Arvind Borde / PHY 11, Week 3: Kinematics in 2d

(1) What's a scalar, and what's an example?

____

Examples: ______________________________

(2) What's a vector, and what's an example?

____

Examples: ______________________________

1

Representing Scalars

That's simple. A scalar is simply expressed as a number (with units):

My mass is $82\,\text{kg}$.

My height is $1.8\,\text{m}$.

2

Representing Vectors

We'll represent vectors with arrows:

$$\leftarrow \quad \nearrow \quad \uparrow \quad \swarrow \quad \downarrow$$

The length of the arrow will represent magnitude:

$$\vec{a} : \rightarrow \qquad \vec{b} : \longrightarrow$$

(3) If $\vec{a}$ and $\vec{b}$ represent forces, which will be the

3 bigger force? __________

Comparing vectors

(4) Are these two vectors equal? Reason?

$$\leftarrow \qquad \longleftarrow$$

____

(5) Are these two vectors equal? Reason?

$$\nearrow \qquad \nwarrow$$

____

4

(6) Are these two vectors equal? Reason?

$$\nwarrow \qquad \nwarrow$$

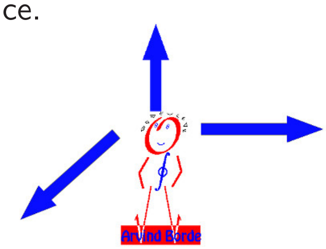
____

____

____

5

Vectors can be added and subtracted. In order to do that we'll move them around, even if the physical quantities they represent are "located" at a specific place.



Net force?

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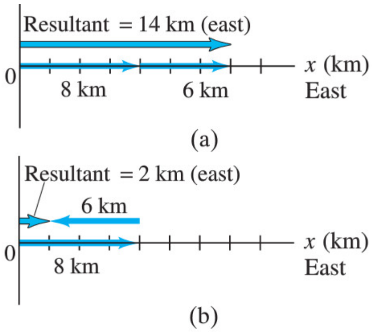
ADDITIONAL NOTES

1-d



7

(7) Add a vector of magnitude 4.5 units pointing North to one of magnitude 6 units pointing North.

____

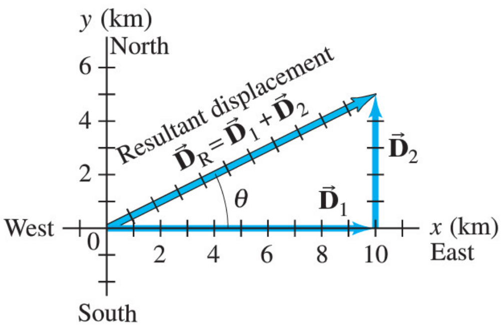
(8) Add a vector of magnitude 4.5 units pointing North to one of magnitude 6 units pointing South.

____

8

2-d: perpendicular vectors (⊥)

Arrange the vectors "tail to nose."



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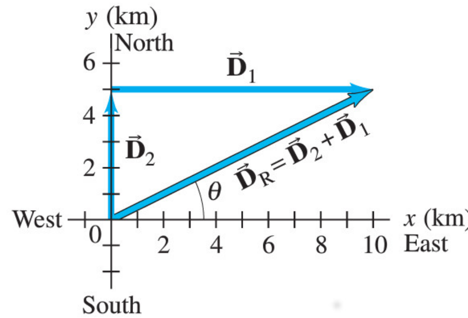
If the vectors are added in reverse order, the resultant is the same.



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The sum of two vectors, perpendicular or not, is called the ________________

We need to know its magnitude and the direction.

You will choose which vector points in the y direction ($\vec{V}_y$), and which in the x ($\vec{V}_x$).

In our example, $\vec{V}_y = \vec{D}_2$, and $\vec{V}_x = \vec{D}_1$.

11

The direction of the resultant here is:

which gives $tan\,\theta = 1/2$ in this example (or $\theta =$), and the magnitude by

$$D_R = \sqrt{D_1^2 + D_2^2} = \sqrt{(10.0\text{ km})^2 + (5.0\text{ km})^2}$$
$$= \sqrt{125\text{ km}^2} = 11.2\text{ km}.$$

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You can use the Pythagorean theorem only when the vectors are *perpendicular* to each other.

ADDITIONAL NOTES

(9) $\vec{A} \perp \vec{B}$. $\vec{A}$ has magnitude 4.0 units and $\vec{B}$ has magnitude 7.3 units. What is the magnitude and direction of their sum?

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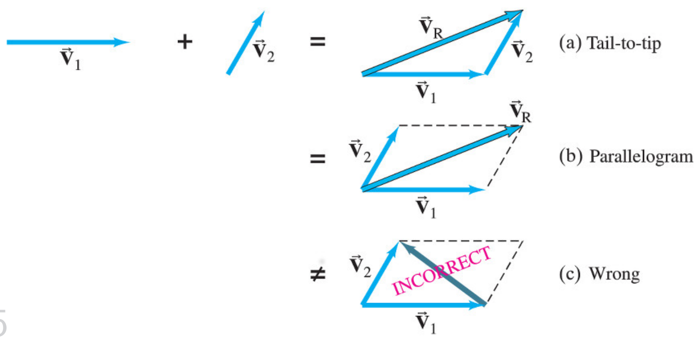
The magnitude is

14

2-d: non-perpendicular vectors

Example:



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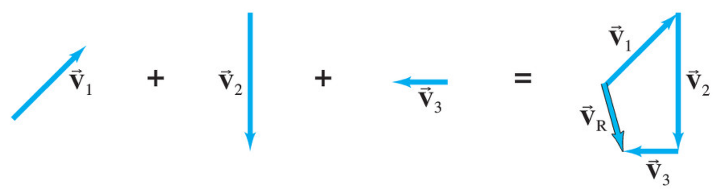
More examples:



Pictures are good, but *components* are better.

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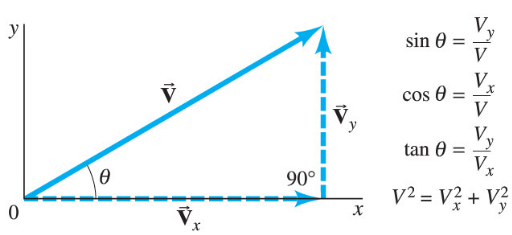
Getting the components of a vector



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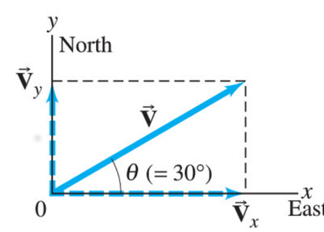
If the vector in the digram below represents a displacement of 500 m in the direction shown, what are its components V_x and V_y?



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ADDITIONAL NOTES

$V_x = V\,cos\,\theta = 500\,cos\,30^\circ =$

$V_y = V\,sin\,\theta = 500\,sin\,30^\circ =$

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Adding with components

If two vectors $\vec{V}_1$ and $\vec{V}_2$ are added, what are the components of the resultant?

20

(**10**) What are the components of the resultant when a vector of magnitude 10 units making an angle of 23° with the x axis ($\vec{V}_1$) is added to a vector of magnitude 6 units making an angle of 57° with the x axis ($\vec{V}_2$)?

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$V_{1x} =$ $V_{1y} =$

$V_{2x} =$ $V_{2y} =$

Therefore,

$V_{Rx} =$

$V_{Ry} =$

22

(**11**) What is the magnitude of the previous resultant, and what angle does it make with the x-axis?

$V_R =$

$tan\,\theta =$

$\theta =$

23

(**12**) What are the components of the resultant when a vector of magnitude 3 units making an angle of 43° with the x axis ($\vec{V}_1$) is added to a vector of magnitude 16 units making an angle of 75° with the x axis ($\vec{V}_2$)?

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ADDITIONAL NOTES

$V_{1x} =$ $V_{1y} =$

$V_{2x} =$ $V_{2y} =$

Therefore,

$V_{Rx} =$

$V_{Ry} =$

25

(13) What is the magnitude of the previous resultant, and what angle does it make with the x-axis?

$V_R =$

$tan\,\theta =$

$\theta =$

26

(14) Which of $\vec{V}_1$ or $\vec{V}_2$ is the resultant, $\vec{V}_R$, more similar to in magnitude and angle with the x-axis? __.

(15) Why? ____________________
____
____

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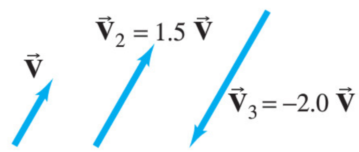
Multiplying vectors by scalars

Multiplying a vector $\vec{\mathbf{V}}$ by a scalar c gives a vector whose magnitude is c times greater and in the same direction as $\vec{\mathbf{V}}$ (or opposite direction if c is negative).



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Multiplying vectors by vectors

It can be done — in two ways:

a) get a scalar answer ("dot product");

b) get a vector answer ("cross product").

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Motion in two dimensions

Time to put these vectors to work for us in 2d motion.

The basic approach is that you treat the x- and y-motions separately, by taking components.

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ADDITIONAL NOTES

TABLE 3–1 General Kinematic Equations for Constant Acceleration in Two Dimensions

x component (horizontal)		y component (vertical)
$v_x = v_{x0} + a_x t$	(Eq. 2–11a)	$v_y = v_{y0} + a_y t$
$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$	(Eq. 2–11b)	$y = y_0 + v_{y0} t + \frac{1}{2} a_y t^2$
$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$	(Eq. 2–11c)	$v_y^2 = v_{y0}^2 + 2a_y(y - y_0)$

We can simplify Eqs. 2–11 to use for projectile motion because we can set $a_x = 0$. See Table 3–2, which assumes y is positive upward, so $a_y = -g = -9.80\,\mathrm{m/s^2}$.

TABLE 3–2 Kinematic Equations for Projectile Motion (y positive upward; $a_x = 0$, $a_y = -g = -9.80\,\mathrm{m/s^2}$)

Horizontal Motion ($a_x = 0$, v_x = constant)		Vertical Motion† ($a_y = -g$ = constant)
$v_x = v_{x0}$	(Eq. 2–11a)	$v_y = v_{y0} - gt$
$x = x_0 + v_{x0} t$	(Eq. 2–11b)	$y = y_0 + v_{y0} t - \frac{1}{2} g t^2$
	(Eq. 2–11c)	$v_y^2 = v_{y0}^2 - 2g(y - y_0)$

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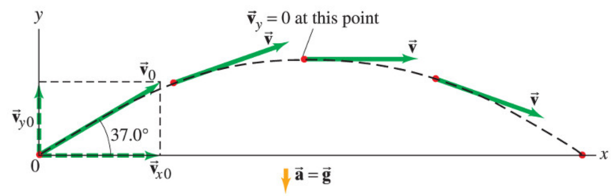
Ok, let's get some 2d action:



A kicked football. A kicked football leaves the ground at an angle $\theta_0 = 37.0°$ with a velocity of 20.0 m/s, as shown . Calculate (*a*) the maximum height, (*b*) the time of travel before the football hits the ground, and (*c*) how far away it hits the ground. Assume the ball leaves the foot at ground level, and ignore air resistance and rotation of the ball.

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Step 1: Resolve initial velocity into components:

$v_{x0} =$

$v_{y0} =$

33

Step 2: Get maximum height:

(16) Vertical velocity, v_y, at the top? ______

(17) Knowing the initial and final vertical velocities, which equation gives you the vertical distance, $h = y - y_0$, traveled? ___

(18) Do it.

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Step 3: Get time to return to ground.

(19) What are heights at the start and the return?

____

(20) Which eqn. gives you t from y, y_0, v_{y0}? ___

(21) Do it.

35

Step 4: Get total horizontal distance.

(22) With $x_0 = 0$, $v_{x0} = 16.0\,\mathrm{m/s}$, and $t = 2.5\,\mathrm{s}$, which eqn. gives x? ___

(23) Do it.

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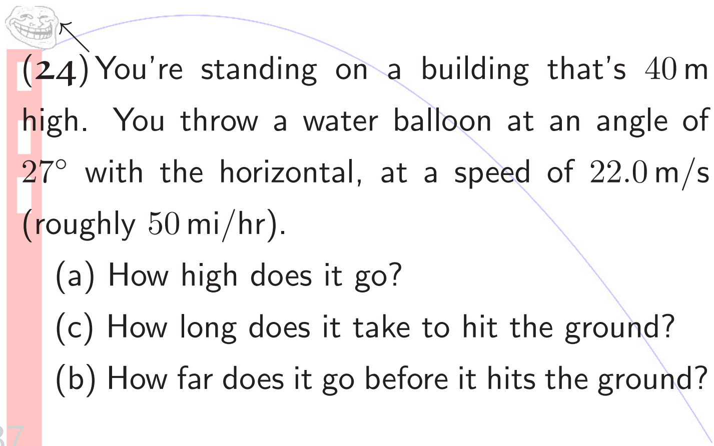
ADDITIONAL NOTES



(24) You're standing on a building that's $40\,\text{m}$ high. You throw a water balloon at an angle of 27° with the horizontal, at a speed of $22.0\,\text{m/s}$ (roughly $50\,\text{mi/hr}$).

(a) How high does it go?

(c) How long does it take to hit the ground?

(b) How far does it go before it hits the ground?

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Step 1: Resolve initial velocity into components:

$v_{x0} =$

$v_{y0} =$

38

Step 2: Get maximum height:

39

Step 3: Get time to get to the ground.

40

(25) Which of these ts is relevant to the balloon hitting the ground after launch? ________

(26) What's with the negative answer?

____

____

____

41

Step 4: Get total horizontal distance.

42

ADDITIONAL NOTES

(27) If you threw your balloon *horizontally* from the 40 m building at a speed of 27 m/s, get a formula for how far it would drop vertically after t s.
The correct equation is ___
We have $v_{x0} =$ ________ and $v_{y0} =$ _______

So, $y =$

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(28) In the previous situation, would it make a difference to the formula for y had you thrown the ballon at a horizontal speed of 2.7 m/s? ______ $2{,}700{,}000$ m/s? ______ 0 m/s? ________

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ADDITIONAL NOTES