Arvind Borde
AST10: Week 11 HW

Solutions

1a) If $H_0 = \dot{a}(t)/a(t)$, then $\diamondsuit$ becomes

$$3H_0^2 + \frac{3k}{a^2(t)} = \frac{8\pi G}{c^4}\rho.$$

1

1b) Solving for ρ:

$$\begin{aligned}\rho &= \frac{c^4}{8\pi G}\left(3H_0^2 + \frac{3k}{a^2(t)}\right) \\ &= \frac{c^4}{8\pi G}(3H_0^2) + \frac{c^4}{8\pi G}\left(\frac{3k}{a^2(t)}\right).\end{aligned}$$

1c) When $k = 0$

$$\rho_c = \frac{c^4}{8\pi G}(3H_0^2).$$

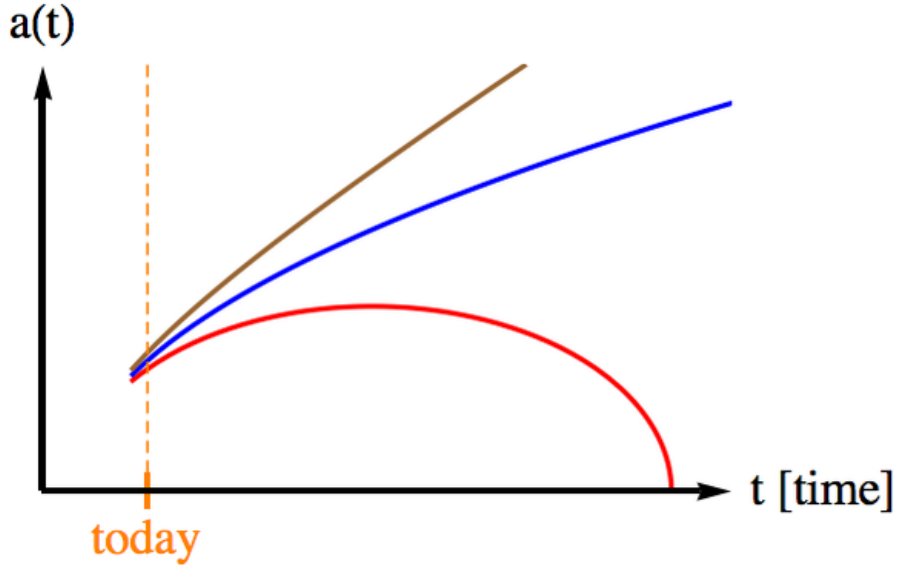
So we can write

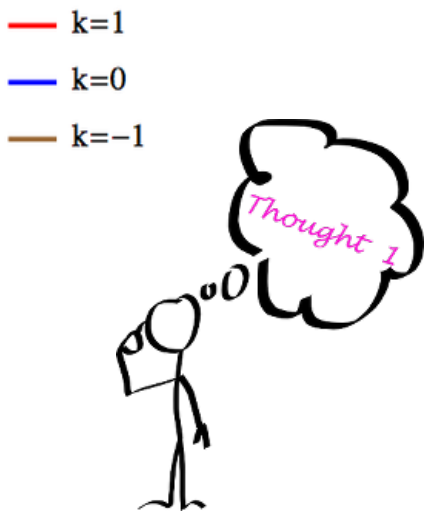
$$\rho = \rho_c + \frac{c^4}{8\pi G}\left(\frac{3k}{a^2(t)}\right).$$

1d) When $k = 1$, $\rho > \rho_c$.

1e) When $k = -1$, $\rho < \rho_c$.

ρ_c is called the critical density of the Universe.
It is the dividing density between
a closed Universe $(k = 1)$ and
an open Universe $(k = 0, -1)$.





2) If $P = -\rho$, then $\rho + 3P = \rho - 3\rho = -2\rho < 0$ (assuming $\rho > 0$).

Then $\heartsuit$ becomes

$$\frac{3\ddot{a}(t)}{a(t)} = -\frac{4\pi G}{c^4}(-2\rho) = \frac{8\pi G\rho}{c^4} > 0.$$

The graph of $a(t)$ would be concave up.

It would no longer be *necessary* that the Universe had a beginning.

3) Static: Fixed; nothing changes.

Steady State: Universe can evolve (and expand) but looks the same at all times.

4) The nature of quasars suggests that the early Universe was very different from the present one.