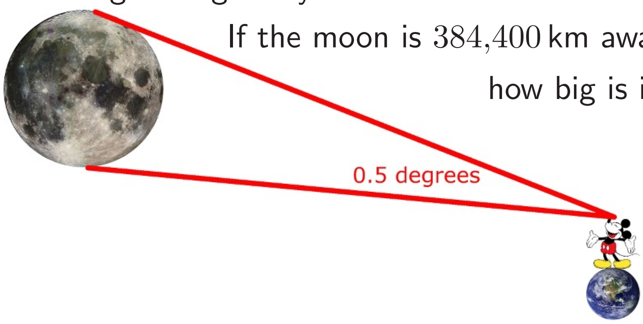
ADDITIONAL NOTES

(3) You are gazing at the moon. It occupies a half degree angle in your field of view as shown. If the moon is $384{,}400$ km away, how big is it?



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Assume rotation without slipping below.

(4) If your car has wheels of radius 30 cm, how far does it move over one full rotation of the wheels?

$d =$

(5) If your wheels are spinning at 2000 rpm how fast are you traveling?

$\omega =$

$v =$

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The angular or rotational acceleration is tangent to the direction of motion. This is different from the centripetal acceleration, which is ______:

$$a_C = \frac{v^2}{r} = \frac{r^2\omega^2}{r} = \omega^2 r$$

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The ______________ of rotational motion is

$$f = \frac{\omega}{2\pi}$$

measured in rev/sec, also called Hertz (Hz): 1 Hz equals 1 rev/sec. It follows that $\omega = 2\pi f$.

The ______________ of rotational motion (time for one revolution) is $T = \dfrac{1}{f}$

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The equations that govern rotational motion under ______________________ mirror their linear counterparts (initial displacement zero):

A] ______________ ______________

B] ______________ ______________

C] ______________ ______________

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(6) Starting from rest, if your car wheels reach an angular speed of 120 rad/s in 1 min, what is the angular acceleration?

Use $\omega =$ ______________.

So ______________________,

or $\alpha =$ ______________.

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ADDITIONAL NOTES

The relationship of *dynamical* linear quantities to their rotational counterparts is not always so simple, but here's a list of the two sets:

Linear		Rotational	
Inertia:	m,	Moment of inertia:	$I\ (= mr^2)$
Force:	F,	Torque:	τ
Momentum:	p,	Angular momentum:	L

The last two are vectors.

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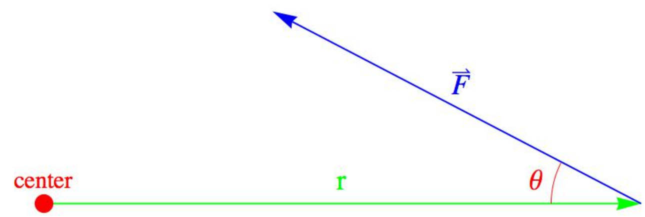
Torque is the rotational version of force:

$$\tau = rF\, sin\, \theta$$



The greater the torque, the "easier" it is to "make rotation."

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(7) You push a $120\,\text{cm}$ door with a $35\,\text{N}$ force perpendicular to it. What's the torque exerted if
(a) you push furthest from the hinge?

____

(b) you push at the midpoint?

____

(c) you push at the hinge?

____

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(8) With the numbers in the previous problem what is the torque if push furthest from the hinge, but at an angle (with the door) of

(a) $60°$? ______________________________

(b) $1°$? ______________________________

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Just as the inertia of a body is a measure of its resistance to force, the moment of inertia of a body is a measure of its resistance to ______________.

The formula given above, $I = mr^2$, is for a point mass a distance r from the center of rotation.

For a solid object, the distribution of the mass plays an important role.

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Think of an extended object as being made up of many small masses.

Suppose you have two objects of the same shape, size and total mass, but object A has most of its mass concentrated at its center, and object B has most of its mass at its edges.

(9) Which do you expect to have the greatest I?

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ADDITIONAL NOTES

Within the rotational world the relationship of rotational quantities parallels that of linear ones:

$p = mv$:	$L = I\omega$
$F = ma$:	$\tau = I\alpha$
$F = \Delta p/\Delta t$:	$\tau = \Delta L/\Delta t$
$KE = \frac{1}{2}mv^2$:	$KE_{\text{rot}} = \frac{1}{2}I\omega^2$
$W = Fd$:	$W = \tau\theta$
$P = \Delta W/\Delta t$:	$P = \tau\theta/\Delta t = \tau\omega$

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The total KE of a moving object is a sum of linear and rotational:

$$KE_{\text{total}} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

As with linear momentum, the angular momentum, $L = I\omega$, of an isolated system is conserved.

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(10) What's the KE_{rot} of an object with $I = 3 \times 10^{-3}$ kg·m^2 that's spinning at 8000 rpm?

Note: It's common to be given rotational speeds in rpm. You must convert to rad/s.

$\omega =$

$\text{KE}_{\text{rot}} =$

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ADDITIONAL NOTES