

AST 10

# Introductory Astronomy II Workbook

**Arvind Borde**

This workbook contains work done by: \_\_\_\_\_

Signature: \_\_\_\_\_



# Introductory Astronomy II

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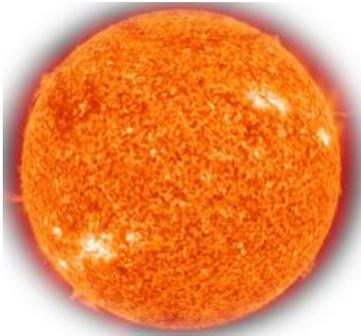
# Arvind Borde / AST 10, Week 1: A Tour of the Universe



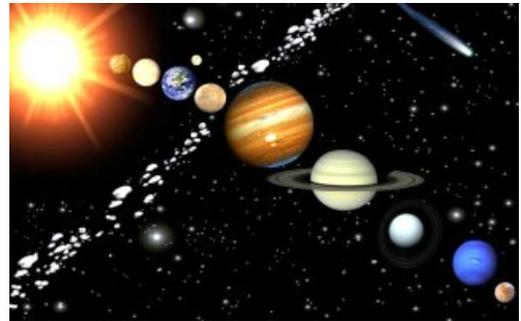
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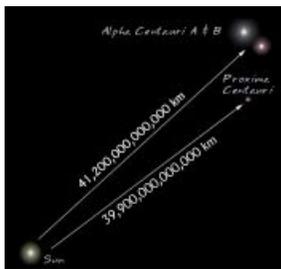
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## ADDITIONAL NOTES

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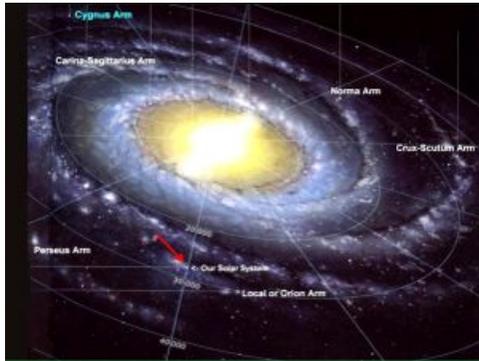
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We believe the Milky Way has between \_\_\_\_\_ stars.

But stars are not the only things we see in the night sky. We also see \_\_\_\_\_.

Seven of these were recorded by Ptolemy, a Greco-Egyptian astronomer and mathematician, in his book the *Almagest* around 130 A.D.

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(1) Does this distance allow the Orion nebula to be in the Milky Way, or force it to be outside it?

\_\_\_\_\_  
\_\_\_\_\_

10



Closeup of the Orion Nebula: It's a

11

Till the 1920s, it was not known if all nebulae were within, or close to, the Milky Way.

The debate on this question was called \_\_\_\_\_

Here's another nebula, first recorded as "a little cloud" by an Arab astronomer, Al Sufi, in his *Book of Fixed Stars* around 964 A.D.

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ADDITIONAL NOTES

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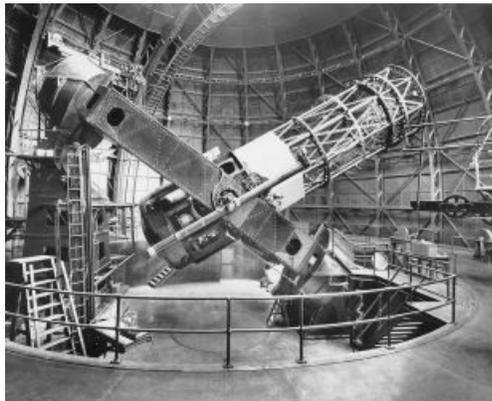
The Andromeda nebula

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The American astronomer \_\_\_\_\_ systematically studied nebulae in the 1920s.



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By studying stars of variable brightness, \_\_\_\_\_, in Andromeda, Hubble found evidence that it was about 9 million million million miles away.



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(2) Why is this significant? \_\_\_\_\_

\_\_\_\_\_

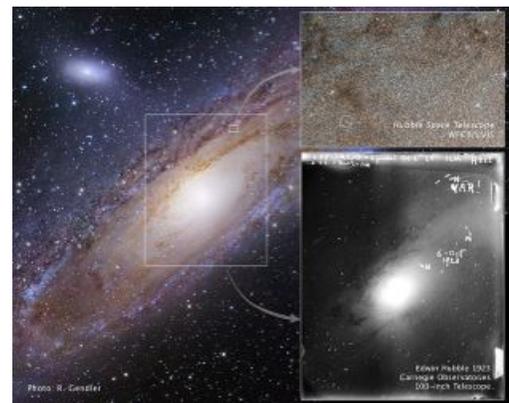
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ADDITIONAL NOTES

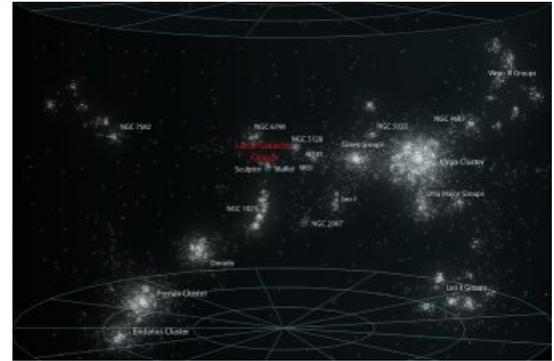
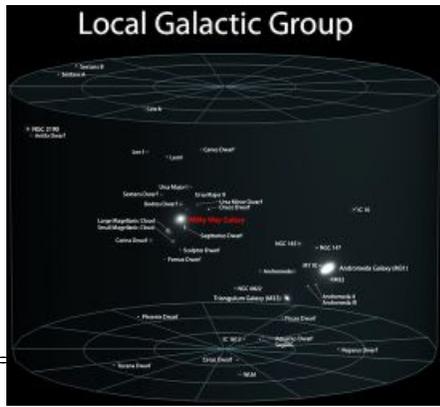
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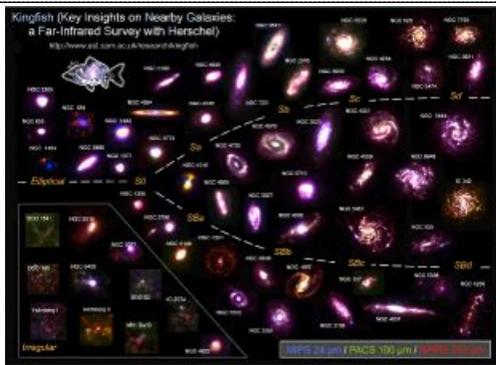
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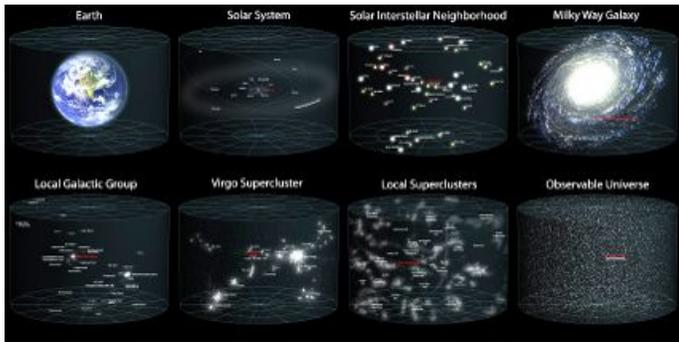


Lots and lots of galaxies.

According to present estimates, there are at least \_\_\_\_\_, possibly up to a \_\_\_\_\_ galaxies.

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Scale

Where we stand.

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ADDITIONAL NOTES

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How do we know distances in astronomy?

For the nearby, rocky planets we simply bounce radio waves off them.

(3) If the distance to such a planet is  $d$ , the speed of radio waves is  $c$ , and the time taken for a signal to go there (or return) is  $t$ , how are these three related?

25 In practice, we measure round trip time,  $2t$ , then halve it.

(5) What are these men up to?



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We then solve equation (1) for  $b_1$  and work it into equation (2), using equation (3) to assist

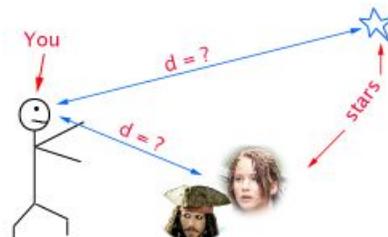
$$b_1 = \frac{d}{\tan \theta_1},$$

and so

$$\begin{aligned} d &= (b - b_1) \tan \theta_2 \\ &= \left( b - \frac{d}{\tan \theta_1} \right) \tan \theta_2. \end{aligned}$$

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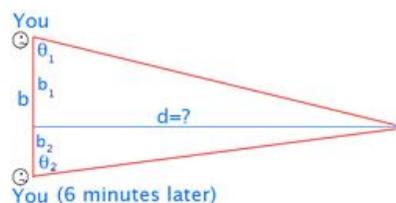
What about other objects? How do we know how far the sun is, or the stars?



(4) Bouncing signals won't work. Why?

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This is what you'd do:



$$\tan \theta_1 = d/b_1 \quad \text{or} \quad d = b_1 \tan \theta_1, \quad (1)$$

$$\tan \theta_2 = d/b_2 \quad \text{or} \quad d = b_2 \tan \theta_2, \quad (2)$$

28 and

$$b_1 + b_2 = b \quad \text{or} \quad b_2 = b - b_1. \quad (3)$$

Then solve for  $d$ :

$$d = b \tan \theta_2 - \frac{d}{\tan \theta_1} \tan \theta_2.$$

$$d = \frac{b \tan \theta_2}{1 + (\tan \theta_2 / \tan \theta_1)}.$$

(That's how you perceive depth: "parallax.")

This formula works on earth and off it.

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ADDITIONAL NOTES

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(6) If there were no apparent shift in the position of the object, how would  $\theta_1$  and  $\theta_2$  be related?

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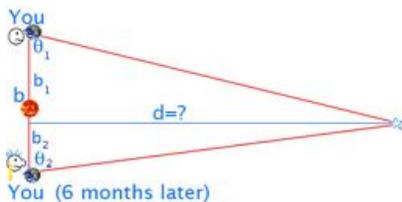
(7) If there were no apparent shift, what would you get for  $d$ ? =====

For objects that are far away, if the baseline is too short, you cannot measure the apparent shift (“parallax”).

If you want to use this method to get distances to the stars, you need a long enough baseline,  $b$ , to give parallax.

(8) So, what do we use?

31



$$d = \frac{b \tan \theta_2}{1 + (\tan \theta_2 / \tan \theta_1)}$$

This gives the distance from the earth’s orbital plane to nearby stars. But, how do we know  $b$ , the diameter of the earth’s orbit?

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Once we know that, we can get distances to the most distant planets, and to nearby stars.

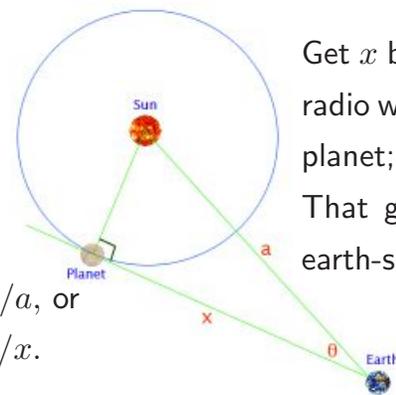
Trigonometry gives us the first rungs in what we call the “Cosmic Distance Ladder.”

Other rungs use “standard candles” – comparing the known (or \_\_\_\_\_) brightness of an object to its \_\_\_\_\_

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Trigonometry rides to the rescue (again):

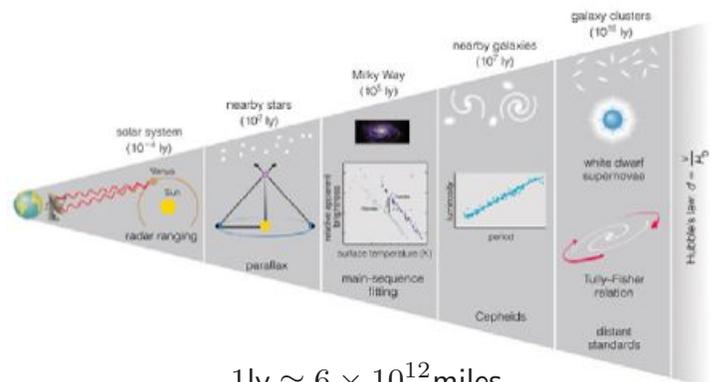


Get  $x$  by bouncing radio waves off the planet; measure  $\theta$ . That gives  $a$ , the earth-sun distance.

$$\cos \theta = x/a, \text{ or}$$

$$a = \cos \theta / x.$$

34



1ly  $\approx 6 \times 10^{12}$  miles  
(6 trillion miles)

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ADDITIONAL NOTES

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\_\_\_\_\_

We use the metric system and decimals over fractions, for the most part, and powers of ten to express quantity. Occasionally we will introduce a new word, such as “light-year.” That’s the

\_\_\_\_\_

Another common distance measure, more useful within the solar system, is the “AU” short for

\_\_\_\_\_

37

Express as decimals

(9)  $3/10$  \_\_\_\_\_

(10)  $4/5$  \_\_\_\_\_

(11)  $1/3$  \_\_\_\_\_

(12)  $3/100$  \_\_\_\_\_

38

Powers of ten: the basics

(13) What do we *choose*  $10^5$  to represent?

\_\_\_\_\_

(14) What do we *choose*  $10^7$  to represent?

\_\_\_\_\_

39

In that spirit

(15) What do we *choose*  $10^{-5}$  to represent?

We *choose* this, because it’s convenient, to define

\_\_\_\_\_

(16) What is  $10^{-2}$ ?

40

Multiplying powers of ten

(17) If you multiply powers of ten, e.g.,  $10^7 \times 10^5$ , what answer do you get?

\_\_\_\_\_

What are

(18)  $10^{13} \times 10^{15}$ ? \_\_\_\_\_

(19)  $10^{-3} \times 10^5$ ? \_\_\_\_\_

41

Dividing powers of ten

(20) If you divide powers of 10, e.g.,  $10^7 \div 10^5$ , what answer do you get?

\_\_\_\_\_

What are

(21)  $10^{13} \div 10^{15}$ ? \_\_\_\_\_

(22)  $10^3/10^{-5}$ ? \_\_\_\_\_

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ADDITIONAL NOTES

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One final definition:  $10^0 =$  \_\_\_\_\_.

These definitions & rules are chosen to give consistent answers: all roads lead to the same power-of-ten-heaven. For example, what is  $10^2 \times 10^{-2}$ ?

- 1) Add powers: \_\_\_\_\_.
- 2) View negative power as a reciprocal:

43

Distances and masses are the key to understanding the Universe, versus, say, shapes, colors, etc.?

Why?

To answer this we need ...

44

**... A crash course in the world**

The whole world consists of two entities:  
\_\_\_\_\_ and \_\_\_\_\_.

Examples of matter are your chairs, your bodies, and the stars.

Examples of interactions are the \_\_\_\_\_  
\_\_\_\_\_.

45

These four are the only known interactions, also called forces.

Unlike the fundamental forces, it may appear that matter is more complicated. Take our bodies.

(23) What are we mainly made of?

\_\_\_\_\_.

But there's other stuff: bones, flesh, hair, etc.

46

Similarly, if you look around the room, you'll see many different substances.

It's been known since the 1800s that the complexity of the material world is based on just a few basic things combining in different ways. These "basic things" are called \_\_\_\_\_.

47

(24) Name some elements.

\_\_\_\_\_

(25) Roughly how many elements are there?

\_\_\_\_\_  
\_\_\_\_\_

(26) Elements come in basic "pieces." What are they called? \_\_\_\_\_

48

ADDITIONAL NOTES

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But that is not the end of the story. Each atom has structure and is itself made up of three more basic things.

(27) What are the constituents of an atom called?

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49

Protons and neutrons form the “nucleus” of the atom, and electrons swirl in a cloud around it.

Protons have positive electric charge, electrons an equal negative charge and neutrons are neutral. The electron cloud is “held in place” by the electric forces between them and the protons.

50

The simplest atom is that of hydrogen. It consists of a single proton and a single electron.

The nucleus is roughly  $10^{-13}$ cm in radius and the electron cloud about  $10^{-8}$ cm.

(28) That’s factor of about 100,000. Why?

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51

Can you subdivide further?

Not for electrons: they appear to have no internal structure.

But there’s one step further for protons and neutrons: they have internal constituents called

=====.

52

At the micro level both matter and interactions are represented by particles, called \_\_\_\_\_. (That’s the plural. The singular is quantum.)

There are \_\_\_\_\_ and \_\_\_\_\_, distinguished by their \_\_\_\_\_. (Like a top.)

=====

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53

**All the known quanta**

Fermions (matter)

6 “quarks”:  
up, down, charm, strange, top, bottom  
(combinations give the proton, neutron, etc.)

6 “leptons”:  
electron, muon, tau and their neutrinos

54

ADDITIONAL NOTES

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\_\_\_\_\_

\_\_\_\_\_

Bosons (interactions)

Electromagnetism: Photon

Strong: 8 Gluons (hold the nucleus together)

Weak:  $W^+$ ,  $W^-$ ,  $Z^0$  (radioactive decay)

Gravitation: Graviton???????

These 24-odd particles make up the world and all its interactions, along with a final particle called the Higgs boson.

55

The weak and strong forces are short range. They drop to zero outside the nucleus. They play no direct role in the structure of the solar system or the Universe.

Electromagnetism is long range, but most large objects are electrically and magnetically neutral. So this force, too, is irrelevant over large distances.

56

That leaves gravity. It and it alone determines the large scale structure of the Universe. It explains why the moon goes around the earth, why planets move around the sun. Understanding gravity is intertwined with understanding the Universe.

Gravity depends on \_\_\_\_\_ and \_\_\_\_\_. They're the two fundamental things we need to understand in order to understand the Universe.

57

\_\_\_\_\_ yet in some ways understand the least.

It's a magical force that doesn't, unlike electromagnetism or the nuclear forces, exist in the fabric of the Universe – it \_\_\_\_\_.

58

The most immediate experience we have of gravity is that things fall.

If the earth were at the center of the Universe, as was thought, one could attribute the tendency of things to fall as their natural tendency to go to the center of the Universe because of their weight. That was the view of Aristotle, and co. (~300 BC).

59

According to this view, the “heavens” were fixed (apart from the wandering planets) and objects fell because they were trying to get to the center of the Universe.

In the Aristotelian view heavier objects would fall more quickly.

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ADDITIONAL NOTES

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Galileo spoiled all that in the 1600s by dropping things.

Around the same time it became clear that the “heavens” were more complicated than had been thought: planets had moons that went around them, for example.

The earth was not the center of everything.

61

In the late 1600s, Isaac Newton fixed everything – the shenanigans in the heavens and why things fall to earth by

**Newton’s Law of Universal Gravitation**

$$F_{\text{grav}} = G \frac{m_1 m_2}{d^2},$$

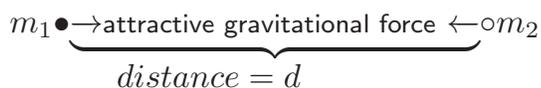
$$G = 6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}$$

Gravitational constant

62

In English:

Every object (mass  $m_1$ ) attracts every other object (mass  $m_2$ ) by a force proportional to the product of their masses and inversely proportional to the square of the distance between them.



63

Why is this a universal law? Because \_\_\_\_\_

It applies to you and the earth, to a piece of chalk and the earth, to a piece of chalk and another piece of chalk, to the earth and the moon, to the sun and Jupiter, ...

The law uses “proportionality.” What’s that?

64

If  $y = x$  then the quantity  $y$  equals  $x$ .

If  $y = 5x$  then  $y$  is \_\_\_\_\_  $x$ .

We write this as

$$y \propto x$$

This is true whenever  $y = kx$  for any fixed  $k$

(“\_\_\_\_\_”).

65

In Newton’s gravitational law

$$F_{\text{grav}} = G \frac{m_1 m_2}{d^2}$$

the quantity  $G$  is the proportionality constant.

\_\_\_\_\_ – same value for any two objects:

$$G = 6.67 \times 10^{-11} \text{N} \cdot \text{m}^2 / \text{kg}^2$$

(this is the value when you measure mass in kilograms and distance in meters).

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ADDITIONAL NOTES

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We also need \_\_\_\_\_ . If

$$y = k \times \frac{1}{x} = \frac{k}{x}$$

where  $k$  is fixed, then  $y$  is said to be inversely proportional to  $x$ .

67

Assume  $k > 0$ . If

(29)  $y = kx$ , as  $x$  goes up,  $y$  \_\_\_\_\_

(30)  $y = kx$ , as  $x$  goes down,  $y$  \_\_\_\_\_

(31)  $y = \frac{k}{x}$ , as  $x$  goes up,  $y$  \_\_\_\_\_

(32)  $y = \frac{k}{x}$ , as  $x$  goes down,  $y$  \_\_\_\_\_

68

Using Newton's law

$$F_{\text{grav}} = G \frac{m_1 m_2}{d^2}$$

(33) Does the gravitational force go up as the masses go up? \_\_\_\_\_

(34) Does the gravitational force go up as the distance increases? \_\_\_\_\_

69

We need to be more specific about *how much* the gravitational force goes up and down by.

The gravitational force on an object of mass  $m$  due to the earth (mass  $M_E$ ) is

$$F = G \frac{m M_E}{d^2}$$

where  $d$  is the distance to the *center of the earth*.

70

In each case below, does the gravitational force on  $m$  go up or down, and by how much?

(35)  $m$  doubles: \_\_\_\_\_

(36)  $m$  triples: \_\_\_\_\_

(37)  $m$  halves: \_\_\_\_\_

\_\_\_\_\_

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How the gravitational force depends on distance is *slightly* trickier:

If the distance between the two objects goes up, the force \_\_\_\_\_  
\_\_\_\_\_.

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ADDITIONAL NOTES

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# Arvind Borde / AST 10, Week 2: Our Home: The Milky Way

The Milky Way is our home galaxy. It's a collection of stars, gas and dust. About it, as for other astronomical systems, we ask: \_\_\_\_\_

\_\_\_\_\_

1

## Star System Example: The Constellations

A constellation is a collection of stars that suggested a pattern to ancient observers of the skies.

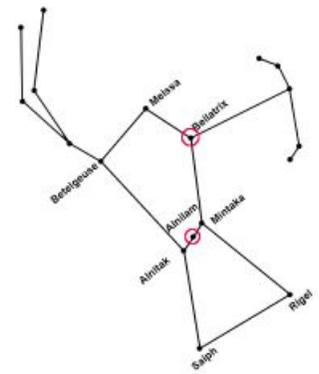
Early recordings of such patterns go back to possible astronomical markings painted on the walls in the cave system at Lascaux in southern France (17,300 years ago).

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## Example: Orion



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What different people saw (in the West):



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In Chinese astronomy, Orion lies across two of the Chinese constellations: the *White Tiger of the West*, and the *Vermilion Bird of the South*.

Native American tribes have variously seen a bison's spine, or two canoes, in the same stars.

In one Hindu myth, the stars of the "belt of Orion" are an arrow in a creation story.

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## ADDITIONAL NOTES

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\_\_\_\_\_

\_\_\_\_\_

Different cultures have seen different patterns in the stars.

(1) Do these star patterns have astronomical significance today? \_\_\_\_\_  
\_\_\_\_\_

(2) How come? \_\_\_\_\_  
\_\_\_\_\_

7

The one remaining serious astronomical significance is that the International Astronomical Union (IAU) has divided the sky into 88 regions, each labelled by a classical Greek constellation, as an aid to locating things.

They can say an object is in the “Orion region” and people will know where to look.

8

So, the constellations consist of stars that may look close, but do not “cohere” (stick) to each other because they may actually be far apart.

What about the Milky Way – that milky band of stars across the sky?

(3) Does anything hold the MW together?  
\_\_\_\_\_  
\_\_\_\_\_

9

(4) What said we about gravity and the Universe?

(5) But gravity can lead to instability. Why?  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

10

(6) What counteracts gravitational collapse on the scale of day-to-day life? \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

11

But, electromagnetism cannot counteract gravity on an astronomical scale.

(7) Why not?  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

12

ADDITIONAL NOTES

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

(8) So, what counteracts gravitational collapse on the astronomical scale?

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Expansion as an antidote to gravitational collapse

A ball thrown up falls back to earth, but a rocket doesn't because it has sufficient velocity (escape velocity):

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Expansion will emerge as an important issue later.

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Rotation as an antidote to gravitational collapse

The Universe is filled with things going around other things: The moon goes around the earth, the earth goes around the sun. The rotation rates are exactly enough to prevent each pair from falling into each other (or from escaping).

In the same way, the

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A mass  $m$  rotating around a central mass  $M_c$  with rotational speed  $v$  will be in stable circular orbit at a radius  $r$  (neither spiral in nor out) if

Follows from equating the force needed for circular orbit (left) with the force of gravitation (right).

16

The mass  $m$  is irrelevant, because it

(9) Cancel like crazy in the previous equation.

(10) Solve for  $v$ .

$$v =$$

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Since the mass of the rotating object is irrelevant, there's a "rotationally correct" speed at any radius, no matter what the mass.

We all know what *that* means.

(11) What?

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ADDITIONAL NOTES

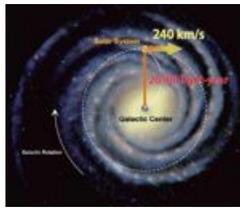
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The sun takes \_\_\_\_\_  
 \_\_\_\_\_ around the center of the MW. Each of  
 the up to 400 G stars of the MW has its own “cor-  
 19 rect” period.

### The Milky Way in detail

We'll look at the

- ▷ \_\_\_\_\_
- ▷ \_\_\_\_\_
- ▷ \_\_\_\_\_
- ▷ \_\_\_\_\_ and
- ▷ \_\_\_\_\_

20 of the Milky Way.

### Shape

We believe that the Milky Way is a \_\_\_\_\_  
 \_\_\_\_\_

There are different types of spiral galaxies, and we  
 are not completely sure *exactly* what type the MW  
 is, although since 2005 we are nearing consensus.  
 \_\_\_\_\_

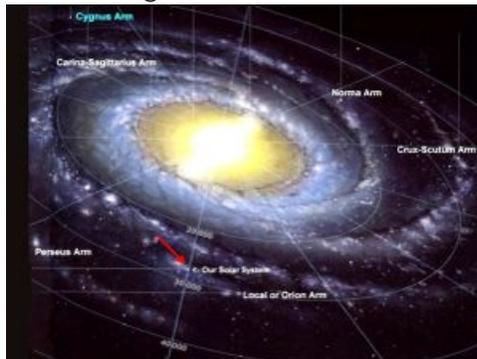
This is an example of what we see of the MW:



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How do we go from what we see to this?



In his study of galaxies, starting in the 1920s, Hubble had classified their shapes in what is called the \_\_\_\_\_.

Hubble incorrectly thought that galaxies evolve along the tuning fork. We believe that not to be the case, but still use his classification, with further refinements.

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### ADDITIONAL NOTES

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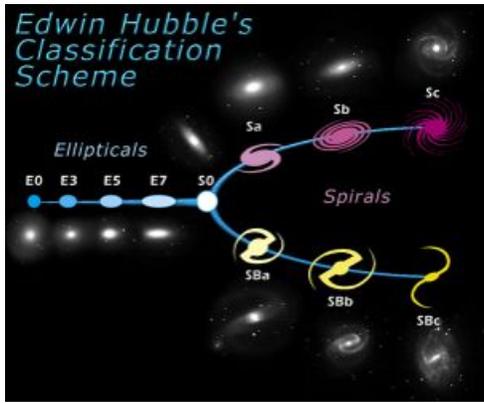
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The “E” stands for \_\_\_\_\_, the “S” for \_\_\_\_\_, and the “SB” stands for \_\_\_\_\_.

The numbers or letters right after tell you the nature of the ellipse, spiral, etc.

Galaxies between two clearcut types, are given composite names. For example, a galaxy of type Sab would lie in-between an Sa type and an Sb.

26

(12) In the diagram, as you go from elliptical to spiral, do the galaxies get flatter or “rounder”?

=====

=====

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27

The Milky Way appears to the eyes as a narrow band across the sky:



(13) Does that suggest a flattened shape for the galaxy or a “rounder” one? \_\_\_\_\_

(14) If so, what overall shape does that suggest for the MW? \_\_\_\_\_

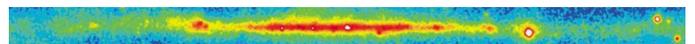
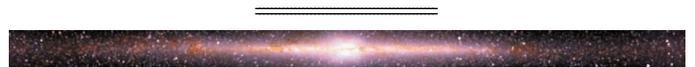
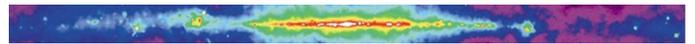
28

The dark band seen in optical (visible to the human eye) pictures across the middle of the Milky Way obscures a good understanding of its structure.

We get around that by examining the Milky Way

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This is a common procedure in astronomy.



(15) Do these pictures suggest a central bulge to the Milky Way? \_\_\_\_\_

29

30

ADDITIONAL NOTES

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Evidence that the Milky Way is a spiral galaxy:

- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_

If it bulges like a spiral, it moves like a spiral, has gas like a spiral, it must be a \_\_\_\_\_

31

We now believe that \_\_\_\_\_

This means that it lies between an Sb and an Sc type, and it may or may not \_\_\_\_\_

Evidence has been mounting since 2005 that it ... \_\_\_\_\_

32

**Structure**

The Milky Way has three main parts:

- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_

33

**The Bulge**

The bulge is the spherical (slightly flattened) center of the galaxy. On the right is a photo of the bulge taken from earth.



34

(16) Does the bulge seem brighter or fainter than the rest of the galaxy? What might you infer from this about the star count there, compared to elsewhere in the galaxy?

\_\_\_\_\_

35

Detailed studies support the idea that the stars in the bulge are closely packed.

The very center of the bulge is called the \_\_\_\_\_

The typical distance between stars here is about \_\_\_\_\_ By contrast, the star nearest to the sun is about \_\_\_\_\_ (And the solar system extends about \_\_\_\_\_ from the sun.)

36

ADDITIONAL NOTES

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In the region of the core of the Milky Way, there's a constellation called \_\_\_\_\_



37

In 1951, at the edge of the constellation Sagittarius, there was discovered a \_\_\_\_\_

Shortly after, it was noted that this radio source lay very close to the precise center of the MW.

Careful study showed that there are three components to "Sgr A:" A East, A West, and A\*, each emitting radio waves for different reasons.

38

Sgr A\* was discovered in 1974 (and named as such in 1982).

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

39

OK, let's tackle our inner freak:

From 1995 onward the Galactic Center Group at UCLA, led by Andrea Ghez, has tracked star motions around the center of the MW:

<http://www.galacticcenter.astro.ucla.edu/about.html>

It appears that Sgr A\* is the location of a supermassive black hole whose mass might be around

40

### The Disk

The disk of the MW includes its \_\_\_\_\_

It was first thought the arms were material.

This would mean that the same material is in each arm throughout. Material arms can be rigid (such as the rotor on a helicopter), or flexible (such as ribbons tied to a rotating central pole).

41

(17) Would the inner part of a rigid arm rotate at the same rate as the outer? \_\_\_\_\_

We observe that the inner part of the arms rotate faster (they complete one revolution around the center of the galaxy more quickly) than the outer. So the arms are not rigid. We might expect this in any case, since the rigidity of day-to-day objects arises from electromagnetic intermolecular forces.

42

### ADDITIONAL NOTES

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

(18) If the arms are flexible and the inner parts rotate faster, what is the eventual outcome?

=====

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The differential rotation rate (different rates nearer center than in the outer regions) along with the long term existence of the arms suggests that the

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We believe they are regions of high density through which the stars pass as they rotate around the center of the Milky Way.

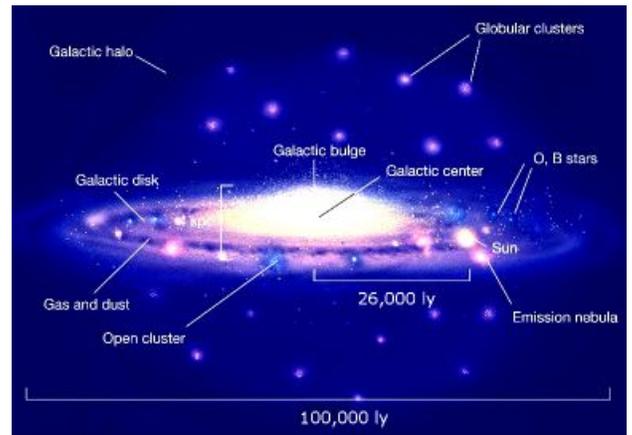
43

44 The arms persist much as traffic jams do.

The Halo

The disk of the Milky Way is surrounded by a roughly spherical region called the halo that contains old stars clustered together in enormous spherical arrangements.

These are called \_\_\_\_\_



45

46

Size

- Bulge: \_\_\_\_\_
- Disk: \_\_\_\_\_  
thickness \_\_\_\_\_
- Halo: \_\_\_\_\_

(19) Why are the hot constituents of the MW disk distributed more thickly than the cold? \_\_\_\_\_

=====

47

48

ADDITIONAL NOTES

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

**Composition**

(20) What do you think the Milky Way is mostly made of? \_\_\_\_\_

\_\_\_\_\_  
\_\_\_\_\_

49

There are two types of stars, \_\_\_\_\_

\_\_\_\_\_

Population I stars are

○ \_\_\_\_\_

○ \_\_\_\_\_

Population II stars are

○ \_\_\_\_\_

50

○ \_\_\_\_\_

Older, Pop. II stars are formed mainly from hydrogen and helium in the early Universe; nuclear reactions have created their metallic elements.

These metals are dispersed by them through stellar winds, flares, and through supernova explosions at the end of some of their lives.

Pop. I stars (and the planets around them) are made from this material, therefore contain metals.

51

(21) Is our sun likely to be Pop. I or Pop. II?

\_\_\_\_\_  
\_\_\_\_\_

52

Which stars are where?

▷ Bulge: \_\_\_\_\_

▷ Disk: \_\_\_\_\_

▷ Halo: \_\_\_\_\_

\_\_\_\_\_

53

The spiral arms contain, in addition to stars, \_\_\_\_\_ where new stars are being formed.

Stars in the MW are often

There are three kinds

○ Associations

○ Open clusters

54

○ Globular clusters

ADDITIONAL NOTES

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

Associations

Associations are groups of stars that have

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They contain the youngest stars.

55



Orion nebula: Star Nursery

56

Open Clusters

Clusters are groups of stars \_\_\_\_\_

Open clusters, mostly in the galactic disk, are \_\_\_\_\_. They contain between 100 and 1,000 stars in a region that's about 10 to 100 ly in diameter.

57



Pleades open cluster

58

Globular Clusters

Globular clusters are \_\_\_\_\_. They contain roughly 100,000–1,000,000 stars in a region about 80 ly in diameter. Globular Clusters are generally \_\_\_\_\_ in shape. The clusters are stable and have survived for billions of years.

59



Group of globular clusters

60

ADDITIONAL NOTES

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(22) Are the stars in a globular cluster closely packed or far apart? \_\_\_\_\_  
\_\_\_\_\_

61

### Formation

There are two competing theories:

- ▷ \_\_\_\_\_
- ▷ \_\_\_\_\_

62

**Outside-In:** The \_\_\_\_\_  
\_\_\_\_\_

The inner material flattened into a disk. As it did the older stars in the halo spewed out metals and therefore the stars in the disk are metal-rich.

**Inside-Out:** The \_\_\_\_\_  
\_\_\_\_\_

63

### Missions to Explore the Structure of the MW

First, disappointing news: \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

- \_\_\_\_\_
- \_\_\_\_\_

64

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_:

<https://voyager.jpl.nasa.gov/mission/status/>

Still we have interesting results from two important missions whose goal was to map the locations of individual stars in the MW.

65

### Hipparcos

- Launched in 1989 by the European Space Agency (ESA).
- Name meant to suggest ancient Greek astronomer, Hipparchus, and is also an acronym for High Precision PARallax COLlecting Satellite.
- Produced a primary catalogue of about 118,000 stars, and a secondary catalogue, called Tycho, of over 2M stars (positions determined to less precision).



66

### ADDITIONAL NOTES

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(1) The Milky Way is a collection of \_\_\_\_\_

(2) What's an example of a star? \_\_\_\_\_

1

Whether the sun is a star has been the subject of speculation for hundreds of years.

Suggested as far back as 450 BCE (Anaxagoras), but it was Giordano Bruno in 1584 who was taken seriously. He said

“There are countless suns and countless earths all rotating round their suns in exactly the same way as the seven planets of our system.”

De L'Infinito Universo E Mondi, 1584  
(On the Infinite, Universe and Worlds)

2

(3) What's the first thing you notice about stars?  
\_\_\_\_\_

Stars shine because they give off energy. We measure energy in a unit called a \_\_\_\_\_.

The \_\_\_\_\_ of a star is the energy it gives off every second; unit is J/s, called a \_\_\_\_\_.

The luminosity of a star is its “actual brightness.”

3

If measured in Watts, the luminosity of the sun is

The earth's entire energy consumption in 2012 was  $5.6 \times 10^{20}$  J. That means that the sun's energy could supply, *every second*, the annual energy needs of a **million** earths.

4

(4) From what did I concoct a “million”?

### What Makes a Star Shine?

**353. Maintenance of the Solar Heat.** — The question at once arises, if the sun is sending off such an enormous quantity of heat annually, how is it that it does not grow cold?

(a) The sun's heat cannot be kept up by *combustion*. As has been said before, it would have burned out long ago, even if made of solid coal burning in oxygen.

(b) Nor can it be simply a *heated body cooling down*. Huge as it is, an easy calculation shows that its temperature must have fallen greatly within the last 2000 years by such a loss of heat, even if it had a specific heat higher than that of any known substance.

As matters stand at present, the available theories seem to be reduced to two, — that of Mayer, which ascribes the solar heat to the energy of meteoric matter falling on the sun; and that of Helmholtz, who finds the cause in a slow contraction of the sun's diameter.

Young, 1900

5

6

### ADDITIONAL NOTES

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(5) What does make a star shine? [What’s the fuel for the energy (light, heat, etc.) it produces?]

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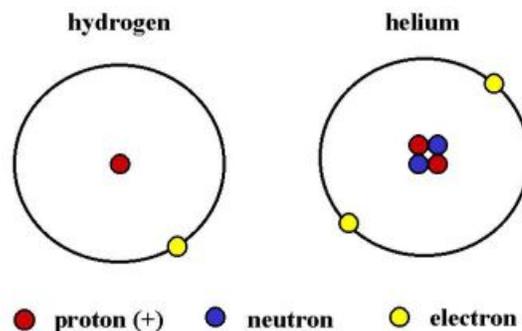
(6) What equation governs the conversion of mass to energy? =====

=====

=====

7

How it happens: =====



8

(7) The masses of a proton and a neutron are roughly the same. An electron is roughly 10,000 times less massive. (So the mass of an atom is essentially the mass of its nucleus.) Based on masses alone, roughly how many Hydrogen atoms would you need to make one Helium? =====

9

(8) Atomic masses are measured in atomic mass units (amu). The mass of Hydrogen is 1.00794 amu and the mass of Helium is 4.00260 amu. How much bigger or smaller are the masses of 4 Hydrogen atoms than one Helium?

10

(9) One amu is  $1.66054 \times 10^{-27}$  kg. Convert the previous answer to kg.

(10) What’s 0.048 as a percentage? =====

(11) What’s  $10^{-9}$  in words? =====

(12) What’s  $10^{-27}$  in terms of  $10^{-9}$ , and in words? =====

11

That’s a tiny amount of missing mass: < 5% of a billionth of a billionth of a billionth of a kg.

(13)  $c \approx 300,000,000$  m/s. What’s  $c^2$  in  $10^n$ ? =====

(14) How many atoms are there, very roughly, under “normal” conditions in one cubic centimeter? =====

12

ADDITIONAL NOTES

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Given the largeness of those numbers, missing mass of  $\approx 0.05 \times 10^{-27}$  kg can lead to a lot of energy ( $E = \underline{\hspace{2cm}}$ ).

(15) How much energy per reaction?

\_\_\_\_\_

(16) How much energy “per cubic cm”?

\_\_\_\_\_

13

Summary: The basic nuclear reaction that powers stars is called \_\_\_\_\_.

Four nuclei of H fuse to form one He:



The reaction releases energy because the mass of the Helium is slightly less than the mass of the four Hydrogen nuclei. The energy *ultimately* leaves the surface as visible light.

14

Fusion is difficult to achieve because it needs a high temperature and density.

(17) Why, do you think?

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

15

Fusion requires a temperature of  $\sim 13,000,000^\circ\text{C}$ .

The temperature at the center of stars like the sun is about  $15,000,000^\circ\text{C}$  and the density is about  $150\text{ g/cm}^3$  (10 times the density of lead).

That’s enough for fusion.

16

The temperature and density decrease as you move outward from the center of the star.

The nuclear “burning” is almost completely shut off beyond the outer edge of the core of stars like the sun at about 25% of the distance to the surface (or 175,000 km from the center).

The temperature here is half its central value and the density drops to about  $20\text{ g/cm}^3$ .

17

Fusion in stars is a three step process:

1) \_\_\_\_\_

\_\_\_\_\_

2) \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

3) \_\_\_\_\_

\_\_\_\_\_

18

ADDITIONAL NOTES

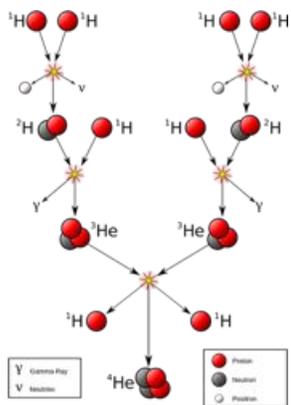
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\_\_\_\_\_

\_\_\_\_\_



19

Solar neutrinos were detected in 1964. The detector was placed 5,000 feet deep in a mine.

(18) Why? \_\_\_\_\_

The detector was simply a 100,000 gallon tank filled with dry-cleaning fluid: the chlorine in the fluid interacts with neutrinos!

Initially, it seemed there were not enough neutrinos. This was solved by assigning them mass.

21

In order fully to understand what it is that our scientists are seeking and the problems that stand in the way of controlled fusion power, we must start with a German scientist named Dr. Frederic Houtermans, one of the first men ever to theorize about hydrogen energy.



**HOUTERMANS** was first to publish solar fusion theory.

I met Dr. Houtermans last summer at the Geneva conference, and he told me about the birth of his idea which later grew into the H-bomb. In 1928 he was deeply absorbed in wondering about the source of energy in stars. "Oh, I was doing other things, too," he told me with a twinkle. "I was courting a very pretty girl." When he was not courting, Houtermans' calculations led him to rule out chemical energy as the source of heat in stars. Instead, he hit upon the idea of energy coming from a thermonuclear process—heat evolved from the violent action of atoms deep inside the fiery furnace of a white-hot star. Convinced that he was on the right track, the young scientist, together with his colleague Robert Atkinson, published his discovery. One night, with the stars shining in the heavens, he pointed toward them and proudly exclaimed to his girl, "I know what makes them shine!"

From "Limitless Power," Ralph Lapp, Life Magazine, October 8, 1956

23

The electromagnetic energy \_\_\_\_\_

\_\_\_\_\_  
 \_\_\_\_\_  
 \_\_\_\_\_

The neutrinos \_\_\_\_\_

\_\_\_\_\_  
 \_\_\_\_\_  
 \_\_\_\_\_

20

History of the idea that fusion makes stars shine:

- 1920: Eddington suggests that H fusing to form He might be the source of energy in stars.
- 1928: Gamow calculates conditions for the electric repulsion between protons in H to be overcome.
- 1929: Houtermans & Atkinson make first calculations of H → He reactions in stars like the sun.
- 1938: Bethe constructs a full theory of this process.
- 1940s onward: Hoyle and others extended these ideas to show \_\_\_\_\_ of heavier elements.

22

To summarize:

What makes the Stars Shine?  
 Jason Socrates Bardi. January 23, 2008  
 Phys. Rev. Focus 21, 3  
<https://physics.aps.org/story/v21/st3>

"Physicists at the turn of the 20th century realized that the existing paradigm for stellar energy production was wrong. The old theory, that the sun's energy was produced by gravitational contraction, could supply only 30 million years of stellar energy, but biologists and geologists were estimating the earth was much older. . . .

24

ADDITIONAL NOTES

\_\_\_\_\_  
 \_\_\_\_\_  
 \_\_\_\_\_  
 \_\_\_\_\_  
 \_\_\_\_\_

“A long series of discoveries beginning with Einstein’s relativity in 1905 led up to Bethe’s discovery of the correct nuclear reactions. Hydrogen fusion seemed like a good candidate because according to  $E=mc^2$ , the small mass difference between the fusing hydrogen and the resulting helium would liberate an enormous amount of energy. Also, spectral analysis in the 1920s revealed that most stars, including the sun, are mostly hydrogen. . . .

25

“Following Bethe’s work, astrophysicists were able to work out many details of stellar physics, which led to a view of stars almost as living beings. As Bethe concluded his Nobel lecture, ‘Stars have a life cycle much like animals. They get born, they grow, they go through a definite internal development, and finally they die, to give back the material of which they are made, so that new stars may live.’”

We’ll study this cycle: birth, life, and death of the sun and other stars.

26

**The Birth of Stars**

- \_\_\_\_\_  
\_\_\_\_\_
- \_\_\_\_\_  
\_\_\_\_\_
- \_\_\_\_\_  
\_\_\_\_\_

27

The mass of the core determines its destiny.

The possibilities for stellar formation are:

- $M < 0.08M_{\odot}$ : \_\_\_\_\_
- $0.08M_{\odot} < M < 0.5M_{\odot}$ : \_\_\_\_\_  
\_\_\_\_\_
- $0.5M_{\odot} < M < 5M_{\odot}$ : \_\_\_\_\_  
\_\_\_\_\_

28

- $5M_{\odot} < M < 7M_{\odot}$ :  
\_\_\_\_\_  
\_\_\_\_\_
- $M > 7M_{\odot}$ :  
\_\_\_\_\_  
\_\_\_\_\_

29

**The Life of Stars (like the sun)**

- 50Myr: \_\_\_\_\_
- 10Gyr: \_\_\_\_\_
- 1 Gyr: \_\_\_\_\_
- 170 Myr: \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

30

ADDITIONAL NOTES

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Luminosity

Not only is your mass your destiny when you're a star, mass is also luminosity (how bright you are):

The apparent brightness is \_\_\_\_\_, where  $d$  is the distance between the star and the observer.

31 (From area of a sphere of radius  $d$ :  $4\pi d^2$ .)

Williamina Fleming worked with a group of women at Harvard in the late 1800s to develop a way to \_\_\_\_\_

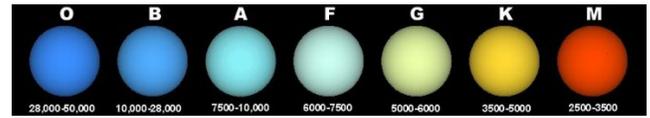
\_\_\_\_\_.

At the time we didn't understand what caused these dark lines, but Fleming and her colleagues still created a complete classification system, with 22 different classes labeled A through P.

33

We said life of stars "like the sun."

(19) Are there others? \_\_\_\_\_



Different types, based on temperature (which determines overall color).

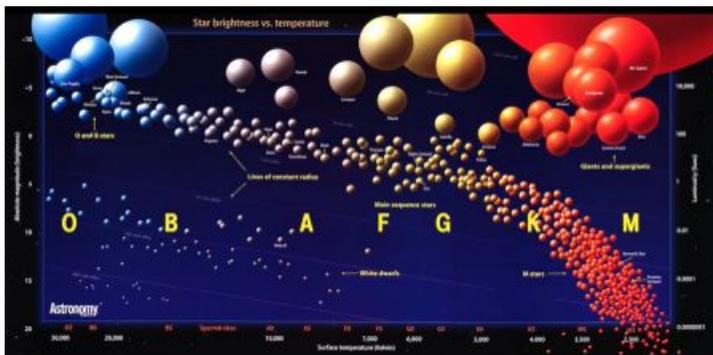
32

Around 1900, Annie Jump Cannon continued this work by combining this classification with new understanding of what caused these dark lines, and their relationship to temperature.

She reordered Fleming's original spectral classes and combined several similar groups to form the Harvard Spectral Classifications – the classification system we use today.

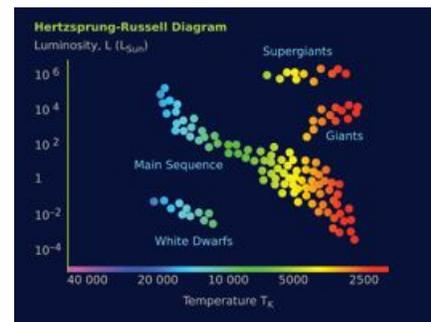
34

The \_\_\_\_\_ Diagram



35

Another view



36

ADDITIONAL NOTES

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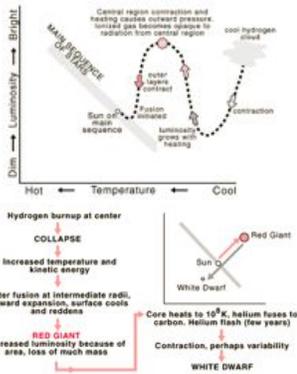


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Life path



37

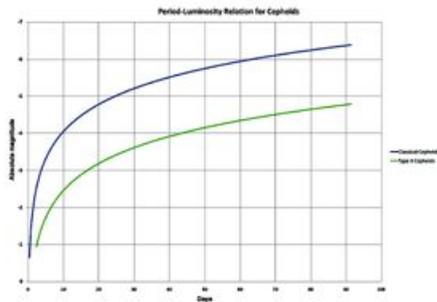
Cepheid Variables

These are stars of variable brightness. Their periods (the time it takes for them to go from maximum brightness to minimum brightness and back again) are \_\_\_\_\_.

If you measure their periods by observing them over the course of a few nights, you can determine their luminosity.

38

Relationship between period and luminosity



The work to arrive at this relationship was done by Henrietta Leavitt at the Harvard College Observatory.

39

Arthur Eddington proposed (around 1917 onward) that Cepheids are single stars that undergo radial pulsations.

Their radius gets smaller and larger over a regular time interval.

40

We believe that energy is stored in the form of ionized (charged) helium during the compression stage of the cycle and then released as the helium recombines during the expansion stage, \_\_\_\_\_

41

ADDITIONAL NOTES

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# Arvind Borde / AST 10, Week 4: The Death of Stars

## Background

Or, were you sleeping?



(1) What are the ingredients of the world?

\_\_\_\_\_

1

(2) What are their basic “quanta” called?

\_\_\_\_\_

(3) How many interactions, and what are they?

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

2

We believe we have a solid understanding of

- Gravitation (classical – Einstein’s Theory)
- Both nuclear forces, and
- Electromagnetism, classical & quantum.

In fact, electromagnetism has been unified with the nuclear forces through the quantum “GUT.”

*Astronomical formulas and understanding come from this fundamental understanding.*

3

(4) Your life is determined primarily by

- (a) your shape.
- (b) your chemical composition.
- (c) your mass.
- (d) complex circumstances.

\_\_\_\_\_

4

(5) A star’s life is determined primarily by

- (a) its shape.
- (b) its chemical composition.
- (c) its mass.
- (d) complex circumstances.

\_\_\_\_\_

5

(6) Your life’s end is determined primarily by

- (a) your shape.
- (b) your chemical composition.
- (c) your mass.
- (d) complex circumstances.

\_\_\_\_\_

6

## ADDITIONAL NOTES

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(7) A star's end is determined primarily by

- (a) its shape.
- (b) its chemical composition.
- (c) its mass.
- (d) complex circumstances.

=====

7

The length of the main-sequence lifetime of a star of mass  $M$  is given by the formula

NOTE: As said, formulas such as this one follow from our basic understanding of gravitation (classical), nuclear physics (quantum) and electromagnetism (classical and quantum).

8

(8) What is the lifetime of the sun?

(9) Does a star that's more massive than the sun live longer or less long than the sun, according to the formula? =====

9

(10) Why, according to your innate intelligence?

=====

10

(11) You end your life in one of three ways:

- (a) =====
- (b) =====
- (c) =====

(12) A star ends its life in one of three ways:

- (a) =====
- (b) =====
- (c) =====

11

We'll discuss each end-state in turn, using four main questions to guide us:

- A] =====
- B] =====
- C] =====
- D] =====

12

ADDITIONAL NOTES

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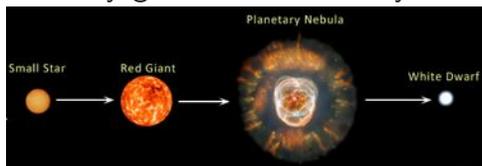
### White Dwarfs

A) Which living stars end up as White Dwarfs?

=====

=====

B) How do they get there? This way:



13

○ =====

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○ =====

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15

○ =====

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○ =====

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○ =====

=====

17 Is this all fiction?

17

Details, details...

○ =====

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○ =====

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○ =====

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○ =====

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14

When the core has finished its helium, it isn't hot enough to be able to "burn" its carbon.

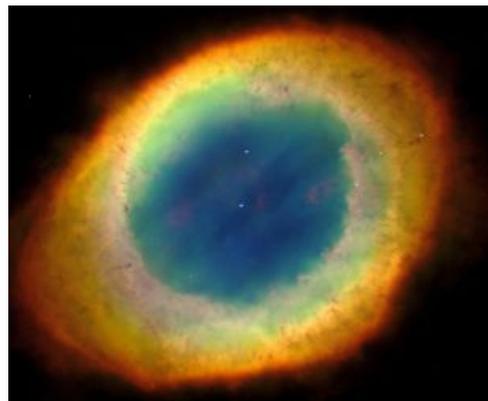
(13) Why "burn," baby, "burn," in quotes? =====

=====

(14) What now? What force wins in a star when nuclear reactions end? =====

=====

16



Ring nebula, D=2,000ly, d=1ly, 1779.

18

#### ADDITIONAL NOTES

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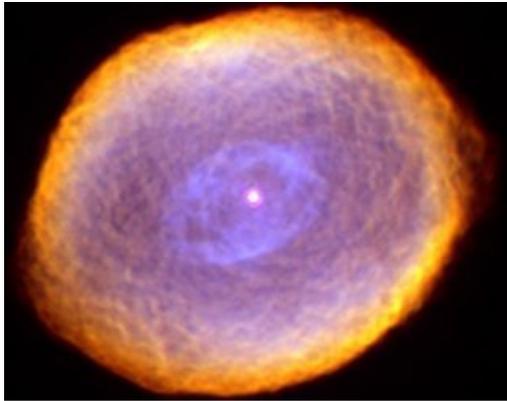
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Spirograph nebula,  $D=2,000\text{ly}$ ,  $d=0.2\text{ly}$ , 1891.

19



Stingray nebula, youngest known planetary nebula,  $D=18,000\text{ly}$ ,  $d=0.16\text{ly}$ , 1994.

20



Helix nebula,  $D=650\text{ly}$ ,  $d=2.5\text{ly}$ , 1824ish.

21



Little Ghost nebula,  $D\sim 3,500\text{ly}$ ,  $d=1\text{ly}$ , 1700s.

22

(15) What nebulae do these seem? =====

Planetary nebulae are temporary (by astro standards). A typical one will be visible for about 25,000 years.

The nebulae expand outward at  $\sim 100,000\text{ km/hr}$  and become too thinly spread out and too far from the light of the parent white dwarf (whose energy

23 it is that illuminates them) to be seen.

### Lists of Planetary Nebulae

- Abell catalogue, 1966: 86
- Strasbourg-ESO Catalog, 1992: 1,143  
<http://vizier.u-strasbg.fr/viz-bin/VizieR?-source=V/84&-to=2>
- Current count:  $\sim 3,000$ .

24

### ADDITIONAL NOTES

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## White Dwarf Catalogs

### SDSS DR7 WHITE DWARF CATALOG

S. J. KLEINMAN<sup>1</sup>, S. O. KEPLER<sup>2</sup>, D. KOESTER<sup>3</sup>, INGRID PELLISOLI<sup>2</sup>, VIVIANE PEÇANHA<sup>2</sup>, A. NITTA<sup>1</sup>,  
 J. E. S. COSTA<sup>4</sup>, J. KRZESINSKI<sup>5</sup>, P. DUFOUR<sup>6</sup>, F.-R. LACHAPPELLE<sup>5</sup>, P. BERGERON<sup>7</sup>, CHING-WA YIP<sup>8</sup>,  
 HUGH C. HARRIS<sup>9</sup>, DANIEL J. EISENSTEIN<sup>9</sup>, L. ALTHAUS<sup>9</sup>, AND A. CÖRSICO<sup>9</sup>  
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 Received 2012 June 3; accepted 2012 October 19; published 2012 December 20

#### ABSTRACT

We present a new catalog of spectroscopically confirmed white dwarf stars from the Sloan Digital Sky Survey (SDSS) Data Release 7 spectroscopic catalog. We find 20,407 white dwarf spectra, representing 19,712 stars,

25

How could a white dwarf star, which is only about the size of the Earth, be responsible for such an extreme act? The answer is gravity. When a star reaches its white dwarf stage, nearly all of the material from the star is packed inside a radius one hundredth that of the original star. This means that, for close encounters, the gravitational pull of the star and the associated tides, caused by the difference in gravity’s pull on the near and far side of the planet, are greatly enhanced.

[http://www.nasa.gov/mission\\_pages/chandra/white-dwarf-may-have-shredded-passing-planet.html](http://www.nasa.gov/mission_pages/chandra/white-dwarf-may-have-shredded-passing-planet.html)

27

How can a white dwarf be this dense?

Normal matter is made of atoms, and there’s a limit to how closely they can be packed. The size of electron orbitals give a minimum size to atoms.

White dwarf material is not made of atoms, but is a plasma of nuclei and electrons. There’s no obstacle to placing nuclei closer than normally allowed by electron orbitals.

29

If you don’t believe the pictures of white dwarfs, how about these words:

April 16, 2015

White Dwarf May Have Shredded Passing Planet

The destruction of a planet may sound like the stuff of science fiction, but a team of astronomers has found evidence that this may have happened in an ancient cluster of stars at the edge of the Milky Way galaxy.

... researchers have found evidence that a white dwarf star ... may have ripped apart a planet as it came too close.

26

## Densities

A typical white dwarf is half as massive as the Sun, yet only slightly bigger than Earth.

An Earth-sized w.d. has a density of  $10^9 \text{ kg/m}^3$ .

(Earth’s average density is  $5,400 \text{ kg/m}^3$ .)

28

## C] How does a white dwarf stay “there”?

A white dwarf has the density to put out strong enough gravity to shred a planet.

Yet its self-gravity does not crush itself.

Why does gravitation not win?

30

## ADDITIONAL NOTES

This was resolved by R.H.Fowler in 1926 using the new quantum mechanics.

Electrons obey the “Pauli exclusion principle”: no two electrons can occupy the same state. It follows that they must obey “Fermi-Dirac statistics”, also introduced in 1926, giving an outward pressure, called “electron degeneracy pressure.”

31 This pressure allows white dwarfs to survive.

(16) Are the temperatures of white dwarfs high or low? \_\_\_\_\_

(17) Are the luminosities of white dwarfs high or low? \_\_\_\_\_

(18) How come? \_\_\_\_\_  
 \_\_\_\_\_

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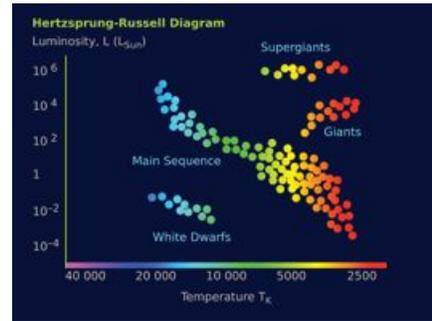
### Neutron Stars

#### A] Which living stars end up as Neutron Stars?

Pre-question: Why do we expect that any do?  
 Why don't all stars end up as White Dwarfs?

35

Remember the \_\_\_\_\_ diagram:



32

#### D] What do they do after?

White dwarfs, having no energy source, shine only with residual heat. As they do, calculations suggest that in 10 (or more) Gyr they'll darken into “black dwarf” stars. We'll know they're there from their gravitational pull, but we will not see them.

(19) We've no direct evidence for black dwarfs. Why not? \_\_\_\_\_

34

- The Milky Way has up to  $4 \times 10^{11}$  stars.
- The total mass of the MW is estimated at about  $8 \times 10^{11} M_{\odot}$ , about 10% visible.
- So, the average mass in visible objects is  $\sim \frac{8 \times 10^{10} M_{\odot}}{4 \times 10^{11}}$

(20) What does this work out to? \_\_\_\_\_

36

#### ADDITIONAL NOTES

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This average mass is skewed in the MW: there are about  $10^{11}$  brown dwarfs, with masses about  $0.05M_{\odot}$ .

That suggests there will be stars that are more massive than the sun.

Most of these will, of course, be only a little more massive and will end up as the sun will.

37

In fact 94–98% of stars in the MW will end up as white dwarfs (or are already there).

In the 1930s it was believed that *all* stars, not just 98% of them, would end up as white dwarfs.

Work that suggested otherwise by Chandrasekhar and others was fought by the most famous astronomer at the time, Arthur Stanley Eddington.

38

From the AIP Oral history archive ([http://www.aip.org/history/ohilist/4551\\_1.html](http://www.aip.org/history/ohilist/4551_1.html)):

Chandrasekhar: “at the end of the meeting, everybody came and said to me, ‘Too bad. Too bad.’ The other astronomers were certain that my work was wrong because Eddington had said so.”

Interviewer: ‘Too bad’ meaning, ‘Too bad that you had got it wrong’?

Chandrasekhar: “Yes.”

39

“In many ways, thinking back over those times, I am sort of astonished that I was never completely crushed by these Stalwarts. You know, none of these people would accept my work, astronomers wouldn’t accept it and finally in 1938, I decided that there was no good my fighting all the time, that I am right and that the others were all wrong. I would write a book. I would state my views. And I would leave the subject. That’s exactly what I did.”

40

Chandrasekhar continued  
“It is hard for people to realize what an incredibly dominating position Eddington had during his life. For example, Shapley told me this: in 1936, they had a tricentennial at Harvard, and, Shapley said, they sent a circular around to American astronomers, to rank astronomers so they could give honorary degrees. And he said that Eddington was the first in every single list he received! ... the fact is that there was not a single astronomer in the thirties who would not with unanimity have said that Eddington is the greatest living astronomer. He had an absolutely dominating position.”

41

In 1983, Chandrasekhar got the Nobel prize for his work from the 1930s.

In 1999, an x-ray telescope was launched into space, as the x-ray counterpart of the Hubble.

It is called the “Chandra.”

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#### ADDITIONAL NOTES

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We now accept that roughly 2% of stars in the MW cannot end as white dwarfs. That actually gives a high number.

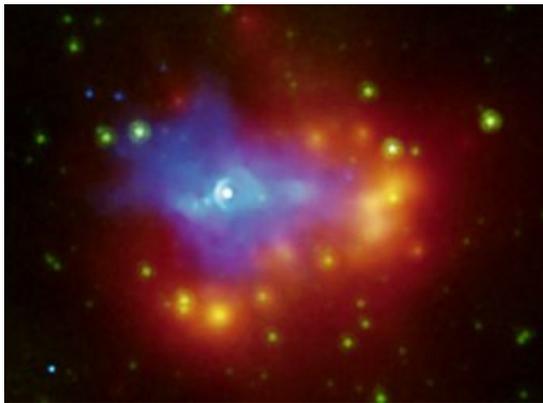
(21) What is 2% of 400 billion? \_\_\_\_\_

Which stars are these?

They're stars with masses greater than  $7M_{\odot}$ . They are mostly on the neutron star track.

About 2,000 neutron stars have so far been detected (mostly as "pulsars"). Here are some:

43



SNR-G54.1+0.3

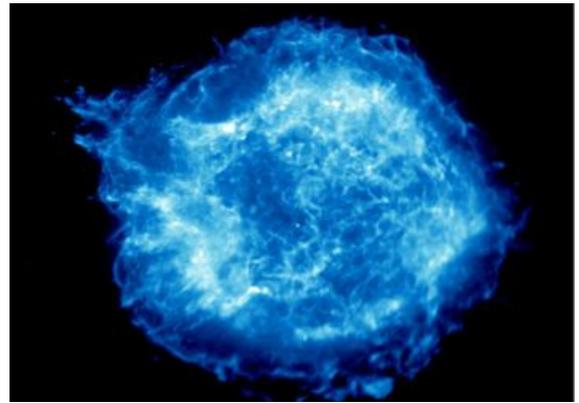
45



Vela

47

44



Cassiopeia A

46



PSR B1509-58

48

ADDITIONAL NOTES

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So neutron stars do exist. They are pretty extreme.

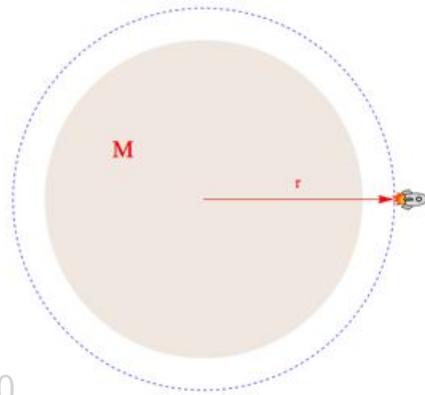
Their density is roughly  $5 \times 10^{14} \text{ g/cm}^3$ . That's the whole human race inside a sugar-cube.

Water has a density of  $1 \text{ g/cm}^3$ ; lead  $\sim 11 \text{ g/cm}^3$ .

The high density leads to a high \_\_\_\_\_: the velocity needed to escape the gravitational pull of an object.

49

Escape Velocity



50

(22) What exactly do we mean by “neutron star”?

\_\_\_\_\_

\_\_\_\_\_

They are “neutron stars.”

51

Background

- ▷ 1920: Neutron predicted by Ernest Rutherford.
- ▷ 1932: Neutron discovered by James Chadwick (Nobel Prize ).

52

▷ 1934: Fritz Zwicky and Walter Baade publish “Supernovae and Cosmic Rays.”

They predict, “With all reserve we advance the view that supernovae represent the transitions from ordinary stars into neutron stars, which in their final stages consist of extremely closely packed neutrons.” They believed that these neutron stars were rapidly spinning, dense remnants of dead stars.

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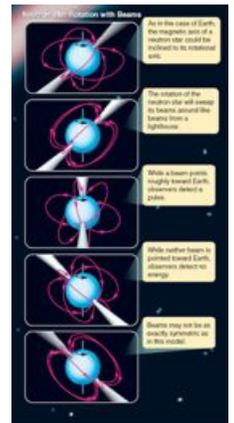
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54



ADDITIONAL NOTES

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The escape velocity from earth is about 11 km/sec.

The escape velocity from a  $2M_{\odot}$  neutron star is about 200,000 km/sec. That’s about  $2/3$  the speed of light,  $c$ .

Can you achieve an escape velocity equal to  $c$ ?

If you were able to compress a neutron star to about half its radius, you would.

55

**B] How do they get there?**

For stars with this much mass, the outer layers are blown away in a \_\_\_\_\_

Why the explosion? \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

56

Massive stars that become supernovas are factories for producing and distributing all the elements needed to make almost everything else.

Inside their cores, nuclear fusion creates nearly all the atoms that make up planets, moons, and us. The carbon in our proteins, the calcium in our bones, the iron in our blood, the oxygen we breathe, etc., were manufactured inside these stars.

57

But stars typically don’t get hot enough to make any atoms heavier than iron.

To make heavier elements like gold, lead, mercury, it was hypothesized that it requires the extreme pressures and heat inside a supernova during those few seconds of the collapse. Then the rebound explosion as the star blows itself apart flings all those elements into space.

58

But there were concerns that even supernova conditions might not be extreme enough to produce elements such as gold. It was suggested that we’d need neutron stars to collide in order for the necessary extreme conditions to arise.

We waited for such an event to have happened.

(23) Why “have happened” not “happen”?

\_\_\_\_\_

\_\_\_\_\_

59

Finally, it happened –  $\sim$  130 million years ago. Two neutron stars collided.

On August 17, 2017, we got signals: gravitational waves (first), and various electromagnetic ones. The event is called GW170817. (Why?)

Analysis suggests that about 10,000 earth masses of precious elements were produced in this single collision.

60

ADDITIONAL NOTES

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# Arvind Borde / AST 10, Week 5: Black Holes

## Which live stars end up as which dead ones?

Bounds on masses:

- White dwarfs:

\_\_\_\_\_

- Neutron stars:

\_\_\_\_\_

Do all stars with initial mass  $> 7M_{\odot}$  have remnants with masses below the neutron star limit?

1

Studies suggest stars that start off \_\_\_\_\_

\_\_\_\_\_

They'll undergo the unstoppable gravitational collapse Chandrasekhar et al had predicted (1930s).

They will get \_\_\_\_\_

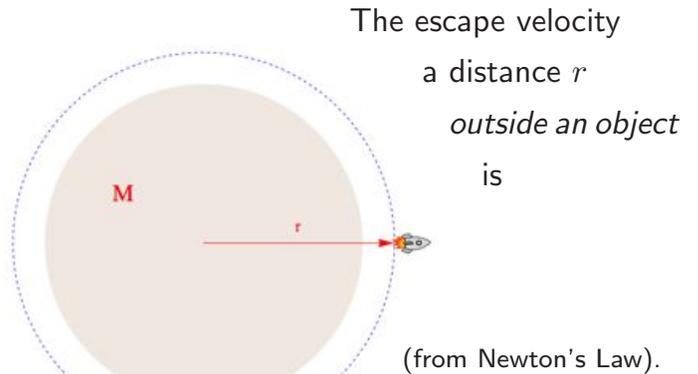
\_\_\_\_\_ These remnants are called

2

We can understand black holes, for now, in terms of \_\_\_\_\_.

Later, we'll see, we'll need a more sophisticated understanding.

3



4

Example: Earth

Mass of earth:  $\sim 6.7 \times 10^{24}$  kg

Radius of earth:  $6.4 \times 10^6$  m.

$G$  (Newton's grav. constant):  $\sim 6.7 \times 10^{-11}$ .

Then the escape velocity from the surface of Earth can be obtained from

$$v_{\text{esc}}^2 = \frac{2GM}{r}$$

5

$$= \frac{2(6.7 \times 10^{-11})(6.7 \times 10^{24})}{6.4 \times 10^6}$$

$$= \frac{2(6.7)(6.7) 10^{-11+24}}{6.4 10^6}$$

$$= \frac{2(6.7)(6.7) 10^{13}}{6.4 10^6}$$

$$= 14.03 \times 10^7 (\text{m/s})^2$$

6

ADDITIONAL NOTES

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Taking the square root gives you

$$v_{\text{esc}} = 11,844 \text{ m/s.}$$

This means that a rocket has to be given a speed of nearly 12 km/s in order to escape from Earth.

(In practice, it achieves escape in stages.)

7

(1) From the formula, for fixed  $M$ ,

$$v_{\text{esc}}^2 = \frac{2GM}{r}$$

does the escape velocity go up or down as  $r$  decreases?

=====

8

If the radius of Earth were somehow compressed to one-tenth,  $v_{\text{esc}} = 37,454 \text{ m/s.}$

to one-hundredth,  $v_{\text{esc}} = 118,440 \text{ m/s.}$

to one-thousandth,  $v_{\text{esc}} = 374,541 \text{ m/s.}$

(2) How compressed would Earth have to be in order for its escape speed to equal the speed of

9 light,  $c$ ?

We can answer this question by “inverting” the equation of the escape velocity to get

$$r = \frac{2GM}{v_{\text{esc}}^2}$$

and using  $v_{\text{esc}} = c = 3 \times 10^8 \text{ m/s:}$

$$\begin{aligned} r &= \frac{2(6.7 \times 10^{-11})(6.7 \times 10^{24})}{(3 \times 10^8)^2} \\ &= \frac{2(6.7)(6.7) 10^{13}}{3^2 10^{16}} \approx 10^{-2} \text{ m} = 1 \text{ cm.} \end{aligned}$$

10

The quantity

is called the \_\_\_\_\_,  $r_s$ , of an object of mass  $M$ . It represents the radius that the object would have to be compressed to in order for light not to escape from it.

For Earth  $r_s = 1 \text{ cm}$ , which is absurdly small.

11 What about for other objects?

Some Schwarzschild Radii

Object	$M$ (kg)	Real $r$ (m)	$r_s$ (m)
Sun	$2 \cdot 10^{30}$	$7 \times 10^8$	$3 \times 10^3$
W. dwarf	$3 \cdot 10^{30}$	$1 \times 10^7$	$4.5 \times 10^3$
N. star	$4 \cdot 10^{30}$	$12 \times 10^3$	$6 \times 10^3$
You	$5 \cdot 10^1$	$5 \times 10^{-1}$	$7 \times 10^{-26}$

(Your  $r_s$  is 7 trillionth of a trillionth of a cm, or about 7 hundred-billionth the radius of a proton.)

12

ADDITIONAL NOTES

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Till the late 1960s the compression needed for most objects to get squeezed under  $r_s$  seemed absurd, even for white dwarfs.

It was only when neutron stars were discovered and studied in 1967 that people realized that very highly compressed objects could exist: \_\_\_\_\_

13

More importantly we have strong and growing astrophysical evidence that not only do stars exist that *could* end up as black holes, stars that *have* ended up as black holes exist. \_\_\_\_\_

(3) Why *necessarily* indirect? \_\_\_\_\_

15

Stars that start at  $18M_{\odot}$ , or above, are left with remnants  $> 3M_{\odot}$ . These undergo unstoppable gravitational collapse, and get compressed under their Schwarzschild radii – become black holes.

How many stars do we know that are as massive?

The count in early 2018 is this:

14

(4) Is gravitational collapse the only way that black holes can form?

Black holes from gravitational collapse require a high density; but that is not necessary for a black hole to form.

16

Varieties of Black Holes

- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_

17

The Densities of Black Holes

The radius of a black hole is

The volume of a sphere is

For a black hole the volume is

The density is

18

ADDITIONAL NOTES

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(5) If the mass doubles how does density change?

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If the sun were to shrink to black hole size, its density would be

That's 20 million billion times the density of water.

19

An object that's two hundred million times as massive as the sun will have a density that's smaller by a factor of  $\frac{1}{4 \times 10^{16}}$

(6) Why? =====

=====

(7) What does the density work out to?

20

Therefore, you can make a black hole out of water, provided the available amount is about 100–200 million times the mass of the sun.

You do not need extreme *concentrations* of matter to form a black hole, as long as you have enough of it ("supermassive amounts").

21

(8) Where do we find such supermassive objects?

=====

Here are some (in solar masses,  $M_{\odot}$ ):

Milky Way:  $\sim$  four million  $M_{\odot}$

Andromeda:  $\sim$  one hundred million  $M_{\odot}$

M87:  $\sim$  six billion  $M_{\odot}$

How do we know they are there? From the dance of nearby objects with an invisible partner.

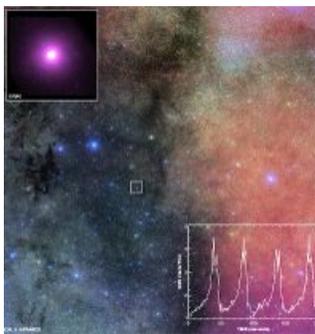
22

**Detection of Black Holes**

- =====
- =====
- =====
- =====
- =====
- =====

23

**GRS 1915+105**



- $14M_{\odot}$  BH-visible binary.
- $D=20,00\text{ly}$ ,  $d=58\text{ly}$ .
- X-ray pulses, 50s.
- Infalling matter.
- Observed: 2001.
- Pic released: 2011.

24

ADDITIONAL NOTES

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Centaurus A Center



- Jet from central BH.
- Top left jet: 13,000ly.
- Observed 2007.
- Announced 2009.

25

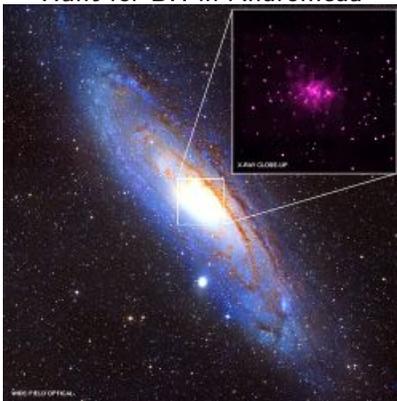
SDSS J1126+2944



- Galaxy/BH merger.
- D=1 billion ly.
- Smaller is intermediate?
- Still being studied.
- Announced 2016.

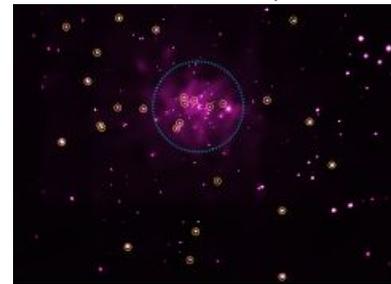
26

Hunt for BH in Andromeda



27

26 candidates identified (12 “strong”)



Data: 1999–2012. Published 2014.

28

Unusually large stellar BH



Galaxy IC 10, studied by Andrea Prestwich and team

29

THE ORBITAL PERIOD OF THE WOLF-RAYET BINARY IC 10 X-1: DYNAMIC EVIDENCE THAT THE COMPACT OBJECT IS A BLACK HOLE

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A. ZEAS,<sup>1</sup> S. H. SAAR,<sup>2</sup> T. P. ROBERTS,<sup>5</sup> AND M. J. WARD<sup>3</sup>

Received 2007 April 13; accepted 2007 September 18; published 2007 October 11

ABSTRACT

IC 10 X-1 is a bright ( $L_x = 10^{38}$  ergs  $s^{-1}$ ) variable X-ray source in the Local Group starburst galaxy IC 10. The most plausible optical counterpart is a luminous Wolf-Rayet star, making IC 10 X-1 a rare example of a Wolf-Rayet X-ray binary. In this Letter, we report on the detection of an X-ray orbital period for IC 10 X-1 of 34.4 hr. This result, combined with a reexamination of optical spectra, allows us to determine a mass function for the system of  $f(M) = 7.8 M_\odot$  and a probable mass for the compact object of 24–33  $M_\odot$ . If this analysis is correct, the compact object is the most massive stellar-mass black hole known. We further show that the observed period is inconsistent with Roche lobe overflow, suggesting that the binary is detached and that the black hole is accreting the wind of the Wolf-Rayet star. The observed mass-loss rate of [MAC92] 17A is sufficient to power the X-ray luminosity of IC 10 X-1.

They found a binary, with one very massive companion invisible.

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ADDITIONAL NOTES

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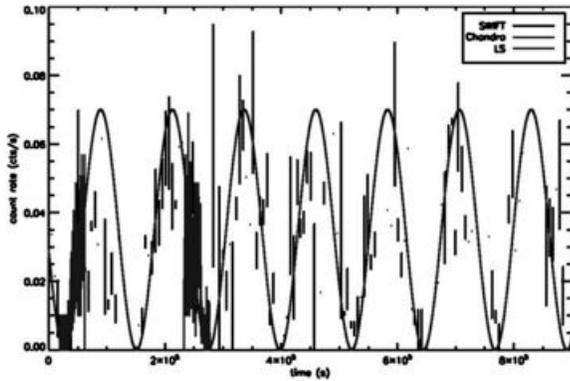
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31

Regular radiation fluctuations were the key.

“... scientists estimate that the black holes for this event were about 29 and 36 times the mass of the sun, and the event took place 1.3 billion years ago. About 3 times the mass of the sun was converted into gravitational waves in a fraction of a second – with a peak power output about 50 times that of the whole visible universe. By looking at the time of arrival of the signals – the detector in Livingston recorded the event 7 milliseconds before the detector in Hanford – scientists can say that the source was located in the Southern Hemisphere.”

<https://www.ligo.caltech.edu/news/ligo20160211>

33

▷ Does gravity affect light?

**Newton’s Law of Universal Gravitation**

$$F_{\text{grav}} = G \frac{m_1 m_2}{d^2},$$

$$G = 6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}$$

Gravitational constant

Every object ( $m_1$ ) attracts every other object ( $m_2$ ) by a force proportional to the product of the masses and inversely so to the square of the distance between them.

35

February 11, 2016

**“LIGO Opens New Window on the Universe with Observation of Gravitational Waves from Colliding Black Holes”**

“... the detected gravitational waves were produced during the final fraction of a second of the merger of two black holes to produce a single, more massive spinning black hole.

“... The gravitational waves were detected on September 14, 2015 at 5:51 a.m. Eastern Daylight Time.”

32

Defects in this whole narrative

The story so far is defective in several ways:

- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_

34

The formula for escape velocity is a consequence of Newton’s Law.

Newton’s law applies to objects with mass.

Does light have mass? If not, the escape velocity calculation cannot be used for it.

But still we expect a black hole not to allow light to escape.

36

ADDITIONAL NOTES

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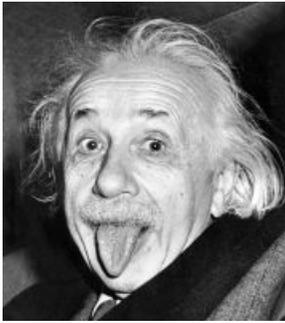


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The story starts with him ...



<http://www.npr.org/blogs/13.7/2011/09/28/140839445/is-einstein-wrong>

1

(1) Who's he?

=====

(2) What's he most known for?

=====

2

## Summary of the Theory of Relativity

Theory developed between 1905 and 1916, primarily by Albert Einstein.

First version (1905), called Special Relativity. Einstein worked for a decade on extending it, till he succeeded in 1915 (published in 1916) with the General Theory. General Relativity has four main ingredients:

3

1. =====

2. =====

3. =====

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4. =====

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4

It's not all words:

Einstein's theory, expressed via equations,

$$G_{ab} \equiv R_{ab} - \frac{1}{2}g_{ab}R = \frac{8\pi G}{c^4}T_{ab}$$

↓ - [g<sup>cd</sup> (∂<sub>a</sub>g<sub>ed</sub> + ∂<sub>e</sub>g<sub>ad</sub> - ∂<sub>d</sub>g<sub>ae</sub>)]

Spacetime Geometry [g<sup>cd</sup>(∂<sub>c</sub>g<sub>bd</sub> + ∂<sub>b</sub>g<sub>cd</sub>) - ∂<sub>b</sub>g<sub>cd</sub>] + Matter [∂<sub>d</sub>g<sub>cb</sub>]

Ricci Curvature, R<sub>ab</sub> + [g<sup>cd</sup>(∂<sub>a</sub>g<sub>bd</sub> + ∂<sub>b</sub>g<sub>cd</sub> - ∂<sub>d</sub>g<sub>ab</sub>)]

Curvature Scalar, R × [g<sup>cd</sup>(∂<sub>e</sub>g<sub>cd</sub> + ∂<sub>c</sub>g<sub>ed</sub> - ∂<sub>d</sub>g<sub>ec</sub>)]

Metric, g<sub>ab</sub>

5

Einstein's interest in the nature of the Universe went back, at least, to 1896 (he was 16). At the time people had hypothesized that the Universe was permeated by a substance called the "ether."

A document he wrote then was titled "On the Investigation of the State of the Ether in a Magnetic Field." (Document 5, Collected Papers of Albert Einstein, Vol. 1.)

6

### ADDITIONAL NOTES

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At about this time, these were his grades:

8. ENTRANCE REPORT OF THE GEWERBESCHULE, AARGAU KANTONSSCHULE  
[ca. 26 October 1895]

[...]  
Entered in autumn:  
[...]

Grade 3:

Einstein, Albert	14/III 1879	Gym. Munich	
G[erman]	3 - 2	Ph[ysics]	2
I[talian]	3	Ch[emistry]	Must do catch-up work
F[rench]	has gr. gaps	[History]	3 - 4
G[ometry]	3	[Natural history]	3
d[escriptive]		[provisionally accepted]	
M[athematics]	2		

Document 8

7

Einstein's first paper on relativity in 1905 led that year itself to . . .

8

**DOES THE INERTIA OF A BODY DEPEND UPON ITS ENERGY-CONTENT?**

By A. EINSTEIN

September 27, 1905

The results of the previous investigation lead to a very interesting conclusion, which is here to be deduced.

The mass-energy paper was short (three pages) at the end of which Einstein concluded

If a body gives off the energy  $L$  in the form of radiation, its mass diminishes by  $L/c^2$ . The fact that the energy withdrawn from the body becomes energy of radiation evidently makes no difference, so that we are led to the more general

Page 3

In other words,  $m = L/c^2$ , or, as we know it,

$$E = mc^2$$

To get to this Einstein had to make a leap on what energy is.

9

10

It was a bold leap, but Einstein was on a roll that year, 1905.

(He would also definitively establish that molecules exist, and produce his work on the photoelectric effect that won him his Nobel prize – and opens doors for you everywhere).

The possibility of releasing large amounts of energy

led to . . .

Albert Einstein  
Old Grove Rd.  
Hassau Point  
Peconic, Long Island  
August 2nd, 1939

F.D. Roosevelt,  
President of the United States,  
White House  
Washington, D.C.

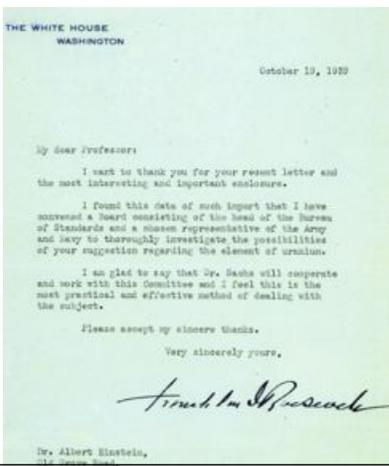
Sirs

Some recent work by E.Fermi and L. Szilard, which has been communicated to me in manuscript, leads me to expect that the element uranium may be turned into a new and important source of energy in the immediate future. Certain aspects of the situation which has arisen seem to call for watchfulness and, if necessary, quick action on the part of the Administration. I believe therefore that it is my duty to bring to your attention the following facts and recommendations:

In the course of the last four months it has been made probable - through the work of Joliot in France as well as Fermi and Szilard in America - that it may become possible to set up a nuclear chain reaction in a large mass of uranium, by which vast amounts of power and large quantities of new radium-like elements would be generated. How it appears almost certain that this could be achieved in the immediate future.

This new phenomenon would also lead to the construction of bombs,

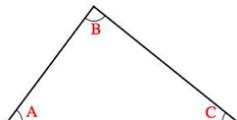
ADDITIONAL NOTES



13

### Introduction to Curved Geometry

A key feature of flat geometry is that the angles of a triangle always add to \_\_\_\_\_



15

Therefore, on a sphere

This is true of any triangle that you draw on a sphere with “straight lines” (lines of shortest distance).

17

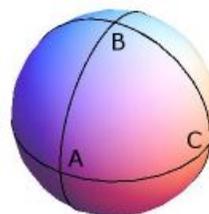
14

Humans are not the only entities that have now figured out that small amounts of mass can lead to vast quantities of energy ( $c^2$ ).

The sun (and other stars) have, as well.

But, on to curved geometry. . .

Now look at a triangle drawn on the surface of a sphere:



(3) What does  $\angle A$  seem to be? \_\_\_\_\_

16

(4) What does  $\angle C$  seem to be? \_\_\_\_\_

Surfaces where the angles of a triangle add to

- \_\_\_\_\_ have \_\_\_\_\_.
- \_\_\_\_\_ have \_\_\_\_\_, or are called \_\_\_\_\_.
- \_\_\_\_\_ have \_\_\_\_\_.

18

### ADDITIONAL NOTES

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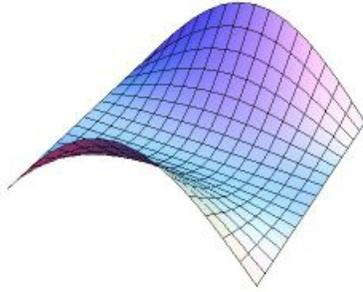


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A \_\_\_\_\_ is an example of a space with negative curvature:



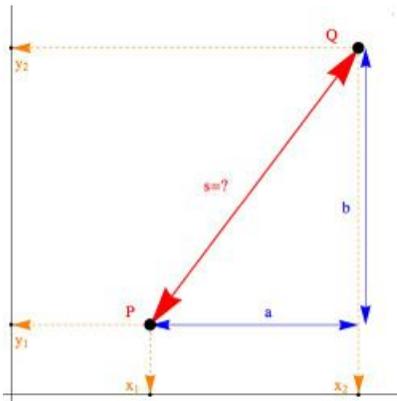
19

Another approach to curved geometry is through the distance formula. From Pythagoras' Theorem we know that the squared distance between two points  $(x_1, y_1)$  and  $(x_2, y_2)$  is

$$s^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

This is the 2-d flat space distance formula. When you use this formula it means that you are working with flat space.

20



2-d flat space distance formula

21

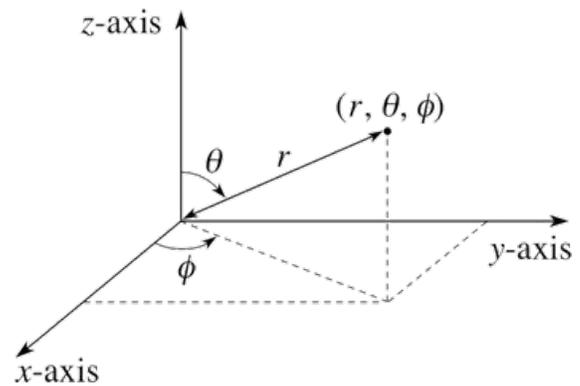
We'll need the notation that “ $dx$ ” means a (very small) difference in the variable  $x$ . The squared distance formula for flat space is then just the sum of squares of coordinate differences:

22

Every geometry, curved or flat, has a characteristic distance formula expressible via squares of the appropriate coordinate differences.

If you're discussing the geometry of a sphere you use a distance formula that defines *that* geometry. The standard coordinates here are basically latitude ( $\theta$ ) and longitude ( $\phi$ ).

23



Spherical coordinates

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ADDITIONAL NOTES

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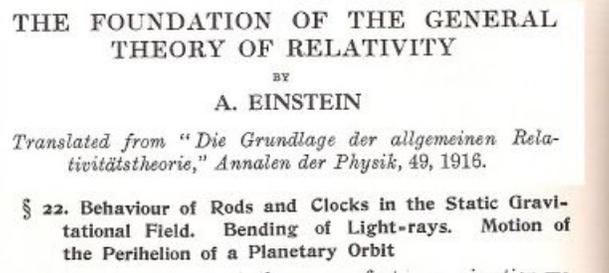
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The squared distance formula on a sphere is

$$ds^2 = r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2$$

25

Einstein made three predictions in his 1916 paper:



We'll discuss them in reverse order.

26

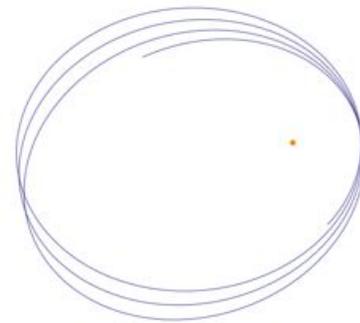
Einstein Test 3

Motion of the perihelion of a planet

The \_\_\_\_\_ of a planetary orbit is \_\_\_\_\_

A planet (Mercury, e.g.) goes around the sun on an elliptical path. But, the path does not close: the perihelion is not at the same point every year.

27 This is called \_\_\_\_\_



A precessing ellipse

28

Till Einstein, we could explain most of the precession, except for a small amount:

0.012° – every hundred years!

Einstein's proposal was that the matter of the sun warps surrounding spacetime geometry. Mercury moves on a straight line on this curved background.

Sounds weird, but you get exactly the extra 0.012°

29 that you need.

How do we describe the warped geometry of spacetime caused by a (roughly) spherical object such as the sun with mass  $m$ ?

Through a spacetime distance formula:

$$ds^2 = f(r)dt^2 - \frac{1}{f(r)}dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2)$$

where

$$f(r) = 1 - \frac{2Gm/c^2}{r} = 1 - \frac{r_s}{r}$$

30

ADDITIONAL NOTES

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The quantity  $r_s = \frac{2Gm}{c^2}$

is called the Schwarzschild radius, in honor of Karl Schwarzschild who arrived at this distance formula almost simultaneously with the final presentation of general relativity. The constant  $m$  has the units of mass, and we interpret it as the mass of the central object that's "creating" this curved geometry.

31

(5) Does  $\frac{2Gm}{c^2}$  appear elsewhere?

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A simple calculation based on Newtonian gravity gives the same expression as the more sophisticated analysis based on curved spacetime.

32

There are two problems with the distance formula that Schwarzschild found.

To see what they are, first answer this:

(6) What number can you not divide by? ==

33

In Schwarzschild's formula,

$$ds^2 = f(r)dt^2 - \frac{1}{f(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

there's the term with  $1/f(r)$  in it.

This would be a problem if there's a value of  $r$  that makes  $f(r) = 0$ .

34

(7) Remembering that  $f(r) = 1 - \frac{r_s}{r}$ , is there a value of  $r$  that makes  $f(r) = 0$ ?

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Therefore, it appears that  $r = r_s$  ought to be disallowed.

35

(8) There's a second problem, this time with  $f(r)$  itself. What value is  $r$  not allowed to have in the formula for  $f(r) = 1 - \frac{r_s}{r}$ ?

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36

ADDITIONAL NOTES

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Places where formulas involve division by zero are examples of “singularities.”

For decades people dismissed these problems.

Schwarzschild's formula was meant to describe the geometry *outside* a spherical object, and for these problems to become real, the object would have to have a radius  $r < 2Gm/c^2$  in the first case and

37  $r = 0$  in the second.

The mass of the earth is about  $6.7 \times 10^{24}$  kg,  $c \approx 3 \times 10^8$  m/sec, and  $G \approx 6.7 \times 10^{-11}$  m<sup>3</sup>/kg·sec<sup>2</sup>. In these units, what is  $r_s$  for the earth?

$$\begin{aligned} r_s &= \frac{2Gm}{c^2} \\ &= \frac{2(6.7 \times 10^{-11})(6.7 \times 10^{24})}{(3 \times 10^8)^2} \\ &= \frac{2(6.7)(6.7) 10^{13}}{3^2} \frac{1}{10^{16}} \approx 10^{-2} \text{m} = 1\text{cm}. \end{aligned}$$

38 That's absurdly small.

Some Other Schwarzschild Radii

Object	$M$ (kg)	Real $r$ (m)	$r_s$ (m)
Sun	$2 \cdot 10^{30}$	$7 \times 10^8$	$3 \times 10^3$
W. dwarf	$3 \cdot 10^{30}$	$1 \times 10^7$	$4.5 \times 10^3$
N. star	$4 \cdot 10^{30}$	$12 \times 10^3$	$6 \times 10^3$
You	$5 \cdot 10^1$	$5 \times 10^{-1}$	$7 \times 10^{-26}$

(Your  $r_s$  is 7 trillionth of a trillionth of a cm, or about 7 hundred-billionth the radius of a proton.)

39

The unrealistically small size of the Schwarzschild radius, coupled with the apparent singularity at  $r = r_s$  led people to believe that all physical objects must have a radius greater than their Schwarzschild radii.

Einstein came to believe this himself in a study he made of the Schwarzschild solution, and that was the dominant view till the 1960s.

40

**ON A STATIONARY SYSTEM WITH SPHERICAL SYMMETRY CONSISTING OF MANY GRAVITATING MASSES**

By ALBERT EINSTEIN  
(Received May 10, 1939)

p. 922

The essential result of this investigation is a clear understanding as to why the “Schwarzschild singularities” do not exist in physical reality. Although the theory given here treats only clusters whose particles move along circular paths it does not seem to be subject to reasonable doubt that more general cases will have analogous results. The “Schwarzschild singularity” does not appear for the reason that matter cannot be concentrated arbitrarily. And this is due to the fact that otherwise the constituting particles would reach the velocity of light.

This investigation arose out of discussions the author conducted with Professor H. P. Robertson and with Drs. V. Bargmann and P. Bergmann on the mathematical and physical significance of the Schwarzschild singularity. The problem quite naturally leads to the question, answered by this paper in the negative, as to whether physical models are capable of exhibiting such a singularity.

p. 936

41

Of course, we now know that such concentrated forms of matter can exist.

We call the *resulting spacetime geometry* \_\_\_\_\_

42

ADDITIONAL NOTES

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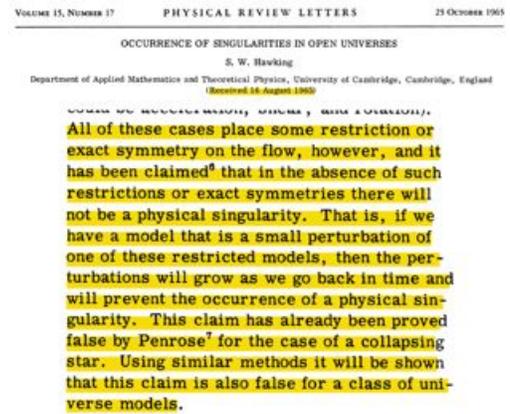
Black Holes gained acceptance in the 1960s through the work of Roger Penrose on the theoretical side, and the discovery of pulsars on the observational.

(Penrose’s work attracted the attention of a young scientist, looking for a reason to live.)

That scientist was \_\_\_\_\_



43



44



Stephen Hawking, mid 1960s (left) and around 2010 (right)

45

“One reason the apparent singularity at  $r = 2M$  was not investigated more thoroughly was that it was generally thought to be unphysical: no ‘real’ body would ever become so compressed that it would be inside its Schwarzschild radius. . .”

– Stephen Hawking in 1979 in “Some Strangeness in the Proportion,” An Einstein Centennial Symposium.

Note: We use  $M$  as shorthand for  $Gm/c^2$ .

46

The calculations of Chandrasekhar (and others) had shown that white dwarf stars that are more massive than about 1.4 solar masses will undergo an unstoppable gravitational collapse. They’ll presumably contract within their Schwarzschild radii.

It’s not completely surprising that that seemed far-fetched. The normal radius of a white dwarf is about 20 thousand times its Schwarzschild radius.

47

It was only with the discovery of neutron stars (“pulsars”) in the 1960s that we had clear confirmation that objects existed with sizes comparable to their Schwarzschild radii.

How are you to visualize all this? We use “ingoing” and “outgoing” light.

48

ADDITIONAL NOTES

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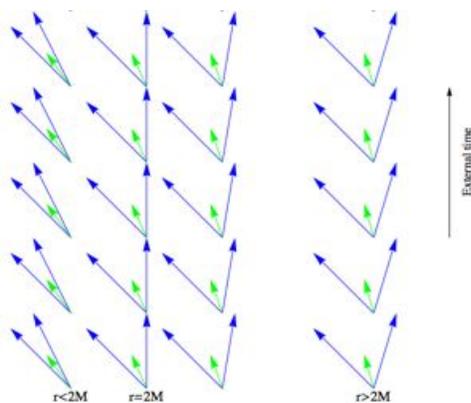
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49

This picture illustrates several important features of the Schwarzschild solution.

- 1) The blue arrows represent in-going out-going light.
- 2) The green arrows “inside” represent the potential directions for material particles.
- 3) “External time” agrees with the green arrows for  $r \gg 2M$  but not for  $r \leq 2M$ .
- 4) The surface at  $r = 2M$  is a “one way membrane”, *built out of pure geometry*.

50

It's the surface of a black hole. You can go in but you can't come out.



Attributed to Jon Carter(?)

51

Viewed this way, a black hole is a real object with an imaginary surface. That surface is called the event horizon.

The event horizon at  $r = 2M = 2Gm/c^2$  is imaginary in the sense that there is no physical object stationary there, no physical gates, no guards. It's a twist of geometry that allows you to go in, but not to come out.

52

### Blackholes: Friends or Frenemies?

A black hole at the center of our galaxy sounds bad news. Should we run?

Well outside a black hole you have none of the getting-sucked-in-and-never-being-seen-again-by-your-friends black hole headaches. If the sun should suddenly decide to go blackhole, we'd experience no gravitational setbacks.

53

(9) I said “getting-sucked-in-and-never-being-seen-again-by-your-friends,” not “getting-sucked-in-and-never-seeing-your-friends-again.” Why?

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(10) What was I getting at in the previous slide when I underlined gravitational?

54

### ADDITIONAL NOTES

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What can you do near and far from a B.H.?

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56

But there may be topological escapes:

PHYSICAL REVIEW D                      VOLUME 55, NUMBER 12                      15 JUNE 1997

**Regular black holes and topology change**

Arvind Borde\*

Institute of Cosmology, Department of Physics and Astronomy, Tufts University, Medford, Massachusetts 02155  
and Department of Mathematics, Southampton College, Long Island University, Southampton, New York 11968  
(Received 9 August 1996)

The conditions are clarified under which regular (i.e., singularity-free) black holes can exist. It is shown that in a large class of spacetimes that satisfy the weak energy condition the existence of a regular black hole requires topology change. [R0556-2821(97)04210-0]

PACS number(s): 04.70.Bw, 02.40.Pc, 04.20.Cv, 04.20.Dw

Mars et al. [1] have recently produced a "Schwarzschild black hole" that obeys the weak energy condition [2], but is nevertheless free of singularities. In this paper I clarify how the singularity avoidance occurs, not only in their model, but in general. My main result is that singularities can be avoided in a large class of black holes only through topology change. In proving this result I will use methods and terminology...

When  $\sigma^2 < (16/27)\pi^2$  in Bardeen's model, there is an event horizon. The new spacetime also obeys the weak energy condition (assuming Einstein's field equation), yet it contains no physical singularities. A similar example (i.e., possessing an event horizon and obeying the weak energy condition, yet nonsingular) has also been constructed by modifying the Reissner-Nordström metric in a region near  $r=0$  and leaving

57

Detection of Black Holes

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58

Einstein Test 2

The bending of light

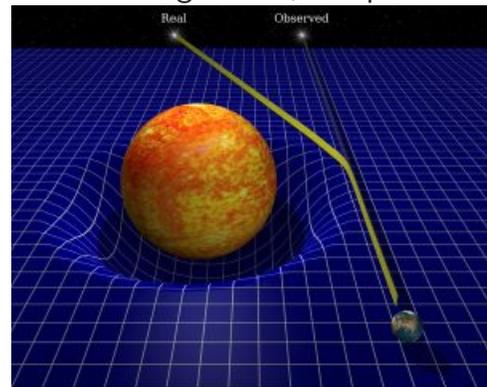
Light bends around objects like the sun.

Really? Does Gravity affect light?

Really (even though in Newtonian gravity, it's just mass that's involved).

59

Light travels in straight lines, except when it bends:



60

ADDITIONAL NOTES

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The English astronomer Arthur Stanley Eddington, and others, proposed a test of Einstein's prediction of the bending of light, to be done during a solar eclipse in Brazil on May 29, 1919.



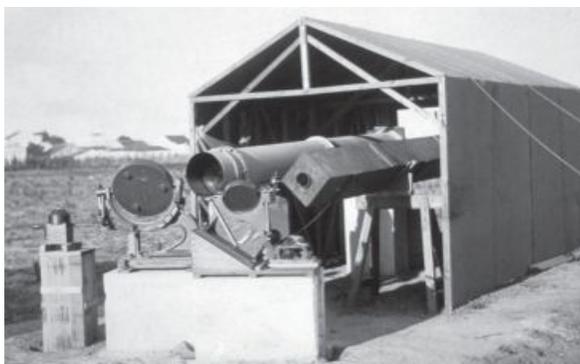
61

(11) Why solar eclipse? \_\_\_\_\_

\_\_\_\_\_

62

An expedition was organized:



63

They took photos with two telescopes in cloudy conditions of about a dozen stars near the sun during the eclipse, then of the same stars at night two months later.

(12) Why did they return two months later?

\_\_\_\_\_

\_\_\_\_\_

64

The team returned to England to compare and analyze the photographic plates.

They asked for a special joint meeting of the Royal Astronomical Society and the Royal Society of London for November 6, 1919, to make an announcement.

65

The prediction from Einstein's theory was that the angular positions of stars near the sun would shift by  $1.75''$ . The Eddington expedition results were

Telescope 1:  $(1.98 \pm 0.12)''$

Telescope 2:  $(1.61 \pm 0.30)''$

Given the small number of stars looked at, these are not completely convincing results.

66

ADDITIONAL NOTES

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Reaction at the meeting was mixed.

One person present called it

“the most important result obtained in connection with the theory of gravitation since Newton’s day.”

But another pointed to a portrait of Newton hanging in the room and urged caution:

“We owe it to that great man to proceed very carefully in modifying or retouching his Law of Gravitation.”

67

Einstein had been kept informed as the data was analyzed. He had always been confident.

On September 27, nearly 6 weeks before the official announcement, he wrote to his mom:

“...joyous news today. ... the English expeditions have actually measured the deflection of starlight from the Sun.”

68

The Press shared Einstein’s enthusiasm. The London Times of November 7, 1919, one day later, carried a long article about the Royal Society meeting, headlined

REVOLUTION IN SCIENCE  
NEW THEORY OF THE UNIVERSE

Three days later The New York Times got into it...

69

LIGHTS ALL ASKEW  
IN THE HEAVENS

Men of Science More or Less  
Agog Over Results of Eclipse  
Observations.

EINSTEIN THEORY TRIUMPHS

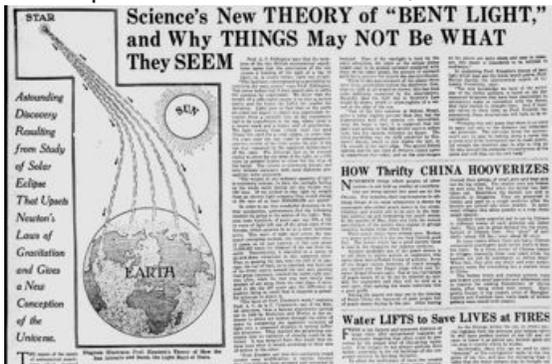
Stars Not Where They Seemed  
or Were Calculated to be,  
but Nobody Need Worry.

A BOOK FOR 12 WISE MEN

No More in All the World Could  
Comprehend It, Said Einstein When  
His Daring Publishers Accepted It.

70

The news spread all over the world, even Vermont:



71

In 1921 Einstein visited New York:



72

ADDITIONAL NOTES

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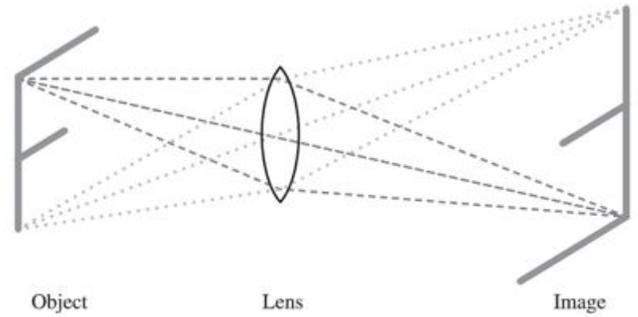
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The bending of light has since been confirmed by many precise experiments to agree exactly with the theory of relativity.

Now, the bending of light can cause lensing.

What's lensing? This...

73



Anything that bends light can cause lensing: “normal” lenses, mirrors, or ... gravitation.

74

Einstein realized this. He presented the results in 1936, but said he thought it impractical to observe:

**LENS-LIKE ACTION OF A STAR BY THE DEVIATION OF LIGHT IN THE GRAVITATIONAL FIELD**

SOME time ago, R. W. Mandl paid me a visit and asked me to publish the results of a little calculation, which I had made at his request. This note complies with his wish.

The light coming from a star  $A$  traverses the gravitational field of another star  $B$ , whose radius is  $R$ . Let there be an observer at a distance  $D$  from  $B$  and at a distance  $s$ , small compared with  $D$ , from the extended central line  $AB$ . According to the general theory of relativity, let  $\alpha$  be the deviation of the light ray passing the star  $B$  at a distance  $b$  from its center.

not decrease like  $1/D$ , but like  $1/\sqrt{D}$ , as the distance  $D$  increases.

Of course, there is no hope of observing this phenomenon directly. First, we shall scarcely ever approach closely enough to such a central line. Second, the angle  $\beta$  will defy the resolving power of our instruments. For,  $\alpha$ , being of the order of magnitude of one second of arc, the angle  $R_0/D$ , under which the deviating star  $B$  is seen, is much smaller. Therefore, the light coming from the luminous circle can not be distinguished by an observer as geometrically different from that coming from the star  $B$ , but simply will manifest itself as increased apparent brightness of  $B$ . The same will happen, if the observer is situated at a small distance  $s$  from the extended central line  $AB$ .

75

In a letter to the editors of the journal Science, Einstein added “Let me also thank for you for your cooperation with the little publication which Mister Mandle squeezed out of me. It is of little value, but it makes the poor guy happy.”

Einstein was wrong on the unobservability of gravitational lensing.

77

DECEMBER 4, 1936

SCIENCE

If we are interested mainly in the case  $q \gg 1$ , the formula

$$q = \frac{l}{x}$$

is a sufficient approximation, since  $\frac{x^2}{l^2}$  may be neglected.

Even in the most favorable cases the length  $l$  is only a few light-seconds, and  $x$  must be small compared with this, if an appreciable increase of the apparent brightness of  $A$  is to be produced by the lens-like action of  $B$ .

Therefore, there is no great chance of observing this phenomenon, even if dazzling by the light of the much nearer star  $B$  is disregarded. This apparent amplification of  $q$  by the lens-like action of the star  $B$  is a most curious effect, not so much for its becoming infinite, with  $x$  vanishing, but since with increasing distance  $D$  of the observer not only does it not decrease, but even increases proportionally to  $\sqrt{D}$ .

ALBERT EINSTEIN

INSTITUTE FOR ADVANCED STUDY,  
PRINCETON, N. J.

76

He based his skepticism on the resolving power of telescopes and the improbability of the alignment needed for lensing to occur:

The lensed star,  $A$ , the lensing star,  $B$ , and the observe must line up in almost a straight line.

78

ADDITIONAL NOTES

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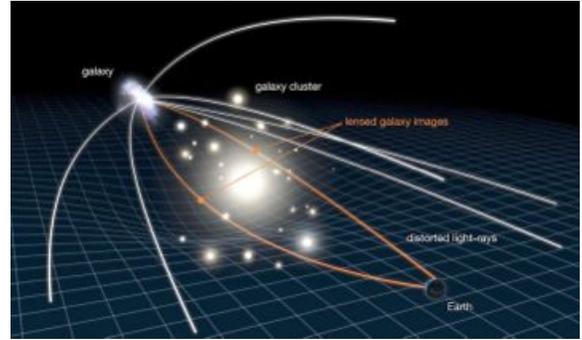
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(13) What did he not foresee?

- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_

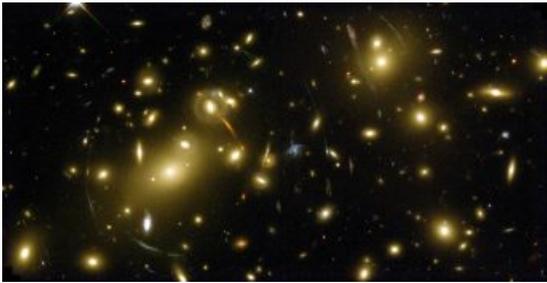
79

Gravitational lensing:



80

Gravitational lensing causes distortions, magnifications, and multiple images (can be formed at different times because of the long travel paths).



81

Einstein Test 1

The behavior of rods and clocks

Space *and* time are warped.

The behavior of time is particularly interesting.

Clocks tick slightly slower on the surface of the earth than on the top of tall buildings or in planes.

Motion also affects the “flow” of time.

82

Tests of Time Alterations

a) In 1971 Keating and Hafele flew four caesium atomic clocks around the world on commercial aircraft, first traveling from east to west, then from west to east. The results of the experiment confirmed the relativistic predictions within 10%. The experiment was repeated in 1996 on a trip from London to Washington and back, a 14 hour journey. The result was within 2 ns of the prediction.

83

b) Muon lifetime [Bailey, J. et al. Nature **268**, 301 (1977)]: Muons with “rest lifetime” of  $2.198 \mu\text{s}$  were sped to high speed ( $.999c$ ). The measured lifetimes at those speeds were found to be  $64.368 \mu\text{s}$ , consistent with relativity.

84

ADDITIONAL NOTES

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c) paper in *Science*, 2010:  
**Optical Clocks and Relativity**  
 C.W.Chou, D.B.Hume, T.Rosenband, D.J.Wineland

*Science*, 24 Sep 2010, Vol.329, Issue 5999, pp.1630-1633

“Observers in relative motion or at different gravitational potentials measure disparate clock rates. . . . We observed time dilation from relative speeds of less than 10 meters per second by comparing two optical atomic clocks connected by a 75-meter length of optical fiber. We can now also detect time dilation due to a change in height near Earth’s surface of less than 1 meter.”

85

d) GPS



86

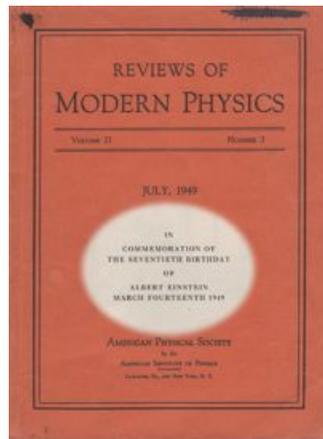
From: [www.losangeles.af.mil/shared/media/document/AFD-100302-043.doc](http://www.losangeles.af.mil/shared/media/document/AFD-100302-043.doc)

3.3.1.1 Frequency Plan. . . . The SV carrier frequency and clock rates – as they would appear to an observer located in the SV – are **offset to compensate for relativistic effects**. The clock rates are offset by  $\Delta f/f = -4.4647 \times 10^{-10}$ , equivalent to a change in the I5 and Q5-code chipping rate of 10.23 MHz offset by a  $f = 4.5674 \times 10^{-3}$  Hz.

3.3.4 GPS Time and SV Z-Count. GPS time is established by the Operational Control System (OCS) and is referenced to Coordinated Universal Time (UTC) as maintained by the U.S. Naval Observatory (UTC(USNO)) zero time-point defined as midnight on the night of January 5, 1980/morning of January 6, 1980. GPS time is the ensemble of corrected composite L1/L2 P(Y) SV times, corrected via the clock corrections in the L1 and L2 NAV data and **the relativity correction**.

87

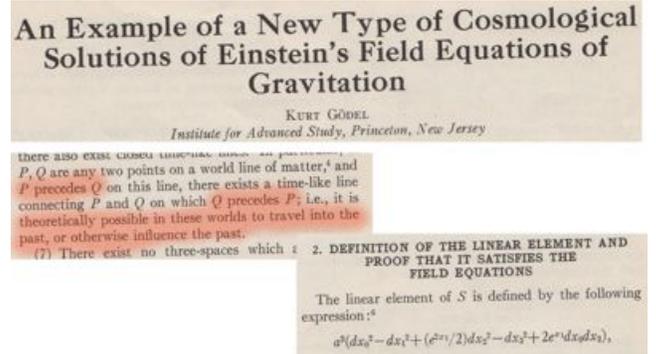
So time can be twisted in relativity.  
 How kinky can time get?



89

88

Pretty kinky. . .



90

ADDITIONAL NOTES

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### Gravitational Radiation

Einstein continued to work on relativity after his paper on the foundations of the theory.

Later that same year he discovered that his theory permitted gravitation to travel as “waves:” ripples in the fabric of spacetime.

He’d have a rocky relationship with grav. radiation.

91

#### 32. “Approximative Integration of the Field Equations of Gravitation”

[Einstein 1916g]

SUBMITTED 22 June 1916  
PUBLISHED 29 June 1916

In: *Königlich Preußische Akademie der Wissenschaften (Berlin). Sitzungsberichte* (1916): 688–696.

#### §2. Plane Gravitational Waves

It follows from equations (6) and (9) that gravitational fields always propagate with velocity 1, i.e., with the velocity of light. Plane gravitational waves, traveling along the positive X-axis, can therefore be found by setting

$$\gamma'_{\mu\nu} = \alpha_{\mu\nu} f(x_1 + ix_4) = \alpha_{\mu\nu} f(x - t). \tag{15}$$

92

He followed in 1918 with a full paper on the subject:

#### 1. “On Gravitational Waves”

[Einstein 1918a]

SUBMITTED 31 January 1918  
PUBLISHED 21 February 1918

In: *Königlich Preußische Akademie der Wissenschaften (Berlin). Sitzungsberichte* (1918): 154–167.

The important question of how gravitational fields propagate was treated by me in an academy paper one and a half years ago.<sup>1</sup> However, I have to return to the subject matter since my former presentation is not sufficiently transparent and, furthermore, is marred by a regrettable error in calculation.

93

Nearly twenty years later he began to have doubts. He wrote to his friend Max Born in 1936:

“Together with a young collaborator, I arrived at the interesting result that gravitational waves do not exist”

and sent a paper, “Do Gravitational Waves Exist?,” to a newish physics journal *The Physical Review*.

94

The paper had mistakes. They were caught by the referee. Einstein was not pleased at being refereed at all:

“Dear Sir,  
We (Mr. Rosen and I) had sent you our manuscript for publication and had not authorized you to show it to specialists before it is printed. I see no reason to address the – in any case erroneous – comments of your anonymous expert. On the basis of this incident I prefer to publish the paper elsewhere.”

95

Although Einstein rejected the referee’s criticism, it seemed to sow doubt.

Rosen and he sent a new paper to the *Journal of the Franklin Institute* with the opposite conclusion: gravitational waves do exist.

#### ON GRAVITATIONAL WAVES.

BY

A. EINSTEIN and N. ROSEN.

ABSTRACT.

The rigorous solution for cylindrical gravitational waves is given. For the

96

#### ADDITIONAL NOTES

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We've been hunting for gravity waves for over 80 years.

They were discovered for the first time in September 2015.

97

Why are gravity waves important to find?

They provide a new window into the Universe.

So far, everything we know is through electromagnetic radiation.

If we can reliably detect gravitational radiation from astronomical events, it will allow us to understand what's going on in a new way.

99



101

Why are gravity waves hard to find?

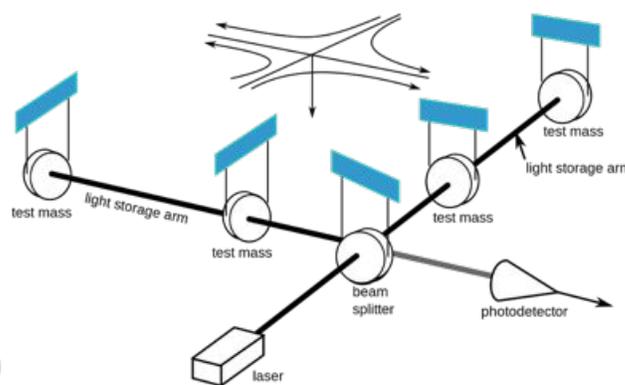
The strengths are expected to be weak. There's a parameter that's used

$$h = \frac{4G(KE-ns)}{rc^4}$$

where  $r$  is the distance, and KE-ns is the non-spherical kinetic energy of the source. Given that  $G$  is small and  $c$  is large, we need huge energies in the source to provide a measurable signal.

98

How were gravity waves found?



100



102

ADDITIONAL NOTES

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**Light**

(1) How do we know the rest of the astronomical Universe exists? (We cannot feel it, smell it, hear it or taste it.) \_\_\_\_\_

(2) What do we need in order to see (besides our eyes)? \_\_\_\_\_

1

**What is Light?**

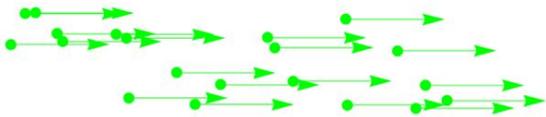
When people say "light" they usually mean visible light.

But it's one form of a larger phenomenon called \_\_\_\_\_ We'll use "light" to cover the whole spectrum.

Light has a dual nature: \_\_\_\_\_ and \_\_\_\_\_

2

**Light as a Particle**



The individual particles ("packets of energy") are called ... \_\_\_\_\_

3

The person who owned this nice pair of legs is responsible for the photon:



(3) Who is it?

4

Einstein used the "particle nature of light" to explain the \_\_\_\_\_.

This is the effect where light, shining on certain materials, sets up electric currents.

For most astronomical uses, however, it's the "wave nature of light" that's more important.

5

**Light as a Wave**

(4) What's a wave? \_\_\_\_\_

The two key attributes of waves are:

- \_\_\_\_\_
- \_\_\_\_\_

The speed of a wave,  $c$ , is related to these two by

6

ADDITIONAL NOTES

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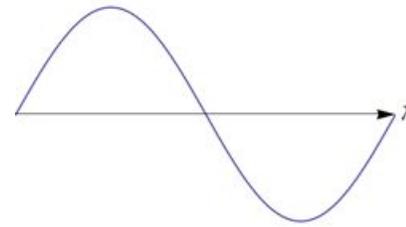
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(5) If two waves (A, B) have the same speed, but A has twice the wavelength of B, how are their frequencies related? \_\_\_\_\_

(6) If two waves (C, D) have the same speed, but C has three times the frequency of D, how are their wavelengths related? \_\_\_\_\_

7 \_\_\_\_\_

Visualizing wavelength

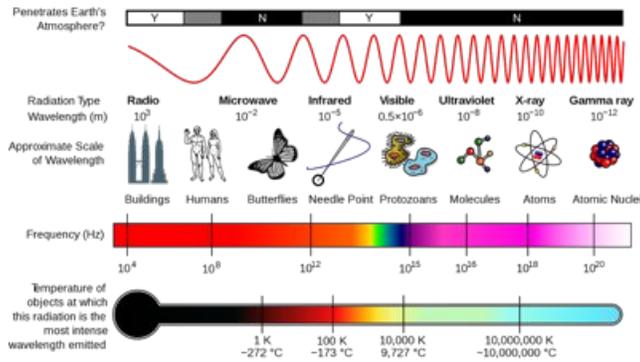


\_\_\_\_\_

\_\_\_\_\_

8

Different forms of e.m. radiation have different  $\lambda$ s:



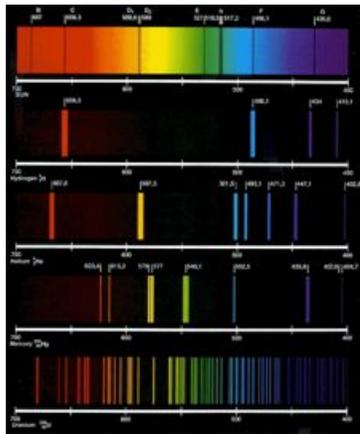
9

Spectral Lines

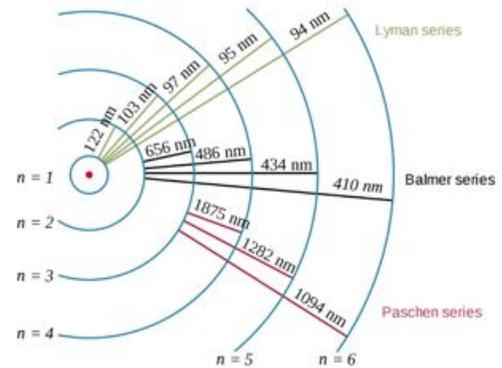
Elements emit radiation at characteristic wavelengths – a sort of signature of the element. These are called its \_\_\_\_\_.

The radiation from an astronomical object can be decomposed, via an instrument called a spectrometer, into its spectral lines. From those we can precisely identify the elements in the object.

10



11



How spectral lines arise.

12

ADDITIONAL NOTES

\_\_\_\_\_

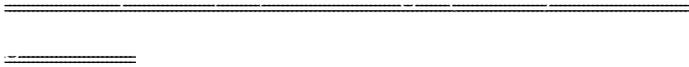
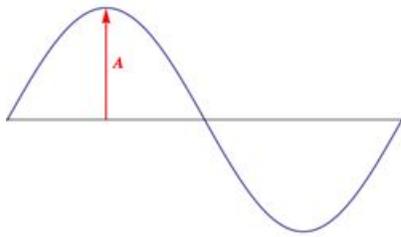
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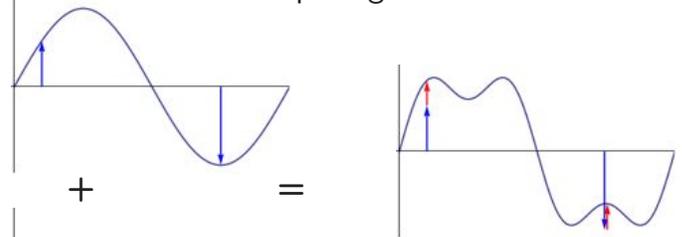
\_\_\_\_\_

Wave Amplitude



13

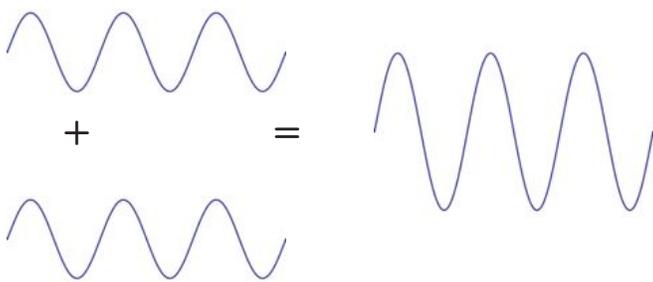
Superposing Waves



When two waves meet their amplitudes add, as shown.

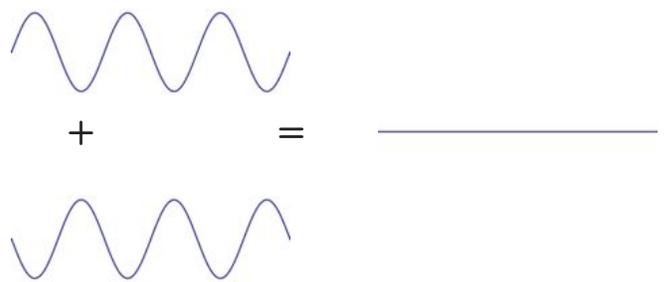
14

Constructive Interference



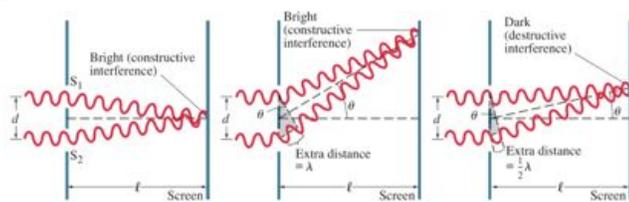
15

Destructive Interference



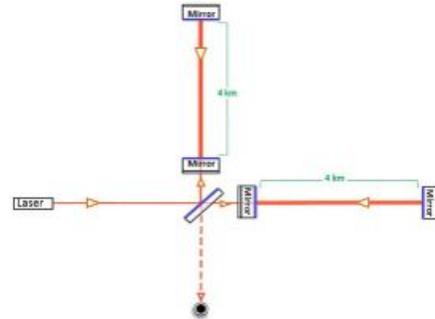
16

These effects, where waves can reinforce each other or cancel each other, lead to patterns of dark and light lines called an interference pattern.



17

A (Michelson) interferometer detects changes in the arm lengths by looking for interference.



18

ADDITIONAL NOTES

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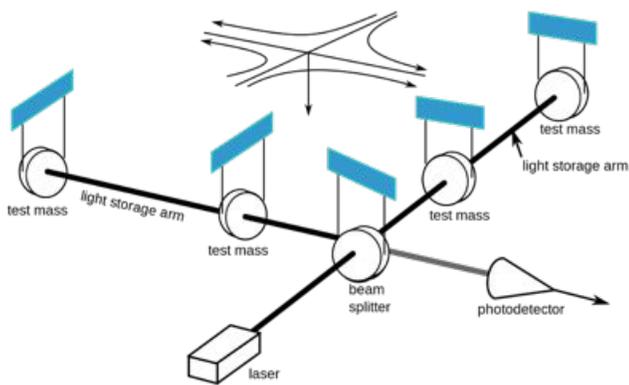


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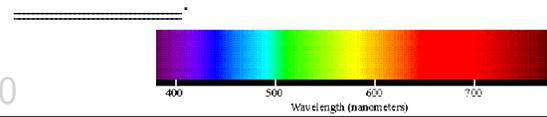
Reminder: How were gravity waves found?



19

### The Doppler Effect

As objects move away from us, the wavelength of signals they emit gets stretched (gets longer) compared to normal. The faster an object moves from us, the longer the wavelength of the radiation from it and the “redder” it looks. This is called



20

Similarly, as objects move toward us, the wavelength of signals they emit gets compressed (gets shorter) compared to normal.

The faster an object moves toward us, the shorter the wavelength of the radiation from it and the “bluer” it looks. This is called \_\_\_\_\_.

(7) Can you explain this with frequencies? \_\_\_\_\_

21



22

### Energy

The energy in an EM wave is proportional to the frequency:

\_\_\_\_\_

As the frequency goes up, the energy goes \_\_\_\_\_

23

It’s the energy in electromagnetic radiation that can be harmful.

(8) Is uv radiation likely to be more or less harmful than infrared? \_\_\_\_\_

(9) Why?

\_\_\_\_\_  
\_\_\_\_\_

24

### ADDITIONAL NOTES

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\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

Telescopy

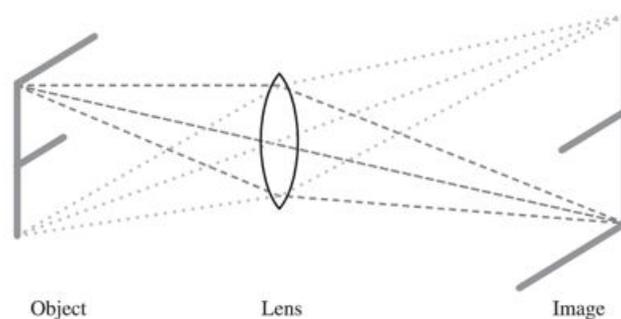
(10) How does light travel?

=====

=====

25

The bending of light causes \_\_\_\_\_



26

On the previous diagram, different rays of light travel along different paths and reunite (“focus”) at the image.

There’s an interesting consequence:

(11) What would happen if part of the lens were taped off? Would part of the image disappear?

=====

27

Refracting Telescopes

Lensing is the basis of class of cameras, telescopes, etc., called \_\_\_\_\_

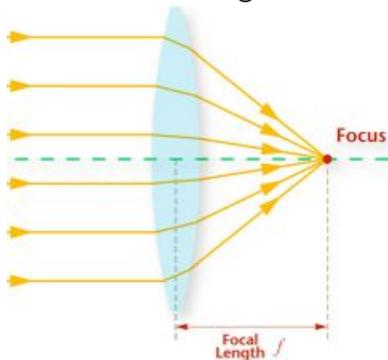
These are devices that work like the eye does.

A key attribute of a lens is its \_\_\_\_\_

This is \_\_\_\_\_

28

Focal length



29

Image Size

If an object “occupies” an angle  $\theta$  (in degrees) in the field of view, then the image size,  $s$ , it makes on a seeing device with focal length  $f$  is

\_\_\_\_\_

=====

$s$  has the same units as  $f$ .

30

ADDITIONAL NOTES

\_\_\_\_\_

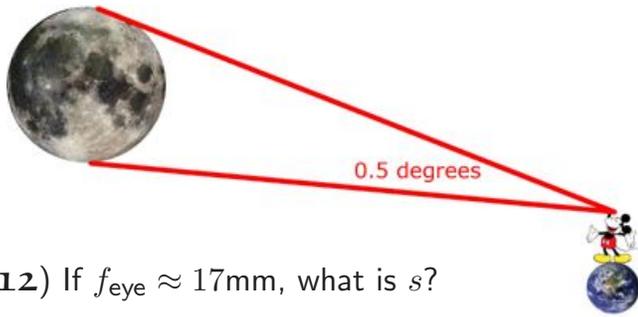
\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_



(12) If  $f_{\text{eye}} \approx 17\text{mm}$ , what is  $s$ ?

31

That's the diameter of the image the moon makes on your retina.

(13) A telescope with a focal length that's a thousand times as long (17m), will make an image that's how large?

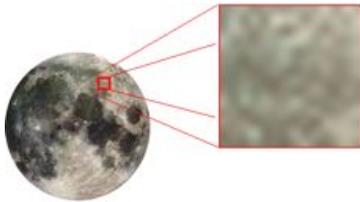
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32

Image Resolution

Simply increasing the size of an image does not mean you'll see more detail.



You also need to increase the \_\_\_\_\_

33

The \_\_\_\_\_ of a device is the \_\_\_\_\_.

The formula (for resolution in degrees) is

where  $D$  is the diameter and  $\lambda$  the wavelength.

=====

=====

34

(14) Do you get better resolution with red light or blue?

=====

(15) Looking at the formula for resolution, what might you change in a device to get better resolution across wavelengths?

=====

35

(16) Visible light has an average wavelength of  $\lambda = 5 \times 10^{-7}\text{m}$ . What is the resolution of a telescope with a 1 m lens (diameter)?

36

ADDITIONAL NOTES

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The angles in angular resolutions are usually very small.

We use fractions of a degree when representing small angles:

$\left(\frac{1}{60}\right)^\circ = 1'$  (one \_\_\_\_\_).

$\left(\frac{1}{60}\right)' = \left(\frac{1}{3600}\right)^\circ = 1''$  (one \_\_\_\_\_).

37

How to make better refracting telescopes

- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_

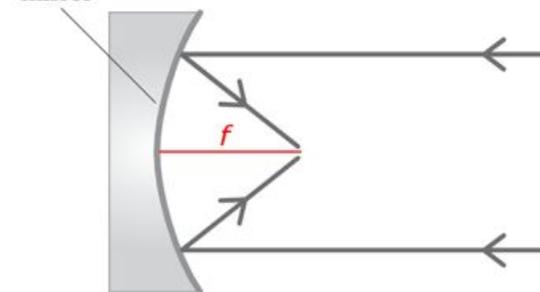
38

Problems with refracting telescopes

- Long tubes tend to \_\_\_\_\_
- Larger lenses are \_\_\_\_\_
- \_\_\_\_\_
- Blurring due to \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_
- Chromatic aberration: \_\_\_\_\_
- \_\_\_\_\_

39

Reflecting Telescopes



40

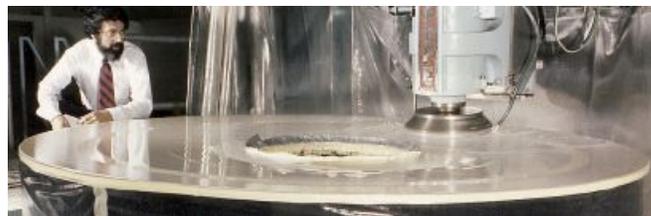
Advantages of Reflecting Telescopes

- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_

41



The mirror backing is often glass, because it can be shaped with high precision.



Polishing the mirror for the Hubble space telescope

42

ADDITIONAL NOTES

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How much precision do you need?

It depends on the wavelength. For observations at wavelength  $\lambda$ , \_\_\_\_\_

\_\_\_\_\_

(17) Do you need a smoother surface for a radio telescope or an optical one?

\_\_\_\_\_

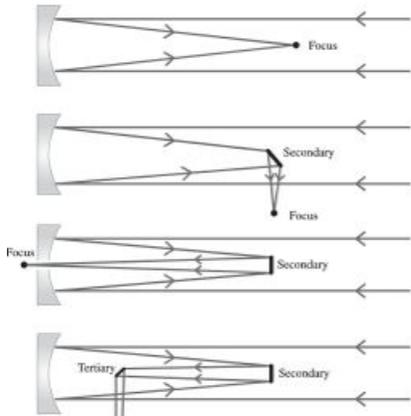
43

The reflective surface depends on use: aluminum for visible wavelength, gold for infra-red, etc.

(18) Looking at gold, what says it's a better reflector at the red(ish) end of the spectrum than the blue? \_\_\_\_\_

(19) There seems a flaw in reflecting telescopes. What? \_\_\_\_\_

44



Different reflector models.

The blocking of light in the middle does not obscure part of the image, for the same reason as it does not for lenses.

It just \_\_\_\_\_

The next few slides show you some telescopes used by astronomers.

45

46

▷ BICEP and Keck Arrays

We travel first to the South Pole:



● What does this telescope do?

Collecting for several years, starting in 2006, low-temperature microwave radiation from the very early Universe (around the “big bang”).

The project is in its third phase, BICEP3.

(An early announcement from BICEP2 in 2014 on the nature of “big bang radiation” was premature.)

47

48

ADDITIONAL NOTES

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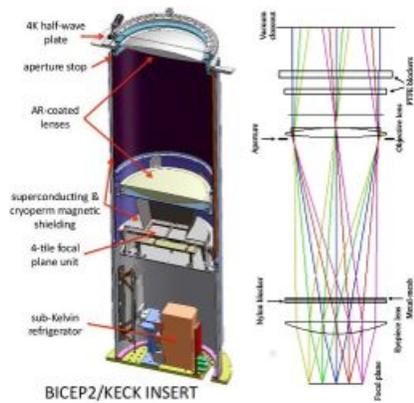


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BICEP2 interior

49



BICEP2 schematics

50

▷ Subaru optical/infrared



51

Located on the summit of Mauna Kea, 4,205m high (dry, stable air).

(At least 13 telescopes from 11 countries there.)



52

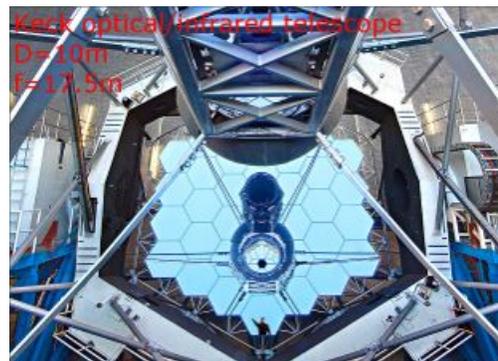
● What has Subaru done?

- August 2013: Direct sighting of a Jupiter-like planet around another star.
- January 2016: Start of hunt for “Planet X,” a possible Neptune sized planet beyond Pluto.
- January 2017: Star forming galaxies in the distant (therefore, early) Universe.

<http://subarutelescope.org/Pressrelease/list.html>

53

▷ Keck



54

ADDITIONAL NOTES

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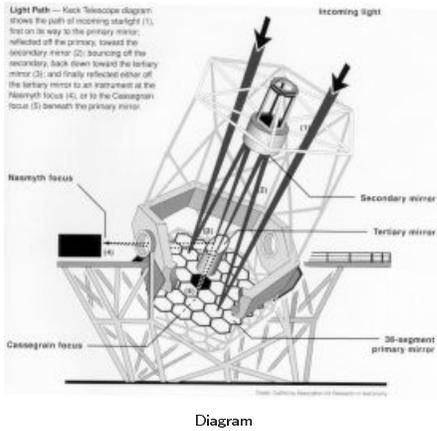
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55

● What has Keck done?

- March 5, 2015: 4-image cosmic lens.
- March 5, 2015: Evidence for a past ocean on Mars from analysis of the atmosphere.
- February 10, 2017: Dwarf star system with carbon, nitrogen, oxygen, and hydrogen.
- Ongoing: Galactic center, UCLA group.

<http://www.keckobservatory.org/recent/type/news>

56

▷ Arecibo



57

A telescope in the ground? Look at rooftops:



58

These are all satellite dishes:

Wavelengths:  $\lambda \sim 1.5\text{--}3\text{cm}$



59

● What has Arecibo done?

- 2017: Limits on g. waves.
- Multi-year: Asteroid hunter.
- 2011: Cold brown dwarfs.
- 1974: Attempted CETI. →
- 1974: T&H, binary pulsar.
- 1964: Mercury rot. 59 days.



60

ADDITIONAL NOTES

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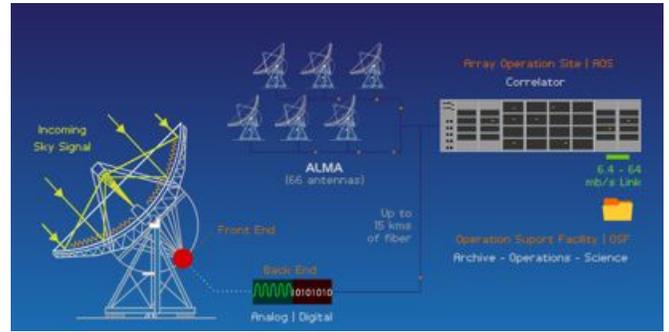


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▷ ALMA array



61



Schematic

62

● What has ALMA done?

- February 2015: Star form., Sculptor Galaxy
- December 2014: Black hole jets blowing away the Hydrogen from entire galaxies.
- November 2014: Formation of stars  $> M_{\odot}$ .



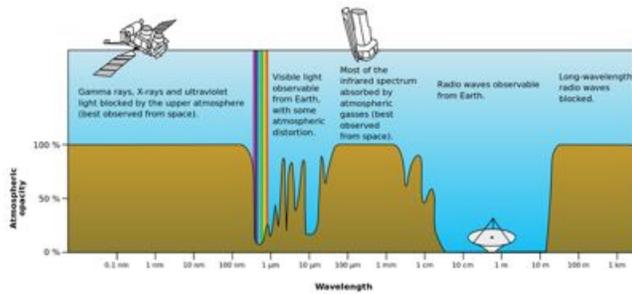
63

These were telescopes based on earth. They have limitations:

- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_

64

At certain wavelengths, we need telescopes based in space:



65

▷ Hubble, 1990 (optical)



66

ADDITIONAL NOTES

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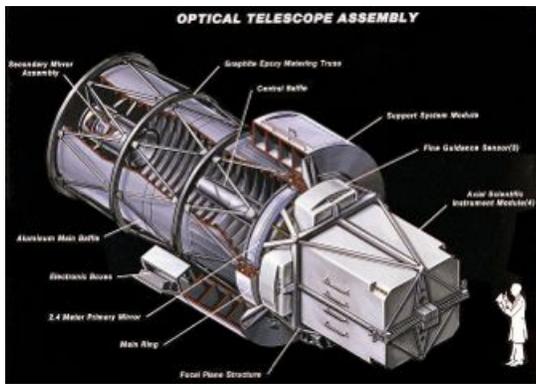
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Inside Hubble

67



Fixing Hubble

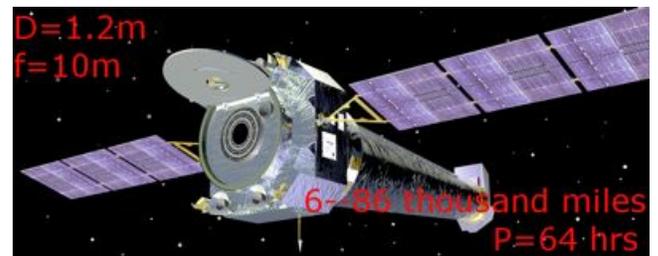
68

● What has Hubble done?

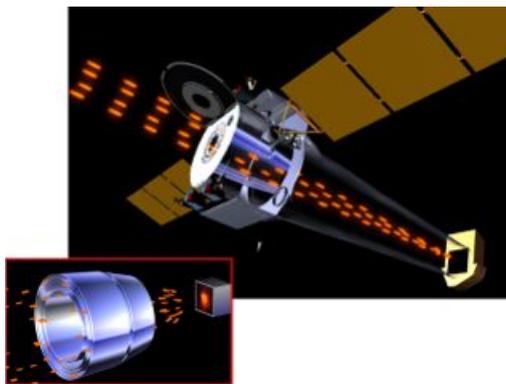
- 2001: Planet forming disks common around stars; found planets.
- 1999–2002: Age of Univ.: ~ 13 giga-yrs.
- 1998–2001: Accelerated expansion of Universe.
- 1997: Black holes at centers of galaxies.
- 1996: 1,000s of pictures of galaxy evolution.

69

▷ Chandra, 1999 (x-ray)



70



The Chandra "mirror"

71

● What has Chandra done?

- January 2015: Bright X-ray flare from MW black hole.
- 2012: Halo of hot gas around MW.
- 2006: Strong evidence for dark matter.
- 2002: Possible evidence for quark stars.
- 2000: Possible mid-sized black holes.

72

ADDITIONAL NOTES

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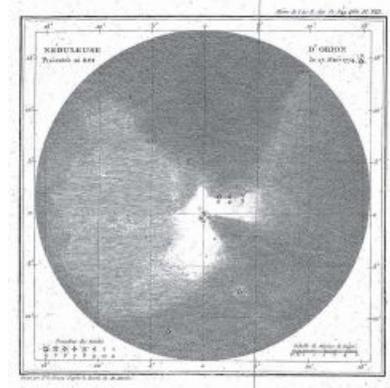
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# Arvind Borde / AST 10, Week 8: Galaxies I

It's been known by us, from the dawn of us, that there are stars in the sky.

There are also \_\_\_\_\_: objects outside the solar system that had a diffuse appearance rather than a pointlike image, as in the case of a star.

1



Drawing from 1771.

2



Photo from 1880.

3

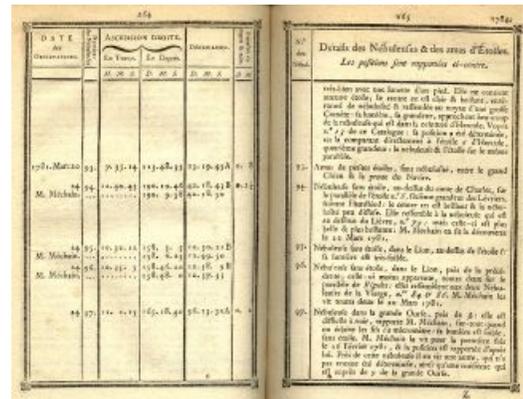
In the late 1700s, the French astronomer Charles Messier catalogued 110 nebulous objects in the sky and carefully noted their positions.

He did so, because comet-hunting was considered important back then, and he wanted astronomers to know which fuzzy objects were *not* comets.

4



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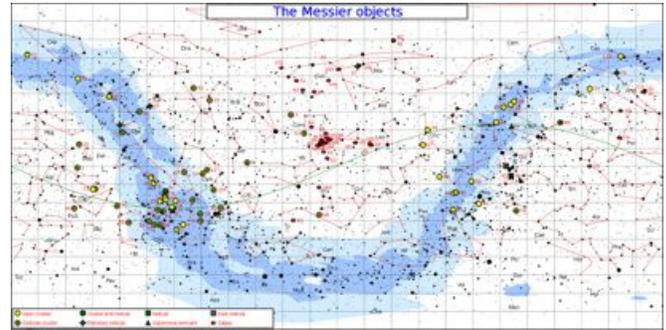
## ADDITIONAL NOTES

Here is #31 in his catalog:

31. 0h 29m 46s (7d 26' 32") +39d 09' 32"  
 (August 3, 1764) 'The beautiful nebula of the belt of Andromeda, shaped like a spindle; M. Messier has investigated it with different instruments, & he didn't recognise a star: it resembles two cones or pyramids of light, opposed at their bases, the axes of which are in direction NW-SE; the two points of light or the apices are about 40 arc minutes apart; the common base of the pyramids is about 15'. This nebula was discovered by Simon Marius, & consequently observed by different astronomers.

7

Here is the full chart:



8

Messier objects through the telescopes of today:



9

But we're getting ahead of the story.

Nebulae were studied in the late 1700s and through the 1800s, notably by the siblings, \_\_\_\_\_  
 \_\_\_\_\_. Like other astronomers, they were initially interested in stars and comets.

On February 26, 1783, Caroline discovered a new nebula; she also independently found M110 (now known to be a dwarf galaxy satellite of Andromeda.)

10

The Herschels eventually catalogued over 5,000 nebulae.

The catalogues were originally published just under William's name, but the observations were made and recorded jointly.

They also resolved some of these nebulae into individual stars. (And, along the way, William discovered Uranus.)

11



The largest (40ft) Herschel telescope.

12

ADDITIONAL NOTES

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The nature of these nebulae (especially spiral) was debated.

Some astronomers said they were in the Milky Way, others argued that they were outside it.

One argument that they were within the MW was the observations of novae in some of them.

13

If a single star outside the MW could be observed brightening, the event would have to be “on a scale of magnitude such as the imagination recoils from contemplating.”

“A Popular History of Astronomy during the Nineteenth Century,” Agnes M. Clerke, page 438.

14

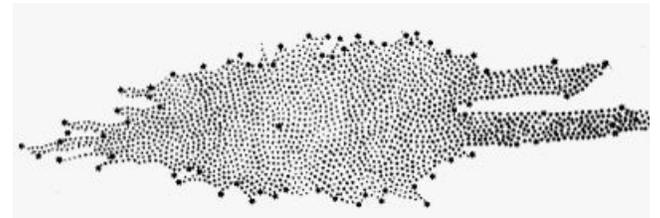
The question relates to the size of the Universe:

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The Milky Way was already known to be vast. Herschel had attempted to map it.



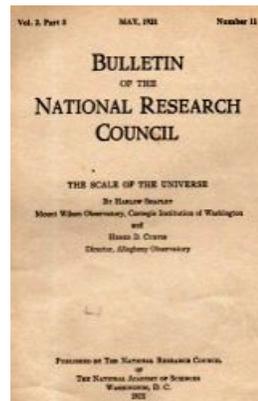
Herschel's map of MW: size estimate 32,000 ly.

Herschel's size was a huge underestimate, but it still meant that if there was more to the Universe than the MW, it would have to be on a scale “such as the imagination recoils from contemplating.”

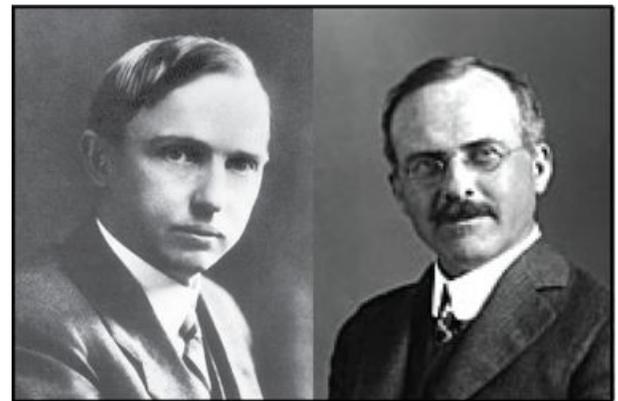
15

The question was debated on 26 April 1920 by two leading astronomers.

The event became known as the Shapley-Curtis “Great Debate.”



17



Shapley

Curtis

18

ADDITIONAL NOTES

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Here's the debate in your language:



19

The two astronomers debated the question focusing on 14 specific points (scoring by Virginia Trimble):

- 1, 2: Stars in globular clusters (S).
- 3: Cepheid variables as distance indicators (S).
- 4: Spectroscopic parallax as distance indicator (S).
- 5: Star counts and MW size (C).

21

- 13: Location of sun – Curtis thought it was at the center of the MW, Shapley not (S).
- 14: Rotational motion of spiral arms (S at the time, C now).

The Great Debate was a draw.

It took later work to settle the question.

23

Who claimed what in da Great Debate?

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- 6: Stellar evolution (C).
- 7: Distribution of spiral nebulae (C).
- 8, 9: Novae brightness and frequency (C).
- 10: Spiral nebulae seemed to recede (C).
- 11–12: Properties of galaxies (C/S).

22

The American astronomer \_\_\_\_\_ systematically studied nebulae in the 1920s.



24

ADDITIONAL NOTES

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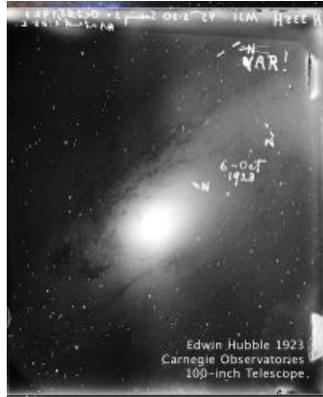


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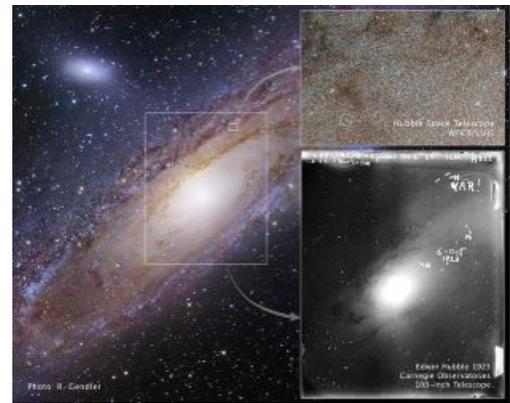


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By studying a class of stars of variable brightness, \_\_\_\_\_, \_\_\_\_\_, in Andromeda, Hubble found evidence that it was outside the MW.



25



26

\_\_\_\_\_

Our galaxy, the Milky way has over 200 \_\_\_\_\_ stars in it.

We believe there to be \_\_\_\_\_ other galaxies, mainly through Hubble telescope pictures.

27

We classify galaxies by their shapes:

- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_

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ADDITIONAL NOTES

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36

ADDITIONAL NOTES

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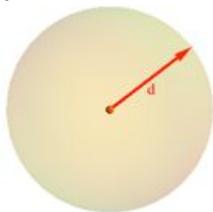


37

Standard candle method is based on the formula

$$A = \frac{L}{4\pi d^2}$$

where  $L$  is the intrinsic luminosity,  $d$  is the distance, and  $A$  is the apparent luminosity (called “flux” by astronomers).



39

### Standard Candles to Get Distances

Using an object whose intrinsic luminosity,  $L$ , you know, you can calculate its distance by measuring the apparent luminosity,  $A$ .

Common standard candles are

- \_\_\_\_\_
- \_\_\_\_\_

41

### Distances to Galaxies

- \_\_\_\_\_: distances to  $\sim 10$  AU. Radar distances depend on knowing the speed of light.
- \_\_\_\_\_: distances to  $\sim 650$  ly.
- \_\_\_\_\_: distances to  $\sim 33$  kly.
- \_\_\_\_\_ determine farther distances.

38

(1) As you increase your distance from a source of light, does its apparent luminosity go up or down?

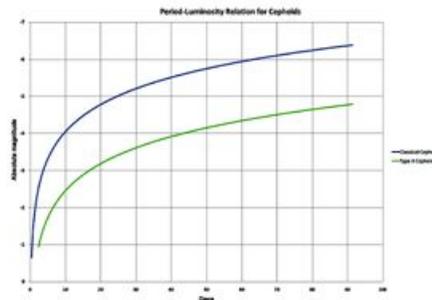
\_\_\_\_\_

(2) If you move away three times as far from a source as you originally were, how much does the apparent luminosity change by? \_\_\_\_\_

\_\_\_\_\_

40

### Cepheid Variables as Standard Candles



42

### ADDITIONAL NOTES

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Method to get distances using Cepheid Variables:

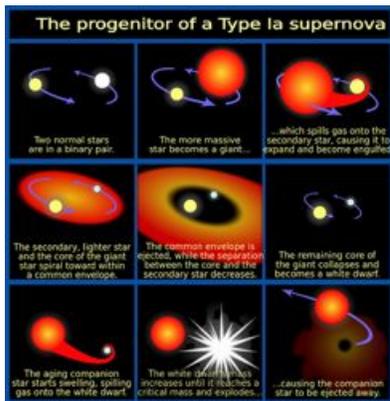
- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_

43

Type 1a Supernovae as Standard Candles

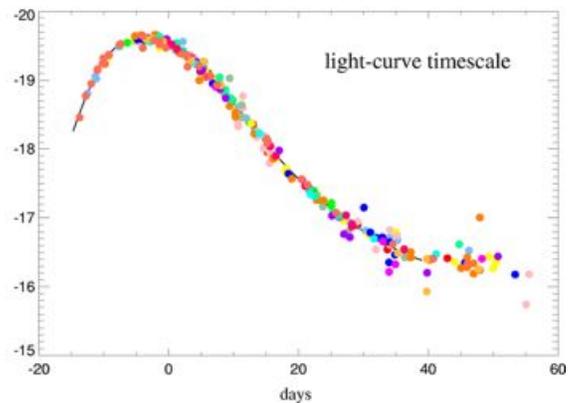
\_\_\_\_\_ They appear suddenly as a very bright star (can outshine an entire galaxy) then fade quickly. There are several types of supernovae. One of these can be used as a standard candle: \_\_\_\_\_

44



45

Luminosities of Type 1A Supernovae



46

What the previous graph shows is that (after some corrections are made), all Type 1a supernovae reach roughly the same peak luminosity.

(3) Why?

\_\_\_\_\_ Since mass determines luminosity, these supernovae all reach the same luminosity.

47

To get distances using Type 1a supernovae:

1. \_\_\_\_\_
2. \_\_\_\_\_

48

ADDITIONAL NOTES

# Arvind Borde / AST 10, Week 9: Galaxies II

## Structure in the Universe

The Universe organizes itself so as \_\_\_\_\_  
\_\_\_\_\_

The new structures are \_\_\_\_\_  
\_\_\_\_\_

(1) How many fundamental forces are there, and what are they? \_\_\_\_\_  
\_\_\_\_\_

▷ Subatomic scale:

Elementary particles called \_\_\_\_\_  
\_\_\_\_\_

Protons and neutrons combine to form \_\_\_\_\_

Nuclei combine with other elementary particles – such as \_\_\_\_\_ – to form atoms.

(2) What fundamental forces bind protons and neutrons? The nucleus? \_\_\_\_\_

(3) What fundamental force binds an atom? \_\_\_\_\_  
\_\_\_\_\_

Scale:

Electrons, protons: \_\_\_\_\_  
(Electron really has no “actual” size.)

Atoms: \_\_\_\_\_

(4) How much bigger is an atom than the electrons and protons that compose it?

▷ Atomic/molecular scale:

\_\_\_\_\_

Scale:

Range: \_\_\_\_\_

(5) What force binds molecules? \_\_\_\_\_

▷ Day-to-day scale:

\_\_\_\_\_

Scale:

Wide range: \_\_\_\_\_

(6) What force binds these objects?  
\_\_\_\_\_

## ADDITIONAL NOTES

1

2

3

4

5

6

▷ Astrophysical object scale:

=====

=====

Scale: Wide range: =====

(7) What forces binds astrophysical objects?

=====

7

▷ Galactic scale

▷ =====

Scale:

Range: =====

( $d_{MW} \approx 10^{21} \text{m} \approx 100,000 \text{ly.}$ )

(8) What “force” binds galaxies? =====

8

In each case, the larger structure has properties and behavior different from its constituents.

The natural question is: =====

=====

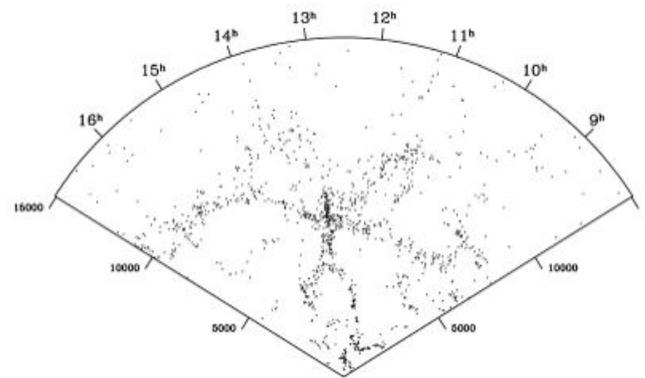
9

The CfA Redshift Survey was started in 1977 at the Harvard-Smithsonian Center for Astrophysics to map the Universe in 3D, in order to get a feeling for what its overall structure is.

A second CfA survey (CfA2) was started by Valerie de Lapparent, John Huchra and Margaret Geller in the winter of 1984/5. Between 1985 and 1995 about 18,000 bright galaxies were studied.

10

The first maps were released in 1986. They show flat wedges of the night sky, where looking further along the wedge from the vertex means you are looking further from earth.



de Lapparent, Geller, Huchra, 1986

12

11

ADDITIONAL NOTES

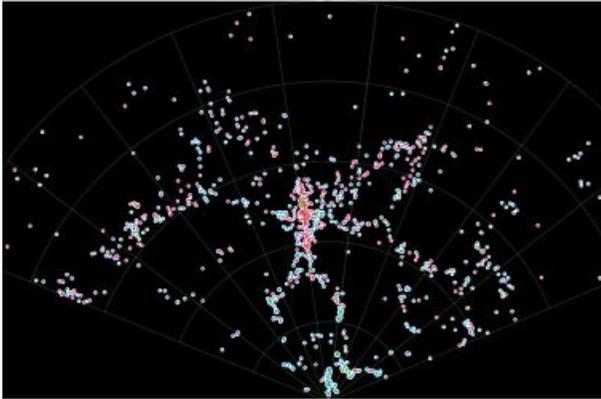
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de Lapparent, Geller, Huchra, 1966

13

\_\_\_\_\_ The organization of the Universe into larger structures stops here.

What was unexpected was that \_\_\_\_\_

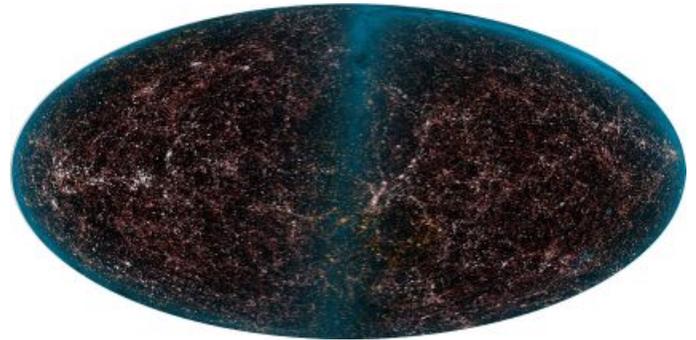
This overall organization has been confirmed by further work from the CfA team, and by other surveys such as the Sloan Digital Sky Survey (2000–).

14

SDSS Map of the Universe



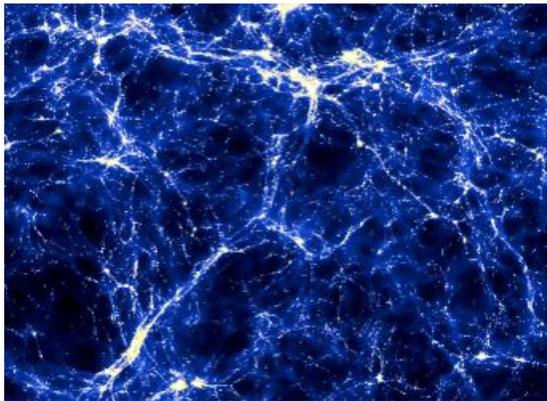
15



A million galaxies

We have a “foamy” picture of the Universe on the largest scales (~ 10 billion light years).

16



Computer model

17

These are 3D maps of the Universe. But there’s a 4th dimension.

(9) What is it? \_\_\_\_\_

18

ADDITIONAL NOTES

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If we look at the ages of galaxies what do we find?

- \_\_\_\_\_  
\_\_\_\_\_

Aside: There is wide ranges of sizes, from smallest known (Segue 2,  $10^{18}$ m) to largest (IC 1101,  $6 \times 10^{22}$ m).

- \_\_\_\_\_  
\_\_\_\_\_

19

### Internal Motions of Galaxies

(10) What force did we say binds a galaxy together? \_\_\_\_\_

(11) Why does gravitation not make a galaxy collapse on itself? \_\_\_\_\_

\_\_\_\_\_  
\_\_\_\_\_

20

(12) If the sun were to get more massive would the planets rotation speeds have to change for the solar system to survive? \_\_\_\_\_

\_\_\_\_\_  
\_\_\_\_\_

21

### The Masses of Galaxies

\_\_\_\_\_  
\_\_\_\_\_

(13) Would you expect stars to orbit faster or slower in a more massive galaxy compared to a less massive one? \_\_\_\_\_

\_\_\_\_\_

22

- \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

- \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

23

(14) Returning to the solar system, must the outer planets travel at slower or faster speeds than the inner? \_\_\_\_\_

\_\_\_\_\_

For a galaxy, though, there's no one object that provides the binding force for the whole system. It's the collective gravity of all of its constituents.

24

### ADDITIONAL NOTES

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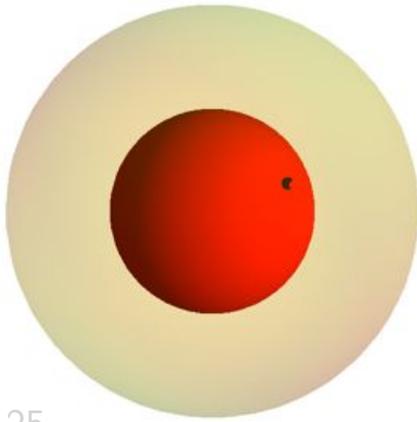
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How does gravity work inside matter?

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=====

The outer layer doesn't.

=====

25

Study how matter is distributed in M33:



26

○ =====

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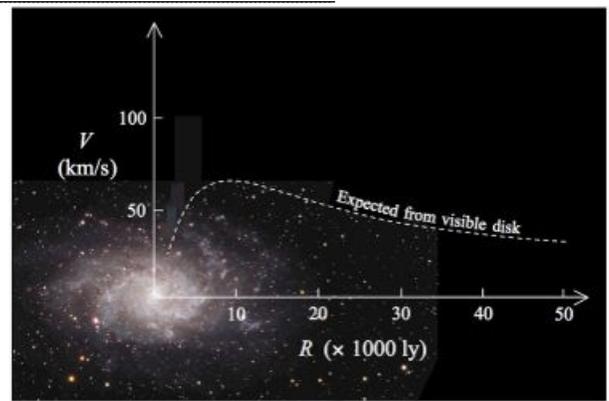
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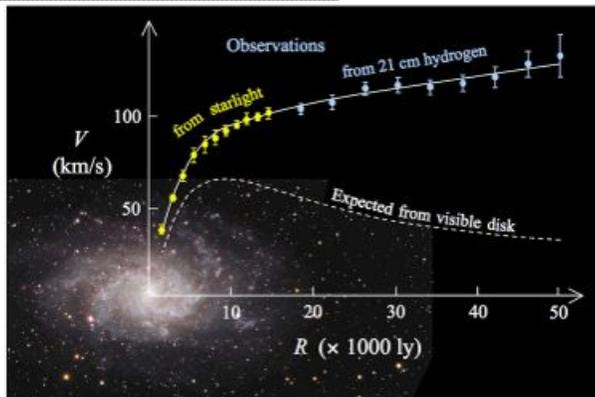
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===== We call it =====.

We'll return to it later.

For now, let's look at special galaxies, called =====

===== These

30

ADDITIONAL NOTES

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- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_

31

Evidence indicates that the activity in galactic nuclei results from the accretion of matter onto supermassive black holes.

- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_

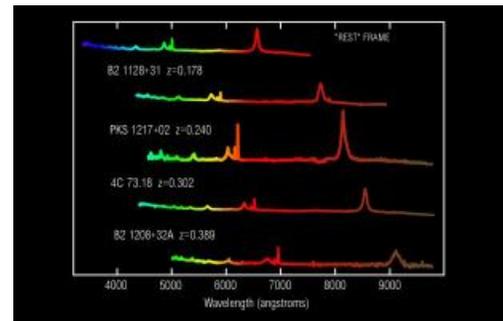
They are much more luminous than the sum of all the stars in the galaxies that contain them

32

### Quasars

- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_

33



34



Sombrero galaxy

35



A ring galaxy

36

### ADDITIONAL NOTES

- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_









# Arvind Borde / AST 10, Week 10: The Expanding Universe

Two strands feed into the narrative that the Universe is expanding:

▷ \_\_\_\_\_ Expansion

and

▷ \_\_\_\_\_ Expansion

1

## Theoretical Expansion

Gravity is a destabilizing phenomenon, even in Newton's theory, because it is (apparently) always attractive.

Newton was aware that his law of gravitation threatened the Universe.

2

He discussed this in letters he wrote in 1692-93.

"Four Letters from Sir Isaac Newton to Doctor Bentley,"  
– Google Books  
edition published by R and J. Dodsley, 1756.

Bentley was not only a "Doctor," he was a Bishop.

3

*To the Reverend Dr. RICHARD BENTLEY, at the Bishop of Worcester's House in Park-street, Westminster.*

S I R,

**W**HEN I wrote my Treatise about our System, I had an Eye upon such Principles as might work with considering Men, for the Belief of a Deity, and nothing can rejoice me more than to find it useful for that Purpose.

December 10, 1692

4

As to your first Query, it seems to me that if the Matter of our Sun and Planets, and all the Matter of the Universe, were evenly scattered throughout all the Heavens, and every Particle had an innate Gravity towards all the rest, and the whole Space, throughout which this Matter was scattered, was but finite; the Matter on the outside of this Space would by its Gravity tend towards all the Matter on the inside, and by consequence fall down into the middle of the whole Space, and there compose one great spherical

December 10, 1692

5

Mass. But if the Matter was evenly disposed throughout an infinite Space, it could never convene into one Mass, but some of it would convene into one Mass and some into another, so as to make an infinite Number of great Masses, scattered at great Distances from one to another throughout all that infinite Space.

December 10, 1692

6

## ADDITIONAL NOTES

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Newton goes on to say how this might explain why the Universe contains what it does, *as known at the time* (stars and planets) . . .

7

thus might the Sun and fixt Stars be formed, supposing the Matter were of a lucid Nature. But how the Matter should divide itself into two sorts, and that Part of it, which is fit to compose a shining Body, should fall down into one Mass and make a Sun, and the rest, which is fit to compose an opaque Body, should coalesce; not into one great Body, like the shining Matter, but into many little ones . . . . . I do not think explicable by meer natural Causes,

December 10, 1692

8

Bentley raised objections and Newton replied:

**harder it is to suppose all the Particles in an infinite Space should be so accurately poised one among another, as to stand still in a perfect Equilibrium. For I reckon this as hard as to make not one Needle only, but an infinite number of them (so many as there are Particles in an infinite Space) stand accurately poised upon their Points.**

January 17, 1693

9

**Yet I grant it possible, at least by a divine Power; and if they were once to be placed, I agree with you that they would continue in that Posture without Motion for ever, unless put into new Motion by the same Power. When therefore I said, that Matter evenly spread through all Space, would convene by its Gravity into one or more great Masses, I understand it of Matter not resting in an accurate Poise.**

January 17, 1693

10

Newton realized that the attractive nature of gravity made systems controlled by gravity alone inherently prone to \_\_\_\_\_

Against gravitational collapse, what supports (1) the atomic nucleus?

(2) molecules?

(3) your nose?

11

12

ADDITIONAL NOTES

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But, what supports and gives stability to

(4) the solar system?  
=====

(5) galaxies?  
=====

(No electromagnetic forces at work here.)

13

The same issue arises in General Relativity.

A year after his fundamental paper laying out that theory, Einstein wrote a paper on cosmology:

**COSMOLOGICAL CONSIDERATIONS ON  
THE GENERAL THEORY OF RELATIVITY**

BY  
**A. EINSTEIN**

*Translated from "Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie," Sitzungsberichte der Preussischen Akad. d. Wissenschaften, 1917.*

14

He saw that the difficulties with the Newtonian theory were serious

If we apply Boltzmann's law of distribution for gas molecules to the stars, by comparing the stellar system with a gas in thermal equilibrium, we find that the Newtonian stellar system cannot exist at all. For there is a finite ratio

p. 178

and that General Relativity would have the same problem: A \_\_\_\_\_

15 What is the obvious resolution? \_\_\_\_\_

That's not what Einstein did. He altered his theory:

**§ 2. The Boundary Conditions According to the General Theory of Relativity**

In the present paragraph I shall conduct the reader over the road that I have myself travelled, rather a rough and winding road, because otherwise I cannot hope that he will take much interest in the result at the end of the journey. The conclusion I shall arrive at is that the field equations of gravitation which I have championed hitherto still need a slight modification, so that on the basis of the general theory of relativity those fundamental difficulties may be avoided which have been set forth in § 1 as confronting the Newtonian theory. This modification corresponds perfectly to the transi-

p. 179–180

16

admitting that

standpoint of present astronomical knowledge, will not here be discussed. In order to arrive at this consistent view, we admittedly had to introduce an extension of the field equations of gravitation which is not justified by our actual knowledge of gravitation. It is to be emphasized,

p. 188

Einstein's change was to introduce the \_\_\_\_\_  
\_\_\_\_\_.

It gave a small effective large scale repulsion to balance the instability caused by the inherently attractive nature of normal gravity.

17

18

ADDITIONAL NOTES

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Einstein's proposal was criticized right away on many grounds.

He came around and dropped the cosmological constant.

But he had missed noticing that his theory in not allowing a static Universe was really \_\_\_\_\_.

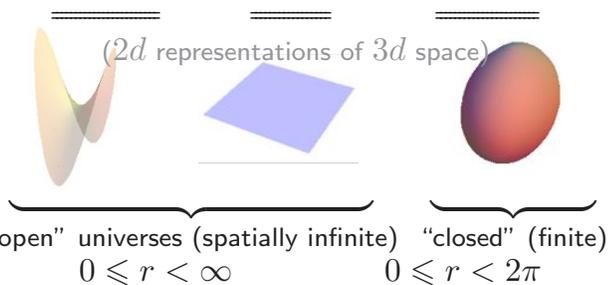
19

That was discovered by Alexander Friedmann in the Soviet Union (1922), and independently by others in France and the U.S.

Assuming the universe is homogeneous (all parts the same) and isotropic (all directions the same), you get three possibilities for the spatial shape of the Universe.

20

The three possibilities, labeled by a variable  $k$ , arise from the \_\_\_\_\_ being \_\_\_\_\_.



21

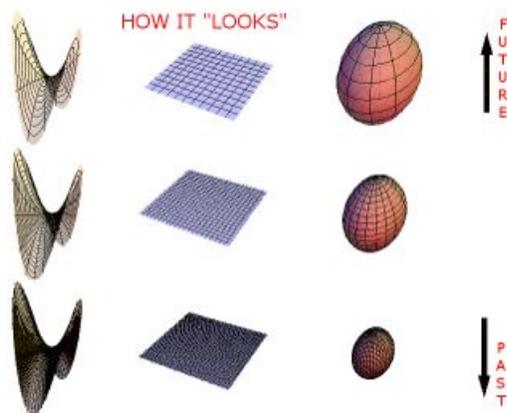
In each case, there's a further function,  $a(t)$ , called the \_\_\_\_\_ of the Universe.

It is positive and its behavior follows from Einstein's theory.

For example, it follows that  $\dot{a}(t)$ , the rate of change of  $a(t)$ , is positive: \_\_\_\_\_.

How may we visualize this?

22



23

How does this connect to what we see?

Consider observers separated by fixed distances  $r$ . The physical distance between them is  $R(t) = a(t)r$ . Even if they stay at fixed coordinates, the distance between them will change because  $a(t)$  does.

24

ADDITIONAL NOTES

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It follows that

$$V_R(t) = HR(t).$$

The last equation is called Hubble’s law.

(6) What does it say in English?

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25

Why “Hubble’s Law”?

That brings us to the *observed* expansion of the Universe.

26

Observed Expansion

By 1920, the year of the Great Debate, it was already known that the nebulae, with some exceptions, showed positive wavelength shifts or “red shifts”.

(One exception was Andromeda.)

27

Such wavelength shifts are interpreted as “Doppler shifts,” shifts due to motion:

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

where  $\Delta\lambda$  is the change in wavelength,  
 $\lambda$  the original (or expected) wavelength,  
 $v$  the speed (away > 0; toward < 0), and  
 $c$  the speed of light.

28

Over the next decade, Edwin Hubble carefully measured the redshifts of a number of galaxies.

He also noted their distances from us:

Distances of extra-galactic nebulae depend ultimately upon the application of absolute-luminosity criteria to involved stars whose types can be recognized. These include, among others, Cepheid variables, novae,

(7) What method (radar bouncing? parallax? standard candles?) was he using to get distances?

29

He announced that he’d found . . .

*A RELATION BETWEEN DISTANCE AND RADIAL VELOCITY  
 AMONG EXTRA-GALACTIC NEBULAE*  
 BY EDWIN HUBBLE  
 MOUNT WILSON OBSERVATORY, CARNEGIE INSTITUTION OF WASHINGTON  
 Communicated January 17, 1929

What was that relation?

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ADDITIONAL NOTES

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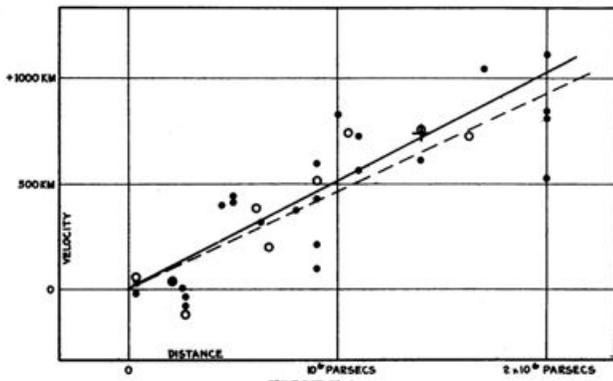
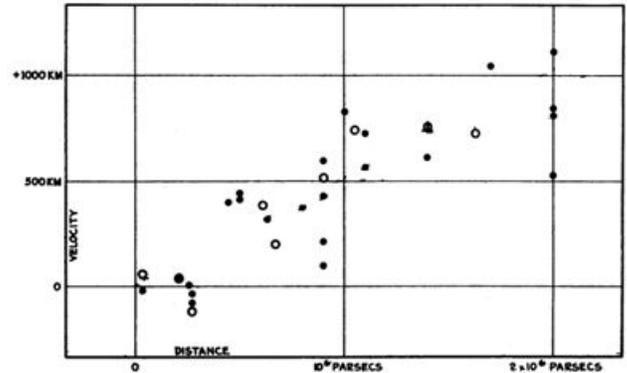


FIGURE 1

Velocity-Distance Relation among Extra-Galactic Nebulae.

31

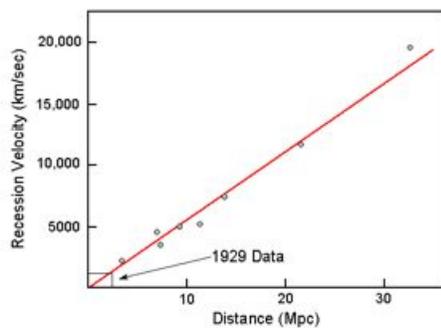
The conclusion was questionable:



32

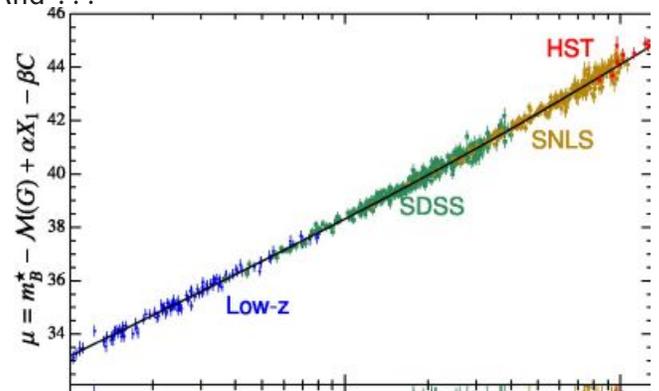
But

Hubble & Humason (1931)



33

And ...



34

We have so much faith in Hubble's Law that if all other methods fail, we use it as a distance indicator: measure the redshift, calculate the velocity, then infer the distance from Hubble's law,

$$V = HR.$$

For that we need to know the present value of Hubble's constant  $H_0$ .

35

Hubble's constant can be viewed as  $V/R$ . It's measured as the speed of recession of distant galaxies (in km/sec) per astronomical distance unit (such as a ly).

The units of  $H$  are  $\frac{1}{\text{sec}}$ .

Therefore, the units of  $1/H$  are seconds.

Hold this thought.

36

ADDITIONAL NOTES

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Let  $\ddot{a}(t)$  be the rate of change of the rate of change of  $a(t)$  (acceleration/decceleration).

Einstein's theory also leads to this equation:

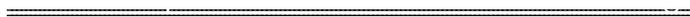
$$\ddot{a}(t) = -\frac{4\pi G}{3c^4}(\rho + 3P)a(t) \quad \heartsuit$$

where  $\rho$  is the energy density of matter, and  $P$  the pressure (think gas).

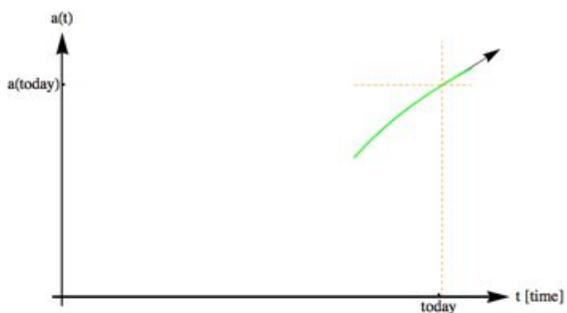
37

Now,  $\rho$  and  $P$  are usually positive.

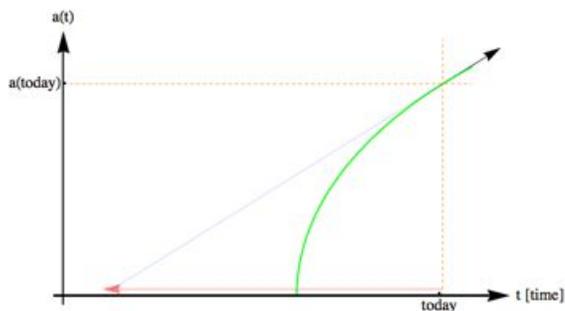
That means  $\ddot{a}(t) < 0$ :



So the graph of  $a(t)$  is increasing today and concave down everywhere:



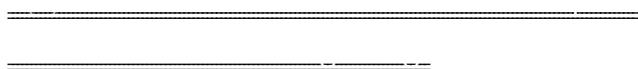
39



The graph of  $a(t)$  cannot wriggle out from under the blue line – the past-projection of  $\dot{a}(t)$  – without going concave up.

40

This has an enormous implication:

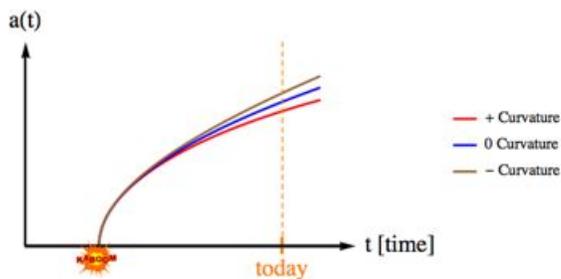


That time is precisely within  $1/H_0$  from today.

Hubble's constant gives us an upper bound on the age of the Universe.

This beginning is popularly called \_\_\_\_\_.

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ADDITIONAL NOTES

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# Arvind Borde / AST 10, Week 11: The Big Bang Theory

The story:

Einstein's theory, expressed via equations,

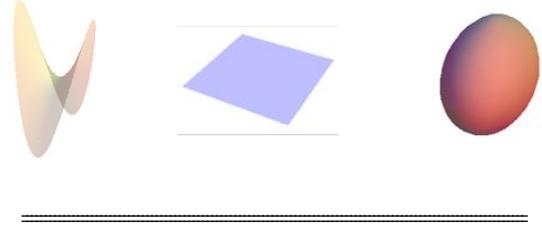
$$2R_{ab} = \partial_c [g^{cd} (\partial_a g_{bd} + \partial_b g_{ad} - \partial_d g_{ab})] - \partial_a [g^{cd} (\partial_b g_{cd} + \partial_c g_{bd} - \partial_d g_{bc})] - [g^{ab} \partial_a \partial_b - \partial_d g_{ae}] \times [g^{ed} (\partial_c g_{bd} + \partial_b g_{cd} - \partial_d g_{cb})] + [g^{ed} (\partial_a g_{bd} + \partial_b g_{ad} - \partial_d g_{ab})] \times [g^{cd} (\partial_e g_{cd} + \partial_c g_{ed} - \partial_d g_{ec})]$$

relates the geometry of spacetime (left side) to matter (right side).

1

Assuming \_\_\_\_\_ and \_\_\_\_\_ for the entire Universe, we get:

1) Three possible geometries for space



2

2) Each is multiplied by the scale factor,  $a(t)$ .

The scale factor obeys two equations:



( $k = -1, 0, \text{ or } 1$ )

$\dot{a}(t)$  = rate of change of  $a(t)$  (velocity)

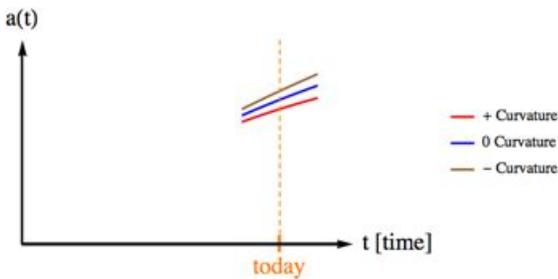
3  $\ddot{a}(t)$  = rate of change of  $\dot{a}(t)$  (acceleration)

The left hand side of the two equations describes the geometry.

On the right hand side we have matter:  $\rho$  is its energy density and  $P$  is its pressure.

As long as \_\_\_\_\_ (true of all known matter), we know that gravity is always attractive, and we have  $\ddot{a}(t) < 0$ .

4

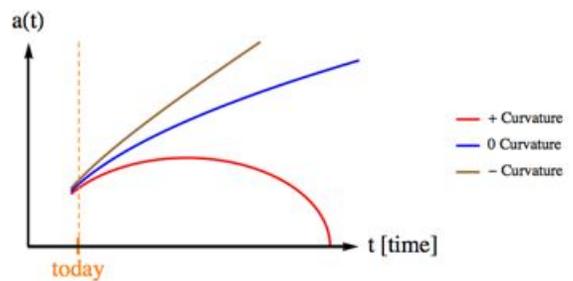


$\dot{a} > 0$ : \_\_\_\_\_

$\ddot{a} < 0$ : \_\_\_\_\_

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Projecting into the future:



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ADDITIONAL NOTES

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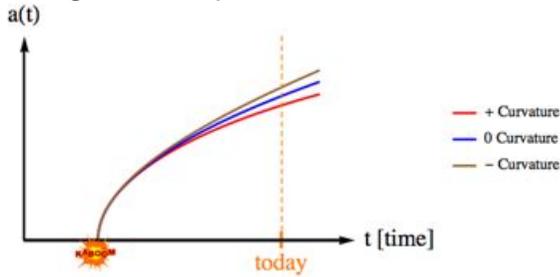


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Projecting into the past:



$a(t)$  zero in past. The Universe had a beginning.

7

This conclusion is based on an important assumption:

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8

Now, ponder the ( $\diamond$ ) equation

$$\frac{3\dot{a}^2(t)}{a^2(t)} + \frac{3k}{a^2(t)} = \frac{8\pi G}{c^4}\rho.$$

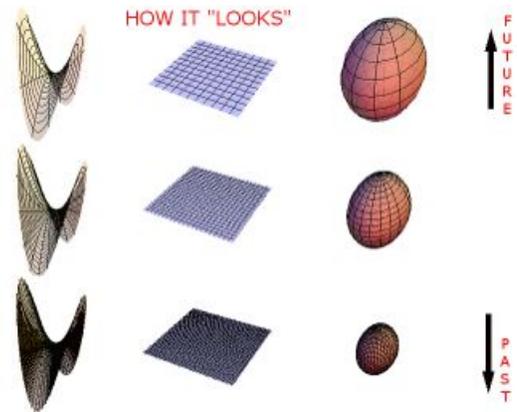
(1) If  $a(t) \rightarrow 0$  what can you say about the left-hand side, and therefore  $\rho$  on the right?

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At the instant of time at which \_\_\_\_\_, the Universe was \_\_\_\_\_ (“the initial or cosmological singularity”). \_\_\_\_\_  
 \_\_\_\_\_ If the density is infinite, so is the temperature. Infinities are problematic.

Still, the Universe was \_\_\_\_\_  
 \_\_\_\_\_

11

The idea that the Universe might have had  $a(t) = 0$  at some point in the past – and therefore a beginning – was difficult for people to accept, and they looked for a way around this conclusion.

a) The behavior of  $\dot{a}(t)$ :

The \_\_\_\_\_ (so  $\dot{a}(t) > 0$ )  
 – we know that from \_\_\_\_\_

12

ADDITIONAL NOTES

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b) The behavior of  $\ddot{a}(t)$ :

The graph *could* be concave up ( $\ddot{a}(t) > 0$ ).

One way: there's a cosmological constant. Its presence would modify the  $\heartsuit$  equation:

$$\frac{3\ddot{a}(t)}{a(t)} = -\frac{4\pi G}{c^4}(\rho + 3P) + \Lambda \quad \heartsuit$$

A large enough  $\Lambda$  would allow  $\ddot{a}(t)$  to be positive.

13

A theory along these lines, called the steady state theory, was proposed and was popular through the 1940s–1960s.

In it, matter is continuously created ( $\sim$  two hydrogen atoms per cubic m every billion yrs) and the Universe does not thin out as it expands. It stays steadily in the same state eternally into the future and, as well, to the past.

14

## 'BIG BANG' THEORY HIT

Experiment Strengthens the View Matter Is Being Formed Continuously

New York Times, December 31, 1956

15

## Radio Signals Suggest Flaw in 'Big Bang' Concept of Universe's Origin

New York Times, November 6, 1960

16

### New Discoveries

During the past year or two a number of strange objects have been discovered in the sky, emitting radio energy with an intensity that defies explanation. Known as quasars, they appear to be billions of light years away. According to Dr. Cyril Hazard, who is on the new team of radio astronomers at Cornell, probably a third of the faint radio sources to be catalogued in the new survey will be quasars.

New York Times, September 27, 1964

If the universe was born in an explosive event, its galaxies and other components should have been much closer together a few billion years ago than they are today. Therefore, if we look back in time, we should find a greater density of such objects.

17

CHICAGO, Nov. 19 — A relatively small astronomical telescope on the moon could enable mankind to see twice as much of the universe, according to a prominent American astronomer.

Such a telescope might even settle a great controversy over the nature of the universe itself—the question of whether the present universe exists in a “steady state” or is still expanding from a “big bang” beginning that took place several billion years ago.

New York Times, November 20, 1964

18

### ADDITIONAL NOTES

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Man apparently stands on the threshold of understanding the basic nature of the universe—whether it was born at a certain time or has always existed, whether it is finite or infinite, whether it is unchanging or ever-changing. Yet on the eve of discoveries that promise to answer these questions, cosmology is in a state of turmoil as became evident last week in reports newly circulated.

A severe blow has been dealt the steady state theory, which says that the universe is essentially eternal and unchanging. Dr.

New York Times, November 14, 1965

19

On the observational side, the discovery of quasars was the game-changer.

The steady state theory requires the Universe to be unchanging and always in the same state.

But quasars have very high red-shift and are, therefore, only at great distances from us, and \_\_\_\_\_  
\_\_\_\_\_. Something had to have changed from those early years.

20

On the theoretical side, the apparently reasonable objection was raised that it is the \_\_\_\_\_  
\_\_\_\_\_ (a(t) = 0). Realistic situations with irregularities, it was argued would “bounce.”

An influential Russian team, led by E.M. Lifshitz claimed that they had found cosmological models without singularities.

21

The game-changer here was a theorem proved by Roger Penrose on black hole behavior.

It attracted the attention of Stephen Hawking, then a Ph.D. student studying the necessity of a cosmological singularity.

22

Within a few months, Hawking had seen that Penrose’s methods could be adapted to the problem he was studying:

VOLUME 15, NUMBER 17      PHYSICAL REVIEW LETTERS      25 OCTOBER 1965

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OCCURRENCE OF SINGULARITIES IN OPEN UNIVERSES  
S. W. Hawking  
Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge, England  
(Received 16 August 1965)

Title

23

He began by posing the question:

At the present time the universe is observed to be expanding. If we assume that there is no creation of matter, this indicates that the density must have been higher in the past. The question then arises, was there some time in the past when the density was infinite (i.e., was there a singularity of space-time), or did the

Para 1

then said what calculations indicated:

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ADDITIONAL NOTES

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imum density and then expand again? This was partly answered by Robertson<sup>1</sup> who showed that if the universe was spatially homogeneous and isotropic, its metric could be written

$$ds^2 = dt^2 - R^2(t)[dr^2/(1-Kr^2) + r^2(d\theta^2 + \sin^2\theta d\varphi^2)], \quad (1)$$

$$K = -1, 0, \text{ or } +1.$$

Robertson showed that provided the matter has normal properties and the Einstein equations without cosmological constant held,

$$R_{ab} - \frac{1}{2}g_{ab}R = -T_{ab},$$

then there would be a physical singularity in any universe whose metric had the form (1). This form restricts the flow of the matter to

Para 1

25

But,

All of these cases place some restriction or exact symmetry on the flow, however, and it has been claimed<sup>6</sup> that in the absence of such restrictions or exact symmetries there will not be a physical singularity. That is, if we have a model that is a small perturbation of one of these restricted models, then the perturbations will grow as we go back in time and will prevent the occurrence of a physical singularity. This claim has already been proved false by Penrose<sup>7</sup> for the case of a collapsing star. Using similar methods it will be shown that this claim is also false for a class of universe models.

End of para 1

26

Hawking concludes that

surface and hence a physical singularity. Thus a universe that is similar on a large scale to the form (1) but has no exact local symmetries will have a singularity. Local irregularities cannot prevent it.

End of para 3

We must take seriously the prediction of a singularity at the start of the Universe. It is called the

Hawking, Penrose and others worked for several years after the papers of 1965 to understand the

\_\_\_\_\_'  
\_\_\_\_\_'  
\_\_\_\_\_'

Their work was very deep, broad and general, but it made one important assumption: gravity is always attractive ( $\rho > 0, P > 0$ ).

27

28

Summary of what we knew by 1980

- o \_\_\_\_\_ How do we know this? \_\_\_\_\_

(Although alternatives have been suggested – such as the “tired light” theory.)

- o \_\_\_\_\_ How do we know this? \_\_\_\_\_

The statements are based partly on observation, and are consistent with Einstein’s theory of relativity with  $\rho > 0$  and  $P > 0$ .

The earliest calculations assumed homogeneity and isotropy, but later work by Penrose, Hawking, and others showed that the conclusions remained true even without symmetries.

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30

ADDITIONAL NOTES

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We don't know exactly when the "beginning" was, but it was somewhere between 13 and 15 billion years ago.

We take that (whenever it was) as the time when time began and measure events from there.

31

**Highlights from History**

Time (s)	What happened?	Temp (°K)
?	Matter forms	?
$2.0 \cdot 10^{-12}$	Electromagnetism	$1.2 \cdot 10^{18}$
$2.0 \cdot 10^{-6}$	quarks combine	$1.7 \cdot 10^{12}$
1.0	neutrinos released	$1.2 \cdot 10^{10}$
$1.8 \cdot 10^2$	Hydrogen forms	$1.2 \cdot 10^9$
$1.2 \cdot 10^{13}$	photons released	$2.9 \cdot 10^3$
⋮	⋮	⋮
$4.4 \cdot 10^{17}$	Today	2.8

32

Are we making all this up? Maybe. But,...

We can estimate the temperature and density as we plot back in time.

↓

We have theories of particle physics that tell us how particles behave at high energies.

↓

We can test this in particle accelerators on earth, where we have achieved effective temperatures of  $(1.4 \cdot 10^{16})^\circ \text{K}$ .

33

Is there anything more definite?

Well, there are the photons released about 370,000 years after the big bang, if it occurred. They would still be in the Universe.

It can be calculated that they would be at a little under  $3^\circ \text{K}$  now, and the expansion of the Universe will have stretched their wavelengths to around 1 cm, the microwave region. Can we find them?

34

Two Princeton cosmologists, Robert Dicke and Jim Peebles, set out in the early to mid 1960s to find this \_\_\_\_\_ (CMBR).

They started building a detector.

Being scientists, they also gave lectures on what they were doing.

35

Meanwhile in nearby AT&T Bell Labs...

In 1960, a 20-foot horn-shaped antenna was built there to be used in communication.

Arno Penzias and Robert Wilson, scientists there, began doing research with it.

They encountered a persistent microwave noise that came from every direction.

36

ADDITIONAL NOTES

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They tested everything they could think of to rule out the source of the radiation racket: the Milky Way, extraterrestrial radio sources, New York City, military testing.

37

Then they found droppings of pigeons nesting in the antenna. They cleaned the mess but the birds came back.

“To get rid of them, we finally found the most humane thing was to get a shot gun... and at very close range [we] just killed them instantly.” (Penzias, interview)

38 But the noise remained, from every direction.

Penzias mentioned their problem in a chance conversation with Bernie Burke, a radio astronomer.

He knew of the efforts that Dicke and Peebles were making, and told Penzias.

Penzias called Dicke.

After the call, Dicke turned to his laboratory colleagues and said simply “we’ve been scooped.”

39

Dicke, Peebles and their collaborators, on the one hand, and Penzias and Wilson, on the other, agreed to publish back-to-back papers.

40

The Dicke-Peebles paper appeared as the first of the two:

**COSMIC BLACK-BODY RADIATION\***

One of the basic problems of cosmology is the singularity characteristic of the familiar cosmological solutions of Einstein’s field equations. Also puzzling is the presence of matter in excess over antimatter in the universe, for baryons and leptons are thought to be conserved. Thus, in the framework of conventional theory we cannot understand the origin of matter or of the universe. We can distinguish three main attempts to deal with these problems.

1. The assumption of continuous creation (Bondi and Gold 1948; Hoyle 1948), which avoids the singularity by postulating a universe expanding for all time and a continuous but slow creation of new matter in the universe.
2. The assumption (Wheeler 1964) that the creation of new matter is intimately related to the existence of the singularity, and that the resolution of both paradoxes may be found in a proper quantum mechanical treatment of Einstein’s field equations.
3. The assumption that the singularity results from a mathematical over-idealization, the requirement of strict isotropy or uniformity, and that it would not occur in the real world (Wheeler 1958; Lifshitz and Khalatnikov 1963).

41

\* This research was supported in part by the National Science Foundation and the Office of Naval Research of the U.S. Navy.

After some background, they pose the question:

Could the universe have been filled with black-body radiation from this possible high-temperature state? If so, it is important to notice that as the universe expands the cosmological redshift would serve to adiabatically cool the radiation, while preserving the

Then go on to say:

While we have not yet obtained results with our instrument, we recently learned that Penzias and Wilson (1965) of the Bell Telephone Laboratories have observed background radiation at 7.3-cm wavelength. In attempting to eliminate (or account for) every contribution to the noise seen at the output of their receiver, they ended with a residual of  $3.5^\circ \pm 1^\circ$  K. Apparently this could only be due to radiation of unknown origin entering the antenna.

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ADDITIONAL NOTES

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The Penzias-Wilson paper sticks to the bare facts of how they did their measurements and what they observed:

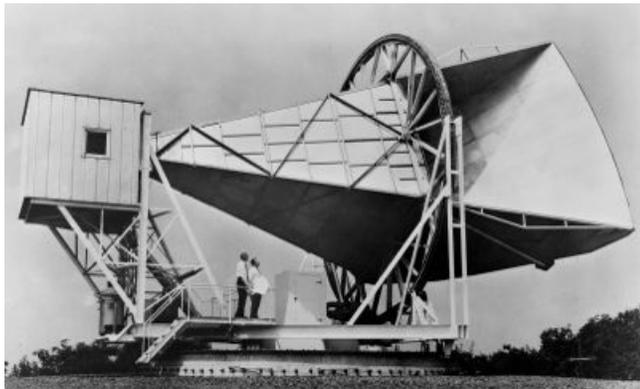
**A Measurement of Excess Antenna Temperature at 4080 Mc/s.**

Penzias, A. A.; Wilson, R. W.

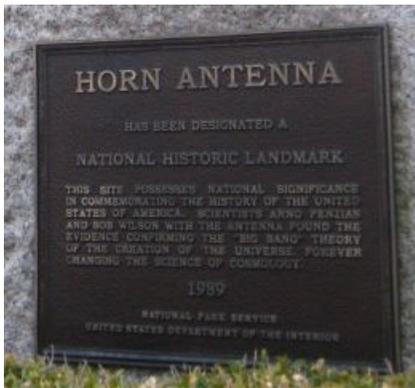
*Astrophysical Journal*, vol. 142, p.419-421

Measurements of the effective zenith noise temperature of the 20-foot horn-reflector antenna (Crawford, Hogg, and Hunt 1961) at the Crawford Hill Laboratory, Holmdel, New Jersey, at 4080 Mc/s have yielded a value of about 3.5 K higher than expected. This excess temperature is, within the limits of our observations, isotropic, unpolarized, and free from seasonal variations (July, 1964 - April, 1965). A possible explanation for the observed excess noise temperature is the one given by Dicke, Peebles, Roll, and Wilkinson (1965) in a companion letter in this issue.

43



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A few years later a Nobel prize was awarded for the discovery.

(2) Who got it? \_\_\_\_\_

\_\_\_\_\_

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46

Apart from who should get credit, the discovery of the CMBR is, in fact, one of the greatest human discoveries ever made.

We are seeing light, once it's free to move around, from creation itself (in some sense).

Over the years it has been studied extensively. It is almost completely isotropic, confirming our assumption about the uniformity of the Universe.

48

ADDITIONAL NOTES

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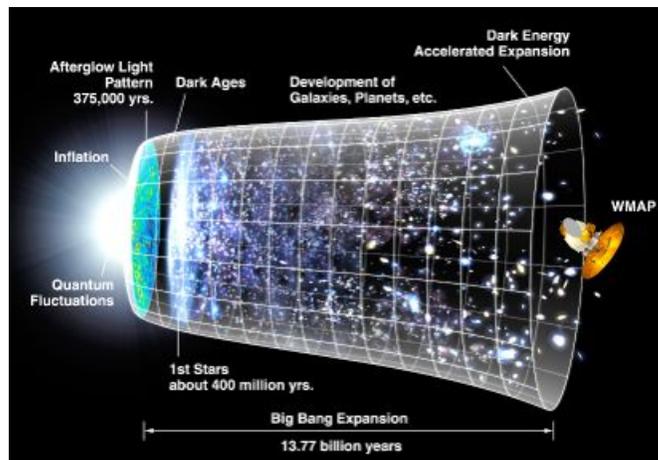
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Observational work is now focused on measuring tiny irregularities in it that may explain \_\_\_\_\_

We need to know in the uniform early Universe, what it was that sowed the seeds of galaxy formation; i.e., why did galaxies form at the particular places that they did, and not elsewhere?

49



50

If the Universe did come into existence, as opposed to having always been there, how did it do so?

People have speculated . . .

51

### Wave function of the Universe

J. B. Hartle

*ni Institute, University of Chicago, Chicago, Il  
tical Physics, University of California, Santa B*

S. W. Hawking

have solved the problem of the initial boundary conditions of the Universe: the boundary conditions are that it has no boundary.<sup>3,7</sup>

52

#### CREATION OF UNIVERSES FROM NOTHING

Alexander VILENKIN  
Physics Department, Tufts University, Medford, MA 02155, USA

Received 11 June 1982

A cosmological model is proposed in which the universe is created by quantum tunneling from literally nothing into a de Sitter space. After the tunneling, the model evolves along the lines of the inflationary scenario. This model does not have a big-bang singularity and does not require any initial or boundary conditions.

The concept of the universe being created from nothing is a crazy one. To help the reader make peace with this concept, I would like to give an example of a compact instanton in a more familiar setting. Let us

53

#### ADDITIONAL NOTES

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Summary of Hot Big Bang Cosmology  
("FLRW"\* model)

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Often called the Big Bang Theory, but it's not really a new theory. \_\_\_\_\_

\_\_\_\_\_

1 \*Friedmann-Lemaître-Robertson-Walker

People often use "big bang" to refer to the "moment of creation:" the \_\_\_\_\_ from which the Universe began.

I'll use it to mean instead the hot dense state from which the explosion occurred. We have \_\_\_\_\_

\_\_\_\_\_

2

We've \_\_\_\_\_ (through the Hawking-Penrose theorems), but they depend on the assumption that gravity is always attractive, enforced through the energy-momentum of matter obeying certain positivity conditions ("\_\_\_\_\_").

(Rather like the positivity of mass enforces attractive Newtonian gravitation.)

3

Observational Successes of the Hot Big Bang Theory

The "theory" is consistent with the observed

- expansion of the Universe.
- CMBR.
- quantities of elements such as hydrogen and helium formed in the early Universe.

4

By '70s, the big bang theory seemed solid.

The origin of the uniformity of the Universe was unknown, but it was indisputable. The uniformity of the CMBR was proof of that.

The density of the Universe was also unknown, but it appeared to be close to the critical density of

$$\rho_c = \frac{c^4}{8\pi G}(3H_0^2).$$

5

Then, at a conference in 1979 to celebrate the 100th anniversary of the birth of Albert Einstein, Dicke and Peebles (Drs. Microwave Background), discussed puzzling aspects of the big bang theory:

**9. The big bang cosmology – enigmas and nostrums†**

\_\_\_\_\_

**R. H. DICKE AND P. J. E. PEBBLES**

6

ADDITIONAL NOTES

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They said

The big bang cosmology that developed out of Einstein's ideas and Hubble's observations has stood the test of time and observation, but even the staunchest advocate would admit that it is at best only a reasonable first approximation that certainly does not tell the whole story. There are in particular some curious and enigmatic features of this cosmology that lead us to think that an important piece of the picture may be missing. It is useful to review and reconsider these curiosities from time to time because they certainly have something to teach us. But what is it?

7

then went on with a quick account of what was understood about the Universe in 1917:

It is not clear how familiar Einstein was with the observational situation in astronomy, or how much attention he paid to it. There was at the time speculation that the spiral nebulae are island universes like the Milky Way galaxy, but there were also some good arguments that these objects must be only minor satellites. It was considered well established, from star counts, that the Milky Way star system is finite and bounded, shaped roughly like a flattened spheroid. The spiral nebulae seemed to be concentrated at the poles of this star system, which would suggest they are related to it. Also, by 1916 van Maanen had found the first tentative evidence of proper motions in some of the larger spirals (van Maanen, 1916). If valid, and if the internal velocities in these systems are less than the velocity of light, it would make them quite close and much smaller than the Milky Way.

8

Although Einstein had been trying to build a larger cosmology, the indications were that the Milky Way was the entire Universe. But:

Over the next two decades it became clear that these indications are misleading, the former because interstellar dust in the plane of the Milky Way obscures the galaxies and the latter because of observational problems. Einstein's vision was remarkably good. In 1924 Hubble showed, by the identification of variable stars of known intrinsic luminosity, that the spiral nebulae are well outside the Milky Way and at least comparable to it in size (Hubble, 1924). Hubble's surveys of the galaxy distribution, begun in 1926 (Hubble, 1926) and continuing through the 1930s, gave the first direct evidence of large-scale homogeneity and isotropy. This has been confirmed by recent observations of the precise isotropy of the radiation background – X-ray, microwave,

9

The observations that the Universe was much bigger than we'd originally thought had also provided evidence of large scale homogeneity and isotropy.

This is remarkable:

The concept of large-scale homogeneity has been with us so long that cosmologists tend to take it as a commonplace, but it is remarkable simply because it stands in such contrast to our experience that things have structure – from the properties of subatomic particles on up to the organization of galaxies in great clouds. Milne (1935) was the first to

10

It's not only remarkable, large scale homogeneity is a problem:

The distant galaxies observed in well-separated parts of the sky are so far apart from each other that there is not time enough since the big bang for a signal to have traveled from one to the other. Observers on Earth can see and compare them, being about half-way in between, and in line with homogeneity it is found that the galaxies are quite similar. By comparing radiation background intensities across the sky it is also found that the temperature and expansion rate are precisely synchronized across the visible universe. Even though the separate parts of the visible universe are not visible to each other they are evolving in very precise unison.

11

Really?

Are structural relations between widely separated parts of the universe a problem? In the past these parts were much closer together. But close proximity in earlier times does not eliminate the problem. Assum-

Yes, really.

This problem, the problem of inexplicable uniformity, is known as the

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ADDITIONAL NOTES

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Horizons in cosmology arise because of the finite age of the Universe.

If the Universe is 14 billion years old, objects that are 15 billion light years away cannot yet have communicated with us.

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Qualitatively, \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

(1) Do you know of an example? \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

15

OK, to stop messing with your minds, why is the initial singularity both a curse and a curse?

1) It's a curse because it cuts off the Universe in the prime of its reverse-youth (and, as collateral damage, causes a potential horizon problem).

2) It's a curse because everything is squeezed so tightly at the initial singularity, it's hard to know what's going on.

17

Further, a patch of the Universe 12 billion light years from us in one direction cannot have communicated with another patch 12 billion light years from us in the opposite direction.

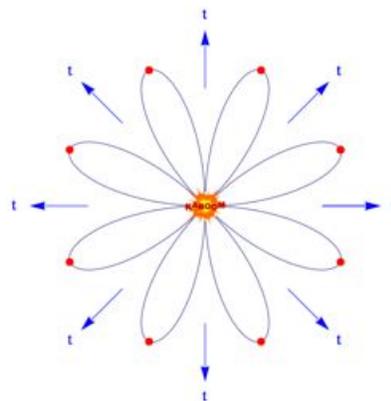
Yet they look virtually identical. \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

14

(2) What about the FLRW model might possibly cause a horizon problem? \_\_\_\_\_

(3) What about the FLRW model might possibly prevent the initial singularity from causing a horizon problem? \_\_\_\_\_

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18

ADDITIONAL NOTES

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This possibility was what Dicke and Peebles were alluding to when they said “in the past these parts were much closer together”:

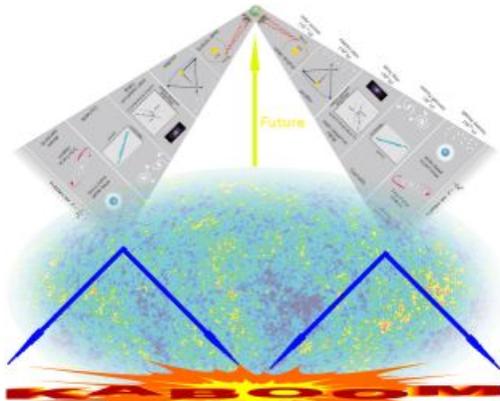
Are structural relations between widely separated parts of the universe a problem? In the past these parts were much closer together. But close proximity in earlier times does not eliminate the problem. Assum-

But, as they go on to say, “close proximity in earlier times does not eliminate the problem.”

19

To recap: The horizon problem of standard FLRW is that \_\_\_\_\_  
\_\_\_\_\_. Different parts of the Universe in different directions look the same (most strikingly the microwave background radiation) even though they can't have communicated. (Whether you think it a problem depends on how content you are with coincidence.)

20



21

OK, we may have one problem. Actually, we have two.

Dicke and Peebles, go right on to say:

The relationships of widely separated parts of the universe are not the only problem. There is a remarkable balance of mass density and expansion rate. In general relativity theory with  $\Lambda=0$  the two are

22

related by the equation

$$H^2 = \left(\frac{1}{a} \frac{da}{dt}\right)^2 = \frac{8}{3}\pi G\rho(t) - \frac{c^2}{R^2 a^2}, \quad (9.1)$$

where  $a(t)$  is the expansion parameter,  $R$  is a constant, and  $|R|a(t)$  is the magnitude of the space curvature (measured in a hypersurface of roughly constant galaxy proper number density, at fixed cosmic time  $t$ ). The present relative value of the two terms on the right side of this equation is poorly known, because the mean mass density,  $\rho$ , is so uncertain, but it is unlikely that the first term is less than 3 per cent of the magnitude of the second. Since  $\rho$  varies as  $a^{-3}$  (or more rapidly if pressure is important) the mass term on the right-hand side dominates the curvature term when  $a$  is less than about 3 per cent of its present value. Tracing the expansion back in time, one finds that at  $t \sim 1$  s, when much of the helium is thought to have been produced, the mass term is some 14 orders of magnitude larger than the curvature term. This means the expansion rate has been tuned to agree with the mass density to an accuracy better than 1 part in  $10^{14}$ . In the limit  $t \rightarrow 0$  this balance

23

Equation (9.1) in the Dicke-Peebles paper should remind you of something.

(4) What is it?

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ADDITIONAL NOTES

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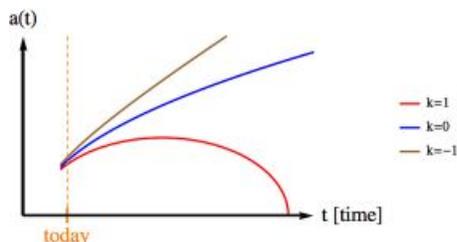
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Remember,  $k = 0$  gives the critical density, the dividing density between

a closed Universe ( $k = 1, \rho > \rho_c$ ) and an open Universe ( $k = -1, \rho < \rho_c$ ).



25

Solving the ( $\diamond$ ) equation for  $\rho$ , we'd obtained

$$\rho = \frac{c^4}{8\pi G} \left( 3H_0^2 + \frac{3k}{a^2(t)} \right).$$

(5) Write the right-hand side in terms of  $\rho_c$ .

26

(6) Solve for  $\rho_c$ .

(7) What is  $\rho_c/\rho - 1$ ?

27

The quantity  $\rho/\rho_c$  is called the \_\_\_\_\_ and denoted by \_\_\_\_\_. So we have

$$(\Omega^{-1} - 1)\rho a^2(t) = -\frac{3kc^4}{8\pi G} = \text{constant}.$$

(8) Writing  $\rho a^2(t)$  as  $\rho a^3(t)/a(t)$ , how does it behave as the Universe expands? \_\_\_\_\_

(9) What does that say about  $(\Omega^{-1} - 1)$ ? \_\_\_\_\_

28 \_\_\_\_\_

Why is that a problem?

The observed density of the Universe is estimated to be between a tenth and twice the critical density of about  $10^{-30}$  g/cc: i.e.,  $0.1 < \Omega < 2$ .

(10) What are the bounds on  $(\Omega^{-1} - 1)$ ?

29

For a rapidly growing quantity to be this small today (after 13 billion years of growth), the density parameter  $\Omega = \rho/\rho_c$  would have had to have been between 0.9999999999999999 and 1.0000000000000001 1 sec after the big bang.

the expansion rate has been tuned to agree with the mass density to an accuracy better than 1 part in  $10^{14}$ . In the limit  $t \rightarrow 0$  this balance

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ADDITIONAL NOTES

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Even small deviations from this would cause the Universe to behave very differently from observed.

A slightly greater value for  $\rho$  would make the Universe to be closed, and recollapse rapidly. A slightly smaller value would give too rapid an expansion.

31

Dicke and Peebles presented the horizon and flatness problems at the Einstein centenary conference in 1979, but had already been giving lectures about them at other venues.

One lecture was by Dicke at Cornell University on November 13, 1978, on the flatness problem.

It was attended by a young particle physicist, Alan Guth.

32

Guth got his Ph.D. from MIT in 1972 and had hopped around in temporary positions since:

- Princeton, 1971–74
- Columbia, 1974–76
- Cornell, 1976–79
- SLAC, 1979–1980

In 1978 Guth felt cosmology wasn't interesting. He went to Dicke's lecture because Penzias & Wilson had just received a Nobel Prize.

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Guth was working on "Grand Unified Theory" at the time – the theory that unifies the strong nuclear, weak nuclear and electromagnetic forces at high energies (temperatures).

In 1970, we'd had a good understanding of just electromagnetism (and gravity).

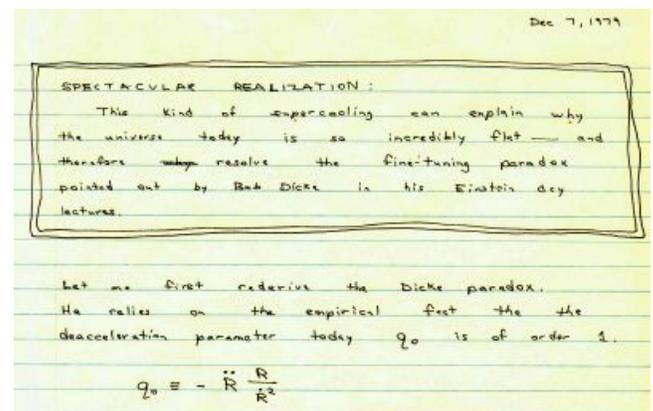
By 1976, we had a unified theory of three of the four fundamental forces.

34

Guth was investigating the existence of magnetic monopoles in GUT: Could they have existed in the early Universe and, if they did, why might we not see them today?

On the night of December 6, 1979, Guth felt he had found the reason why we see no monopoles today, through a mechanism called supercooling.

35 Then he remembered Dicke's talk (a year ago)...



Page from his notebook, provided on March 20, 2014, by Alan Guth

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ADDITIONAL NOTES

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Later that same month, Guth was told over lunch about the horizon problem of cosmology (he didn't know of it at the time). He went home and figured out, that afternoon itself, that his "supercooling" mechanism would solve this problem too.

It was, as he said later, as if he'd found the master key to the Universe: door after door opened with that single key.

37

Guth's career after December 1979:

- January 23, 1980: Announces his ideas in a seminar at SLAC
- January 24, 1980, before lunch: gets invitations to present his ideas from three different universities.
- January 24, 1980, after lunch: Is invited to spend three further years at SLAC; hears that U. Penn, and UC Davis are considering offering permanent professorships.
- January 28, 1980: U. Penn offers the job.
- February-March, 1980: Lectures at ten universities, including Harvard, Princeton, Columbia and Cornell.

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- On his return has offers of professorships from Minnesota, Rutgers, Harvard, Princeton, Maryland, UC Davis and UC Santa Barbara.
- But he wants MIT – and shortly after, he gets it. He's been there ever since.

What was this big idea of Alan Guth's?

**Inflationary universe: A possible solution to the horizon and flatness problems**

Alan H. Guth\*

Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305  
(Received 11 August 1980)

The standard model of hot big-bang cosmology requires initial conditions which are problematic in two ways: (1) The early universe is assumed to be highly homogeneous, in spite of the fact that separated regions were causally disconnected (horizon problem); and (2) the initial value of the Hubble constant must be fine tuned to extraordinary accuracy to produce a universe as flat (i.e., near critical mass density) as the one we see today (flatness problem). These problems would disappear if, in its early history, the universe supercooled to temperatures 28 or more orders of magnitude below the critical temperature for some phase transition. A huge expansion factor would then result from a period of exponential growth, and the entropy of the universe would be multiplied by a huge factor when the latent heat is released. Such a scenario is completely natural in the context of grand unified models of elementary-particle interactions. In such models, the supercooling is also relevant to the problem of monopole suppression. Unfortunately, the scenario seems to lead to some unacceptable consequences, so modifications must be sought.

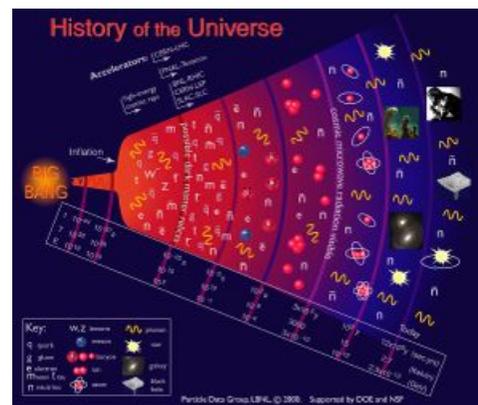
39

**Highlights from History**

Time (s)	What happened?	Temp (°K)
?	Matter forms	?
$2.0 \cdot 10^{-12}$	Electromagnetism	$1.2 \cdot 10^{18}$
$2.0 \cdot 10^{-6}$	quarks combine	$1.7 \cdot 10^{12}$
1.0	neutrinos released	$1.2 \cdot 10^{10}$
$1.8 \cdot 10^2$	Hydrogen forms	$1.2 \cdot 10^9$
$1.2 \cdot 10^{13}$	photons released	$2.9 \cdot 10^3$
.	.	.
$4.4 \cdot 10^{17}$	Today	2.8

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ADDITIONAL NOTES

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Note:

An “eV” is an electron volt, a unit of energy. An eV equals  $1.6 \times 10^{-19}$  J.

A GeV is a billion electron volts.

These units are used by quantum mechanics.

43

Over the 1970s a working model, the standard model, was created that unified the EM and both nuclear forces at an energy scale of about  $10^{16}$  GeV. The theory provides a symmetric, unified description at that scale and explains how the symmetry is broken at lower scales.

Called The Grand Unified Theory, or GUT.

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Guth goes on to say:

Before explaining the puzzles, I would first like to clarify my notion of “initial conditions.” The standard model has a singularity which is conventionally taken to be at time  $t=0$ . As  $t \rightarrow 0$ , the temperature  $T \rightarrow \infty$ . Thus, no initial-value problem can be defined at  $t=0$ . However, when  $T$  is of the order of the Planck mass ( $M_P \equiv 1/\sqrt{G} = 1.22 \times 10^{19}$  GeV)<sup>1</sup> or greater, the equations of the standard model are undoubtedly meaningless, since quantum gravitational effects are expected to become essential. Thus, within the scope of our knowledge, it is sensible to begin the hot big-bang scenario at some temperature  $T_0$  which is comfortably below  $M_P$ ; let us say  $T_0 = 10^{17}$  GeV. At

45

Note that Guth goes freely between temperature, energy and mass. The energy scale that he picks as the starting point is significant: It’s the GUT scale.

He goes on to describe the contents of the Universe as a particle physicist would.

In the standard model, the initial universe is taken to be homogeneous and isotropic, and filled with a gas of effectively massless particles in thermal equilibrium at temperature  $T_0$ . The ini-

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Guth then sets up the FLRW equations:

one of the three discrete values. The evolution of  $R(t)$  is governed by the Einstein equations

$$\ddot{R} = -\frac{4\pi}{3} G(\rho + 3p)R, \quad (2.2a)$$

$$H^2 + \frac{\dot{R}}{R^2} = \frac{8\pi}{3} G\rho, \quad (2.2b)$$

47

The key phrase in Guth’s paper is

behavior differs markedly from the standard model, in which  $H$  falls as  $t^{-1}$ .)

The false vacuum state is Lorentz invariant, so  $T_{\mu\nu} = \rho_0 g_{\mu\nu}$ . It follows that  $p = -\rho_0$ , the pressure is negative. This negative pressure allows for the conservation of energy, Eq. (2.3). From the second-order Einstein equation (2.2a), it can be seen that the negative pressure is also the driving force behind the exponential expansion.

The Lorentz invariance of the false vacuum has one other consequence: The metric described by

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ADDITIONAL NOTES

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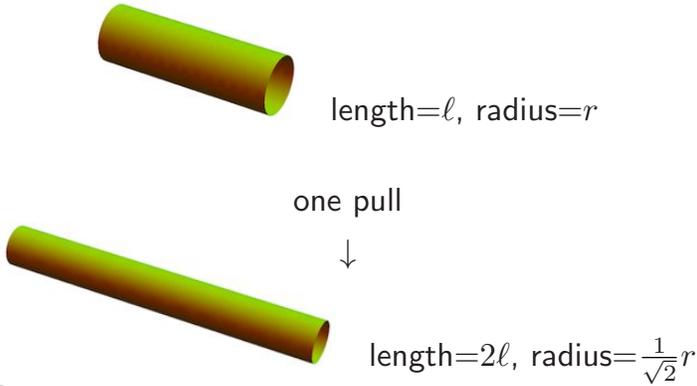


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Exponential Expansion: Handpulled Noodles



49

(11) Why  $1/\sqrt{2}$  in the radius, not  $1/2$ ?

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(12) If the original dough is a cylinder of length  $\ell$  and radius  $r$ , after  $n$  pulls what are the new length and radius?

50

For example, after 10 pulls a cylinder of length 30 cm and radius 3 cm will have over 1,000 strands\* of radius  $< 1\text{mm}$ .

\* Actually, one strand 300 m long ( $\sim 1,000\text{ ft.}$ ).

(13) Can you figure out the radius in your head?

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(14) Ditto for the length (using  $2^{10} \approx 10^3$ )?

51

“Biang” Menu

童子鸡长寿面 [活杀鸡] 13.5  
*Sautéed fresh Young Chicken Long-Pulled Longevity Noodles*  
 Fresh young chicken from local live poultry market diced and sautéed with bell peppers, tomatoes, potatoes and black mushrooms, served with our special long-pulled udun-like noodle (yes, singular, one noodle).  
 Note: chicken bones present

If Biang had a chef who could stretch the dough 100 times, its hypothetical noodle would be 30 trillion light years long, about 2,000 times the size of the known universe.

That’s the power of exponentials.

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Guth was able to show that the problems of big bang cosmology could be solved if you had an exponential scale factor that doubled  $a(t)$  every  $10^{-37}$  seconds.

A hundred doublings would stretch the Universe a million trillion trillion times in ten trillionth-trillionth-trillionth of a second and, Guth showed, would solve our cosmic problems.

53

(15) How does it solve the Horizon Problem?

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(16) How does it solve the Flatness Problem?

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ADDITIONAL NOTES

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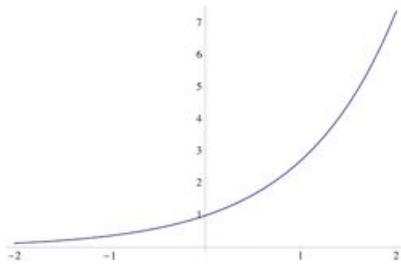


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But, but, but ... exponentials never hit zero



55 (17) What's that to us? \_\_\_\_\_



56

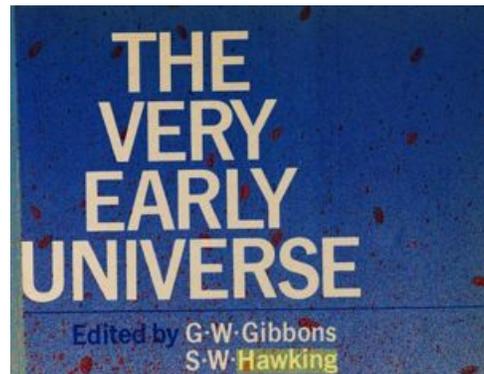
### Inflation Spreads

Cosmic inflation was quickly embraced.

In 1982, Stephen Hawking organized a conference around the very early Universe.

Major advances occurred here in figuring how inflation predicts structure formation: quantum fluctuations give rise to tiny irregularities that act as seeds for larger structures (stars, galaxies, etc.).

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UNIVERSITY OF CAMBRIDGE  
 Department of Applied Mathematics and Theoretical Physics  
 Silver Street, Cambridge CB3 9EW  
 Telephone: Cambridge (0223) 51645

Dr. A. H. Guth,  
 Center for Theoretical Physics,  
 Laboratory for Nuclear Science and  
 Department of Physics,  
 M.I.T.  
 Cambridge - Massachusetts, 02139,  
 United States of America.

14 October, 1981.

Dear Dr. Guth,

As you may know, the Nuffield Foundation has provided funds for a series of workshops on Quantum Gravity and related topics. Next year we are planning to hold a workshop here in Cambridge on the very early Universe (< 1 sec). The dates envisaged are June 21st to July 9th 1982.

The standard model seems to provide a satisfactory account of the evolution of the universe after 1 sec but it assumes certain initial conditions such as thermal equilibrium, spatial homogeneity and isotropy with small fluctuations, partial flatness, and the baryon to entropy ratio. The aim of the workshop would be to discuss how these conditions could have arisen from physical processes in the very early universe on the basis of grand unified theories and quantum gravity. Topics covered would include phase transitions, the generation of baryon number, the production of monopoles, primordial black holes and other long lived particles, the existence and nature of the initial singularity, particle creation and the origin of fluctuations. As in previous years the aim would be to limit the formal programme to about two seminars a day to leave time for informal discussion.

We would like to invite you to take part in the workshop. We could pay your airfare and subsistence, but we would be grateful if you could take advantage of any cheap fares that are convenient to enable us to stretch our limited funds as far as possible.

Yours sincerely,

*Gary Gibbons*  
*Stephen Hawking*

Stephen W. Hawking  
 Gary W. Gibbons

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### ADDITIONAL NOTES

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Where they're looking...



BICEP project, South Pole

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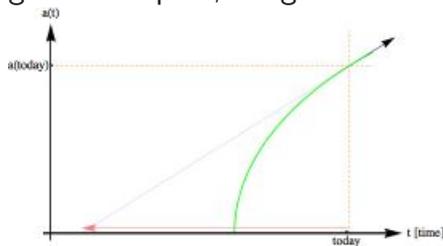
How does Standard Cosmology Imply a Beginning?

Observations support the fact that the Universe is expanding. Therefore, the graph of the scale factor,  $a(t)$ , is increasing at  $t = \text{today}$  and, *if gravity is always attractive*, is concave down everywhere.

We can show based only on these two conditions that  $a(t) = 0$  at a finite time in the past.

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Projecting into the past, we get:



The graph of  $a(t)$  can't wriggle out from under the straight line that's the past-projection of  $\dot{a}(\text{today})$  without going concave up.

65

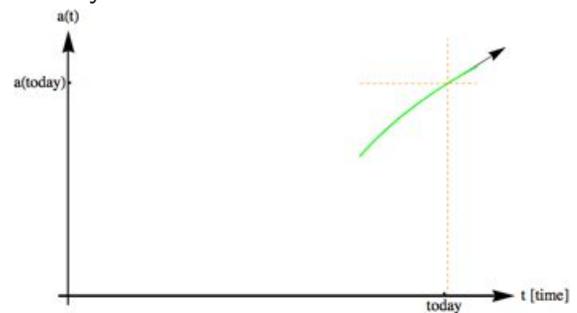
### Revisiting the Initial Singularity

The conclusion that there's an initial singularity is an important one, because it says \_\_\_\_\_  
\_\_\_\_\_. That's one of the most important results of standard modern cosmology.

Many people have resisted this: \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

62

The graph of  $a(t)$  is increasing today and concave down everywhere:



64

So,  $a(t)$  must be zero a finite time in the past in an expanding Universe in which gravity is attractive. As you approach that time, the matter density, \_\_\_\_\_  
\_\_\_\_\_. That's how *calculations* indicate that the Universe had a beginning.

But \_\_\_\_\_  
\_\_\_\_\_

What happens in more realistic situations?

66

### ADDITIONAL NOTES

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Even without symmetry assumptions, the theorems of Hawking and Penrose show that there is an initial singularity (in the form of \_\_\_\_\_) as long as some energy condition holds: weak or strong. (\_\_\_\_\_) That was a very important conclusion. But, it needs to be revisited in inflation for technical and philosophical reasons.

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**1. Technical reasons:**

In inflation, the

1a: \_\_\_\_\_.

1b: \_\_\_\_\_.

Theorems that say there was an initial singularity that depend on these conditions no longer apply.

**2. Philosophical reason:** \_\_\_\_\_

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**2. Philosophical Reason: Inflation is Forever**

Because the mechanism of inflation involves quantum processes, it \_\_\_\_\_. (That would lead to another version of the horizon problem.)

Inflation will end at different times at different places.

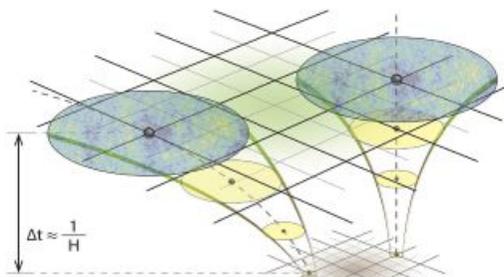
69

Each post-inflationary region will expand at the “normal” cosmological rate, but these regions will be separated by a background that is expanding so fast, they will not, in general, merge.

A “bubble” (“pocket Universe”) such as ours comes from a tiny post-inflationary patch of spacetime in a hot, dense, explosive state. The explosion (“bang”?) would lead to our observed Universe.

70

But, we’d be part of a multiverse:



P.D. Will, “Eternal Inflation and the Multiverse”, 2012  
<https://esc.fnwi.uva.nl/thesis/centraal/files/f252559910.jpg>

71

When inflation ends at a particular place cannot be predicted with certainty. The quantum nature of inflation means we can only assign probabilities.

\_\_\_\_\_  
 \_\_\_\_\_

Inflation is, in general, \_\_\_\_\_: it continues somewhere forever into the future

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ADDITIONAL NOTES

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It's natural to ask if inflation is also past-eternal: does it continue forever into the past?

The explosion from which a post-inflation bubble such as ours evolves would not itself be a singularity, and there would not automatically be a singularity prior to it, just a state of eternal inflation.

In this scenario, \_\_\_\_\_

73

Non-singular Inflationary Cosmology

A short history

In the early 1980s attempts were made to construct inflationary, non-singular cosmologies. They were not successful.

Occasional attempts were made to study if this lack of success was *necessary*.

74

1987: "Geodesic focusing, energy conditions and singularities," A. Borde, *Class. Quant. Grav.*, 4, 343-356 (1987).

"situations where there are repeated violations of the energy conditions are shown to still lead to the focusing of geodesics, . . . The existence of singularities in situations where the energy conditions are violated, as in inflationary cosmological models, is also discussed."

75

1992: "Did the Universe have a beginning?" A. Vilenkin, *Phys. Rev. D* 46, 2355 (1992).

"It is argued that 'eternal inflation' must have a beginning in time. Conditions are formulated for a spacetime to describe an eternally inflating universe without a beginning, and it is shown that these conditions cannot be satisfied. A rigorous proof is given for a two-dimensional spacetime, and a plausibility argument for four dimensions."

76

A Tale of Two Afternoons

Afternoon 1: September 1993

77

- "Eternal inflation and the initial singularity," AB & AV, *Phys. Rev. Lett.*, 72, 3305 (1994).
- "The impossibility of steady-state inflation," AB & AV, Eighth Yukawa Symposium, Japan (1994).
- "Open and closed universes, initial singularities and inflation," AB, *Phys. Rev.*, D50, 3692 (1994).
- "Inflation and initial singularities," AB, Seventh M. Grossmann Meeting, Stanford (1996).
- "Singularities in inflationary cosmology," AB & AV, Sixth Quant. Grav. Seminar, Moscow (1996).
- "Violations of the weak energy condition in inflating spacetimes," AB & AV, *Phys. Rev.*, D56, 717 (1997).

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ADDITIONAL NOTES

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A Tale of Two Afternoons

Afternoon 2: August 2001

79

**I. Introduction.**—Inflationary cosmological models [1–3] are generically eternal to the future [4,5]. In these models, the Universe consists of postinflationary, thermalized regions coexisting with still-inflating ones. In comoving coordinates the thermalized regions grow in time and are joined by new thermalized regions, so the *comoving volume* of the inflating regions vanishes as  $t \rightarrow \infty$ . Nonetheless, the inflating regions expand so fast that their *physical volume* grows exponentially with time. As a result, there is never a time when the Universe is completely thermalized. In such spacetimes, it is natural to ask if the Universe could also be past-eternal. If it could, eternal inflation would provide a viable model of the Universe with no initial singularity. The Universe would never come into existence. It would simply exist.

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The theorem requires one condition, the \_\_\_\_\_  
\_\_\_\_\_:

considering spacetimes for which the past region obeys an *averaged expansion condition*, by which we mean that the average expansion rate in the past is greater than zero:

$$H_{av} > 0. \tag{1}$$

With a suitable definition of  $H$  and the region over which the average is to be taken, we show that the averaged expansion condition implies past-incompleteness.

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The BGV Theorem

**Inflationary Spacetimes Are Incomplete in Past Directions**

Arvind Borde,<sup>1,2</sup> Alan H. Guth,<sup>1,3</sup> and Alexander Vilenkin<sup>1</sup>

<sup>1</sup>*Institute of Cosmology, Department of Physics and Astronomy, Tufts University, Medford, Massachusetts 02155*

<sup>2</sup>*Natural Sciences Division, Southampton College, New York 11968*

<sup>3</sup>*Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139*

(Received 5 October 2001; revised manuscript received 24 January 2003; published 15 April 2003)

Many inflating spacetimes are likely to violate the weak energy condition, a key assumption of singularity theorems. Here we offer a simple kinematical argument, requiring no energy condition, that a cosmological model which is inflating—or just expanding sufficiently fast—must be incomplete in null and timelike past directions. Specifically, we obtain a bound on the integral of the Hubble parameter over a past-directed timelike or null geodesic. Thus inflationary models require physics other than inflation to describe the past boundary of the inflating region of spacetime.

80

After reviewing the background

More general theorems showing that inflationary spacetimes are geodesically incomplete to the past were then proved [8]. One of the key assumptions made in these theorems is that the energy-momentum tensor obeys the weak energy condition. Although this condition is satisfied by all known forms of classical matter, subsequent work has shown that it is likely to be violated by quantum effects in inflationary models [9,10]. Such violations must occur whenever quantum fluctuations result in an increase of the Hubble parameter  $H$ —i.e., when  $dH/dt > 0$ —provided that the spacetime and the fluctuation can be approximated as locally flat. Such upward fluctuations in  $H$  are essential for the future-eternal nature of chaotic inflation. Thus, the weak energy condition is generally violated in an eternally inflating universe. These violations appear to open the door again to the possibility that inflation, by itself, can eliminate the need for an initial singularity. Here we argue that this is not the case. In fact,

We state our main claim.

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metric. We define the Hubble parameter as [14]

$$H \equiv \frac{\Delta u_r}{\Delta r} = \frac{-v_\mu(Du^\mu/d\tau)}{\gamma^2 - \kappa}. \tag{8}$$

Since  $\mathcal{O}$  is a geodesic, we have  $(Dv^\mu/d\tau) = 0$ , and therefore

$$H = \frac{-d\gamma/d\tau}{\gamma^2 - \kappa} = \frac{d\tau}{d\gamma} F(\gamma(\tau)), \tag{9}$$

where

$$F(\gamma) = \begin{cases} \gamma^{-1} & \text{null observer } (\kappa = 0), \\ \frac{1}{2} \ln \frac{\gamma+1}{\gamma-1} & \text{timelike observer } (\kappa = 1). \end{cases} \tag{10}$$

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ADDITIONAL NOTES

As in Sec. II, we now integrate  $H$  along  $\mathcal{O}$  from some initial  $\tau_i$  to some chosen  $\tau_f$ :

$$\int_{\tau_i}^{\tau_f} H d\tau = F(\gamma_f) - F(\gamma_i) \leq F(\gamma_f). \quad (11)$$

In the null case  $F(\gamma_f) = \gamma_f^{-1}$ , which is equal to the value of  $d\tau/dt$  at  $t_f$ , normalized in Sec. II to unity.

Equation (11) therefore reproduces exactly the results of Eqs. (4) and (6), but in a much more general context. Again we see that if  $H_{av} > 0$  along any null or noncomoving timelike geodesic, then the geodesic is necessarily past-incomplete.

85

*IV. Discussion.*—Our argument shows that null and timelike geodesics are, in general, past-incomplete in inflationary models, whether or not energy conditions hold, provided only that the averaged expansion condition  $H_{av} > 0$  holds along these past-directed geodesics. This is a stronger conclusion than the one arrived at in previous work [8] in that we have shown under reasonable assumptions that almost all causal geodesics, when extended to the past of an arbitrary point, reach the boundary of the inflating region of spacetime in a finite proper time (finite affine length, in the null case).

What can lie beyond this boundary? Several possibil-

86

Whatever the possibilities for the boundary, it is clear that unless the averaged expansion condition can somehow be avoided for all past-directed geodesics, inflation alone is not sufficient to provide a complete description of the Universe, and some new physics is necessary in order to determine the correct conditions at the boundary [16]. This is the chief result of our Letter. The result depends on just one assumption: the Hubble parameter  $H$  has a positive value when averaged over the affine parameter of a past-directed null or noncomoving timelike geodesic.

87

### Spreading the Message

Alan Guth spoke at Hawking’s 60th birthday in 2002, partly on the fluctuations that led to structure formation, partly on the BGV theorem.

In the historical part of his talk, Guth mentioned how in the early days of inflation, he was excited that Stephen Hawking had organized a conference on the early Universe and invited him.

88

### On the BGV theorem, Guth asked

“... can inflation by itself be the complete theory of cosmic origins? Can inflation be eternal into the past as well as the future, allowing a model which on very large scales is steady state, eliminating the need for a beginning? The answer I believe is no ... Borde, Vilenkin, and I have proven a rigorous theorem, ... [that] shows that the simplest type of inflationary models still require a beginning, even though they are eternal into the future. The difficulty is that we have no way of discussing the class of all possible inflationary models, so we cannot say that our theorem applies to all cases.”

89

And so word got out, but it sometimes got to unexpected places ...

90

### ADDITIONAL NOTES

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Review Symbols

- $a(t)$ : \_\_\_\_\_
- $P$ : \_\_\_\_\_
- $\rho$ : \_\_\_\_\_
- $G$ : \_\_\_\_\_
- $c$ : \_\_\_\_\_
- $\Lambda$ : \_\_\_\_\_
- $\Omega$ : \_\_\_\_\_

1

Early Indications of Missing Matter

- 1844: Motion of the star Sirius suggested mutual orbital motion with an invisible companion of comparable mass (Bessel). In 1862, companion identified as a faint white dwarf (Clark).
- 1930s: Velocities of stars near the sun (Oort), and velocities of stars in other galaxies (Zwicky) greater than explainable by the total visible mass. Zwicky was ignored.

2

- 1940s, 50s and 60s: Other evidence accumulated that the behavior of astronomical systems (motions, etc.) was not consistent with the amount of matter that was visible.
- 1970s: The scope of the problem was finally established by the careful observations of Rubin and Ford (under-recognized at the time) , and the theoretical work of Ostriker, Peebles, and others.

3

Dark Matter

Modern evidence suggests that \_\_\_\_\_  
 \_\_\_\_\_  
 \_\_\_\_\_

What is the evidence? Among others,

- \_\_\_\_\_
- \_\_\_\_\_

4

Galactic Rotation and Dark Matter

All the stars of a galaxy rotate around the galactic center.

But there's a puzzle in how they rotate. This was known since the 1930s, but was firmly established by the work of Rubin and Ford from the late 1960s to the late 1970s.

5

They first studied Andromeda ...

ROTATION OF THE ANDROMEDA NEBULA FROM A SPECTROSCOPIC  
 SURVEY OF EMISSION REGIONS\*  
 VERA C. RUBIN† AND W. KENT FORD, JR.†  
 Department of Terrestrial Magnetism, Carnegie Institution of Washington and  
 Lowell Observatory, and Kitt Peak National Observatory‡  
 Received 1969 July 7; revised 1969 August 21

... and found curves such as these:

6

ADDITIONAL NOTES

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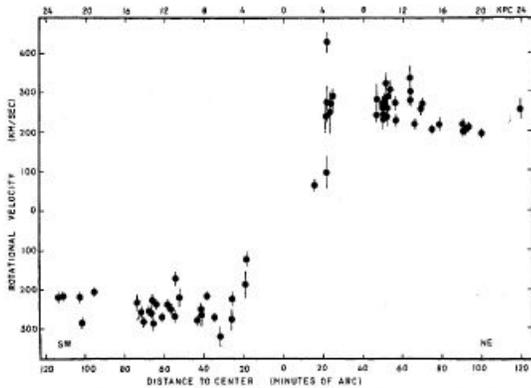


FIG. 3.—Rotational velocities for sixty-seven emission regions in M31, as a function of distance from the center. Error bars indicate average error of rotational velocities.

7

(1) What does the graph on the previous page show about rotational velocities as you go out from the center of Andromeda?

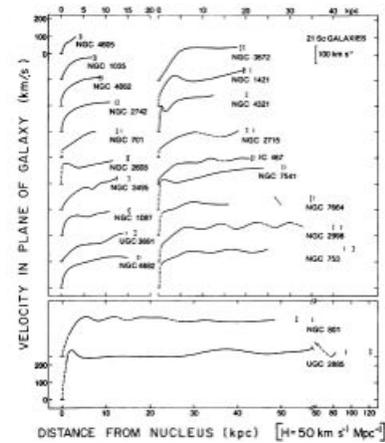
8

Over the next decade they carefully studied another 21 galaxies . . .

ROTATIONAL PROPERTIES OF 21 Sc GALAXIES WITH A LARGE RANGE OF LUMINOSITIES AND RADII, FROM NGC 4605 ( $R = 4$  kpc) TO UGC 2885 ( $R = 122$  kpc)

VERA C. RUBIN,<sup>1,2</sup> W. KENT FORD, JR.,<sup>1</sup> AND NORBERT THONNARD  
 Department of Terrestrial Magnetism, Carnegie Institution of Washington  
 Received 1979 October 11; accepted 1979 November 29

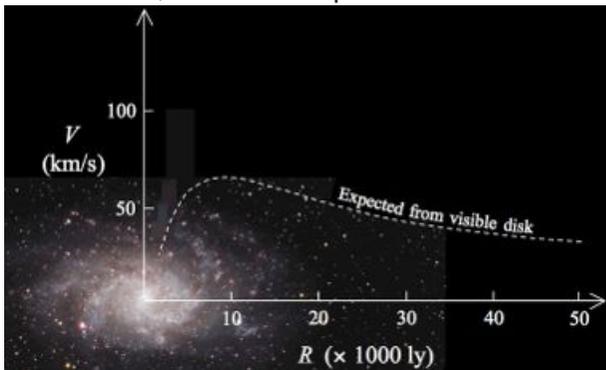
. . . and found similar behavior:



9

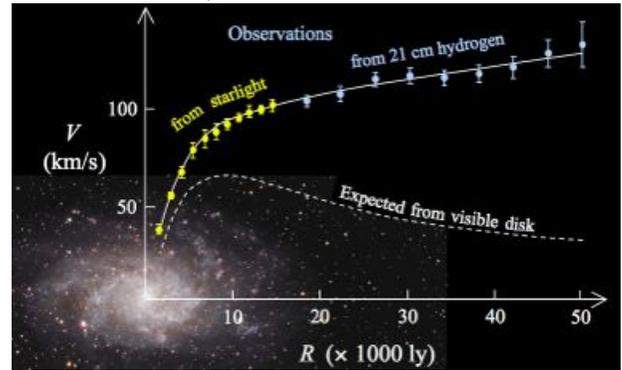
10

Galactic rotation, what we expect:



11

Galactic rotation, what we observe:



12

ADDITIONAL NOTES

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The rotations of stars around the centers of galaxies are more rapid than we expect from the visible matter in them.

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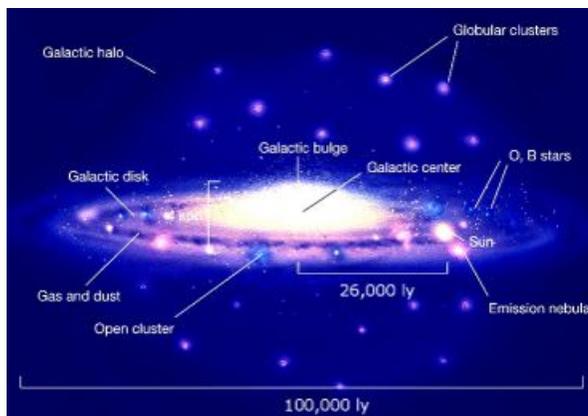
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13



14

(2) Why, based on the rotation curve, do we expect the dark matter to be scattered throughout the halo, not just concentrated in the black hole at the center of a galaxy?

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15

From the New York Times, December 27, 2016:

### Vera Rubin, 88, Dies; Opened Doors in Astronomy, and for Women

Vera Rubin, who transformed modern physics and astronomy with her observations showing that galaxies and stars are immersed in the gravitational grip of vast clouds of dark matter, died on Sunday in Princeton, N.J. She was 88.

Her work helped usher in . . . the realization that what astronomers always saw and thought was the universe is just the visible tip of a lumbering iceberg of mystery.

16

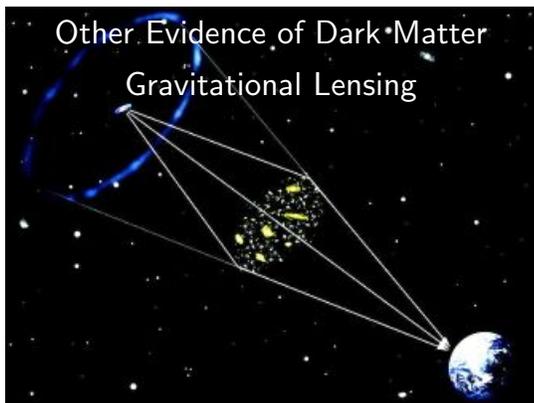
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There appears to be invisible – or “dark” – matter causing the extra bending.



17

18

### ADDITIONAL NOTES

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What is dark matter not?

We're more certain of this than what it is...

- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_

19

What might dark matter be?

Various proposals:

- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_

20

The Search for Dark Matter

- The Alpha Magnetic Spectrometer (AMS-02) on the International Space Station is analyzing cosmic rays for evidence of unusual particles.
  - The China Dark Matter Experiment (CDEX), a search for dark matter WIMP particles, at the China Jinping Underground Laboratory, 7,900 ft deep.
- ... plus IceCube at the S.P. and others.

21

Dark Energy

Evidence suggests that \_\_\_\_\_

(3) So, you HICOPS, how much of the Universe is visible? \_\_\_\_\_

22

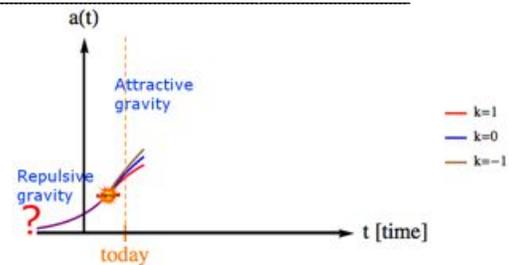
What is the evidence for dark energy?

We need to review the expansion of the Universe:

- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_

23

- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_



24

ADDITIONAL NOTES

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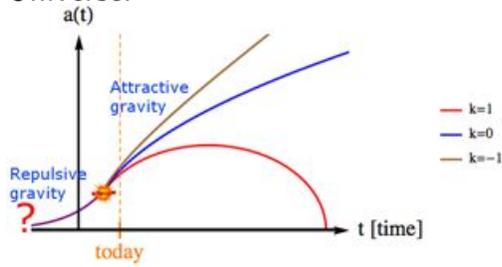


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▷ If the expansion is decelerating, this is the future of the Universe:



Measuring the deceleration rate is important.

25

The Supernova Cosmology Project (SCP), started in 1988 by Saul Perlmutter, had the aim of measuring the deceleration of the Universe - using \_\_\_\_\_ (SNe Ia) as standard candles.

SNe Ia: Supernova from the explosion of an accreting white dwarf or white dwarf merger.

26

SNe Ia are rare – \_\_\_\_\_. To get statistically significant results a large sample is needed.

Perlmutter's group observed thousands of galaxies over two to three nights then imaged the same patches of the sky about three weeks later.

They found batches of about a dozen or so new SNe at a time.

27

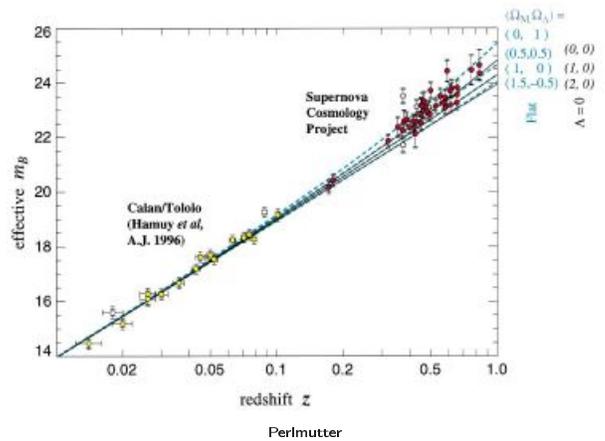
From the success of this strategy Brian Schmidt in Australia and others founded in 1994 a competing collaboration – the High-z Supernova Search Team (HZT).

Over the next few years, the HZT led by Schmidt and the SCP led by Perlmutter searched for supernovae independently, often but not always at the same telescopes.

28

The two teams published breakthrough papers in 1998 announcing that they had found that the expansion of the Universe \_\_\_\_\_.

29



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ADDITIONAL NOTES

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We do not see the energy that drives this acceleration. To what do we attribute this dark energy?

31

We report measurements of the mass density,  $\Omega_M$ , and cosmological-constant energy density,  $\Omega_\Lambda$ , of the universe based on the analysis of 42 type Ia supernovae discovered by the Supernova Cosmology Project. The magnitude-redshift data for these supernovae, at redshifts between 0.18 and 0.83, are fitted jointly with a set of supernovae from the Calán/Tololo Supernova Survey, at redshifts below 0.1, to yield values for the cosmological parameters. All supernova peak magnitudes are standardized using a SN Ia light-curve width-luminosity relation. The measurement yields a joint probability distribution of the cosmological parameters that is approximated by the relation  $0.8\Omega_M - 0.6\Omega_\Lambda \approx -0.2 \pm 0.1$  in the region of interest ( $\Omega_M \leq 1.5$ ). For a flat ( $\Omega_M + \Omega_\Lambda = 1$ ) cosmology we find  $\Omega_M^{flat} = 0.28^{+0.04}_{-0.08}$  (1  $\sigma$  statistical)  $^{+0.03}_{-0.04}$  (identified systematics). The data are strongly inconsistent with a  $\Lambda = 0$  flat cosmology, the simplest inflationary universe model. An open,  $\Lambda = 0$  cosmology also does not fit the data well: the data indicate that the cosmological constant is nonzero and positive, with a confidence of  $P(\Lambda > 0) = 99\%$ , including the identified systematic uncertainties. The best-fit age of the universe relative to the Hubble time is  $t_0^{flat} = 14.9^{+1.1}_{-1.0}$  (0.63/h) Gyr for a flat cosmology. The size of our sample allows us to perform a variety of statistical tests to check for possible systematic errors and biases. We find no significant differences in either the host reddening distribution or Malmquist bias between the low-redshift Calán/Tololo sample and our high-redshift sample. Excluding those few supernovae that are outliers in color excess or fit residual does not significantly change the results. The conclusions are also robust whether or not a width-luminosity relation is used to standardize the supernova peak magnitudes. We discuss and constrain, where possible, hypothetical alternatives to a cosmological constant.

33

(4) How might the cosmological constant accelerate expansion?

35

MEASUREMENTS OF  $\Omega$  AND  $\Lambda$  FROM 42 HIGH-REDSHIFT SUPERNOVAE

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 AND  
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 (THE SUPERNOVA COSMOLOGY PROJECT)  
 Received 1998 September 8; accepted 1998 December 17

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OBSERVATIONAL EVIDENCE FROM SUPERNOVAE FOR AN ACCELERATING UNIVERSE AND A COSMOLOGICAL CONSTANT

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 Received 1998 March 13; revised 1998 May 6

ABSTRACT

We present spectral and photometric observations of 10 Type Ia supernovae (SNe Ia) in the redshift range  $0.16 \leq z \leq 0.62$ . The luminosity distances of these objects are determined by methods that employ relations between SN Ia luminosity and light curve shape. Combined with previous data from our High- $z$  Supernova Search Team and recent results by Riess et al., this expanded set of 16 high-redshift supernovae and a set of 34 nearby supernovae are used to place constraints on the following cosmological parameters: the Hubble constant ( $H_0$ ), the mass density ( $\Omega_M$ ), the cosmological constant (i.e., the vacuum energy density,  $\Omega_\Lambda$ ), the deceleration parameter ( $q_0$ ), and the dynamical age of the universe ( $t_0$ ). The distances of the high-redshift SNe Ia are, on average, 10%–15% farther than expected in a low mass density ( $\Omega_M = 0.2$ ) universe without a cosmological constant. Different light curve fitting methods, SN Ia subsamples, and prior constraints unanimously favor eternally expanding models with positive cosmological constant (i.e.,  $\Omega_\Lambda > 0$ ) and a current acceleration of the expansion (i.e.,  $q_0 < 0$ ). With no prior

34

The observational work led to Nobel prizes for some team members in 2011.

Over the last seventeen years other observations of the CMBR, large scale structure of galaxies, etc., have confirmed that around 68% of the content of the Universe is in the form of dark energy. We don't have a good understanding of what the energy is underlying a possible  $\Lambda$ .

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ADDITIONAL NOTES

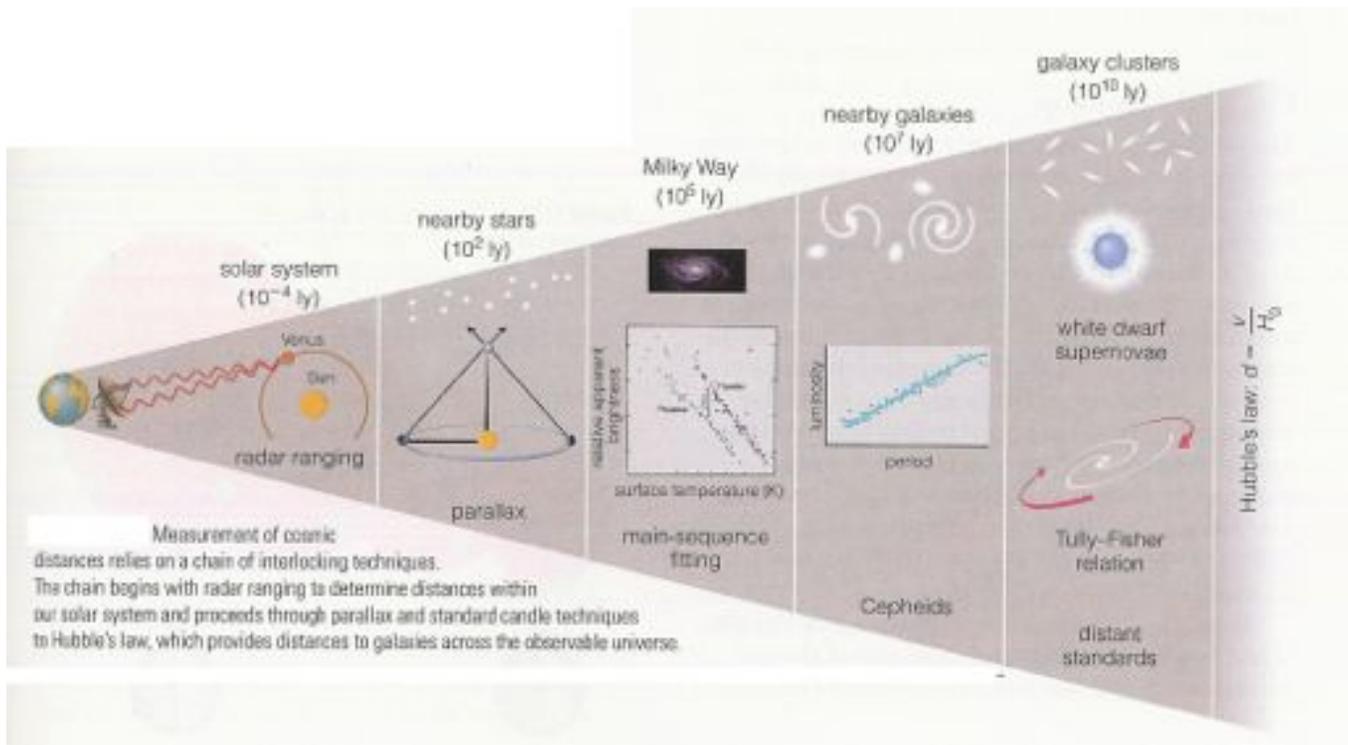




Arvind Borde

# AST 10: Homework 1

- 1] Assuming that light travels at about 300000 km/sec, how long in km would a light-second be? A light-minute?
- 2] If your friend says “You look light-years younger” what, if anything, does that mean?
- 3] Looking at far-away objects gives us a good picture of what the Universe is like at this instant of time. True or false? Give reasons either way.
- 4] Are all galaxies spiral in shape?
- 5] If you bounce a radiowave off the head of your friend and it takes 5-millionth of a second to return to you, how far is your friend from you? (Use the information given in one of the problems above.)
- 6] What do your eyes have in common with methods used by astronomers to figure the distance to nearby stars.
- 7] Does the parallax method work better for relatively nearby objects or for very distant ones?
- 8] What is one specific place in this cosmic distance ladder diagram that standard candles are used? What does “standard candles” mean?



Your name: \_\_\_\_\_

Arvind Borde

# AST 10: Homework 1b

1. Matter is one of two basic entities that make up the world. What is the other?
2. What are the four basic forces? Which of these is most important in determining the large-scale structure of the Universe? Why?
3. Calculate (a)  $10^3 \times 10^{-3}$ , and (b)  $10^3 \div 10^{-3}$ .
4. If  $y \propto x$  and  $x$  triples, what will happen to  $y$ ?
5. If  $y \propto 1/x$  and  $x$  triples, what will happen to  $y$ ?
6. If  $y \propto 1/x^2$  and  $x$  triples, what will happen to  $y$ ?
7. If you unexpectedly gain mass overnight to double what you were, would the gravitational force between you and the earth change, and by how much?

Your name: \_\_\_\_\_

Arvind Borde

# AST 10: Homework 2

1. What is one system that's stable against gravitational collapse because of rotation and one system that's stable because of expansion?
2. What is "gravitational collapse" anyway?
3. Does an Sc galaxy have more or fewer spiral arms than an Sa?
4. As you go from E to S galaxies do they get flatter or "rounder"?
5. Why do we think the MW has a flattened shape?
6. Why do we think the MW is spiral?

Your name: \_\_\_\_\_

Arvind Borde

# AST 10: Homework 2b

1. What are the three parts of the Milky Way?
2. What are two parts of the Milky Way where stars might only be light days away from each other?
3. What is Sagittarius A\* and why should we care?
4. What do we think is the mass of the black hole at the center of the Milky Way?
5. How do we figure out this mass?
6. Are the arms of the Milky Way "material"? Give reasons.
7. How far out does the halo of the Milky Way seem to extend?
8. Is the sun a Population I star or Population II. Why?
9. How many stars does the Milky Way contain?
10. What are globular clusters? Where are they?

Arvind Borde

# AST 10: Homework 3

1. Let's check how much energy there is in 1 gm of matter. The unit of energy below will be an "erg." A 100 Watt bulb uses  $10^9$  ergs/second. A gallon of gas yields  $12 \times 10^{14}$  ergs of energy.

- The speed of light is  $c = 3 \times 10^{10}$  cm/sec. Calculate  $E = mc^2$  for  $m = 1$  gm.
- How many seconds would that power a 100 W bulb?
- How many seconds are there in a year? Convert the answer in (b) to years.
- How many gallons of gas is the answer in (a) equivalent to?

2. We said in class that the sun emits  $3.9 \times 10^{26}$  Joules of energy every second. In these units, the speed of light is  $3 \times 10^8$  meters/sec.

a) Using this value of  $c$ , and rearranging  $E = mc^2$ , calculate how much mass the sun must lose every second to account for this energy output. (The unit will automatically be kg.)

b) Rounding off the value we found in class, each  $4\text{H} \rightarrow \text{He}$  process loses  $5 \times 10^{-29}$  kg. How many such processes do you need to power the sun?

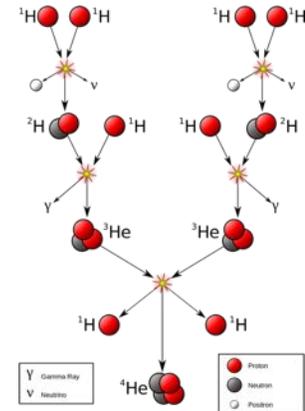
3. As said in the class, fusion in stars is a three-step process:

i) Two protons collide to produce deuterium (a variant of hydrogen), a positron (an anti-electron), and a neutrino.

ii) A proton collides with the deuterium to produce a another helium variant (helium-3) and a gamma ray (high-frequency electromagnetic wave).

iii) Two helium-3s collide to produce a normal helium nucleus, releasing two protons.

- At what stage(s) is energy released that can escape from the star?
- At what stage is light released that can escape from the star?
- Which form of energy gets out quicker?



4. In which part of a star do the nuclear reactions that "fuel" it occur? (Throughout the star? The outer layers? The inner core?) Why just in that region?

Arvind Borde

# AST 10: Homework 3b

1. The Hertzsprung-Russell diagram uses the temperature in degrees Kelvin, related to degrees Celsius by

$$^{\circ}K = ^{\circ}C + 273^{\circ}$$

- a) What is the freezing point of water in  $^{\circ}K$ ?  
b) What temperature is it in  $^{\circ}C$  when it's  $0^{\circ}K$ ?
2. Are stars likely to form from gas clouds that are 1% of  $M_{\odot}$ ? 50% of  $M_{\odot}$ ? 700% of  $M_{\odot}$ ? If not, why not? If stars do form in these cases, which (if any) are likely to create heavy elements such as iron?
3. Would you say our sun is (a) in its youth? (b) in middle age? (c) in old age? Why?
4. We said that the apparent brightness of a star is  $\propto L/d^2$ , where  $L$  is the absolute (or true) brightness. If a star is a hundred times fainter than you'd expect, were it 10 ly from us, is it really nearer us than that or farther? How far is it?

Arvind Borde

# AST 10: Homework 4

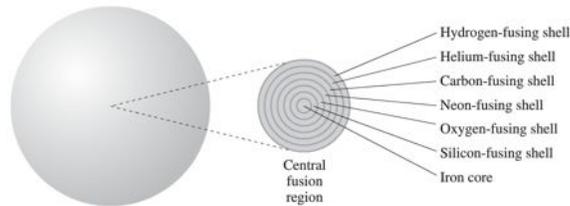
1. What are the three end states of stellar life and what are the *initial* masses and the masses of *the remnants* that lead to these?

2. The lifetime,  $t$ , on the “main sequence” (see the Hertzsprung-Russell diagram from Week 3) of a very massive star is

$$t \approx 30 \left( \frac{M}{7M_{\odot}} \right)^{-3} \text{ Myr.}$$

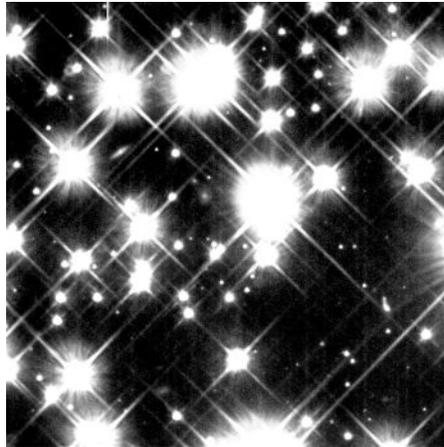
- What is the lifetime of a star with  $M = 7M_{\odot}$ ?
- How does that compare with the lifetime of the sun?
- What is the lifetime of a star with  $M = 14M_{\odot}$ ?
- How does that compare with the lifetime in part a)?
- Based on this as the mass increases, does the lifetime of a massive star go up or down?
- Can you think of a reason why the answer in part e might be what it is?

3. Here’s what goes on in the core of a massive star toward the end of its life



As you go outward, are you fusing lighter or heavier elements? Why might that be the case?

4. Here’s a Hubble space telescope image of a globular cluster. The faint images in it are believed to be white dwarfs. How many do you count?



NASA and H. R icher (University of British Columbia)

Arvind Borde

# AST 10: Homework 4b

1. Look through your notes and fill in columns B and C in the table below with the densities and the units we used ( $\text{kg/m}^3$ ,  $\text{g/cm}^3$ , etc.) for each object. In each case, use powers of 10, not "billions," etc.

Densities

Column A Object	Column B Density	Column C Units	Column D Densities in $\text{gm/cm}^3$
Water			
The Earth			
Lead			
A white dwarf			
A neutron star			

2.  $1 \text{ kg/m}^3$  converts to  $10^{-3} \text{ gm/cm}^3$ . Why?

3. Whether or not you can explain the previous conversion, use it to fill in Column D in the table above. You'll now have all the densities in the same units, and you can see how they compare.

4. Remember that density =  $\frac{M}{V}$  where  $M$  is the mass and  $V$  the volume.  
What is volume in terms of  $M$  and density?

5. Using  $M_{\odot} \approx 2 \times 10^{33} \text{ gm}$  and the previous result, what are the volumes of a one solar-mass (a) white dwarf, and (b) neutron star?

6. As you may remember, the formula for volume is  $V = \frac{4}{3}\pi r^3$ . What is  $r$  in terms of  $V$ ?

7. Use the answers of the previous two questions to get the radii of a one solar mass (a) white dwarf, and (b) neutron star. The answer will be in cm. Convert it to km (one km is 100,000 cm).

8. Here are the dimensions of some common things:

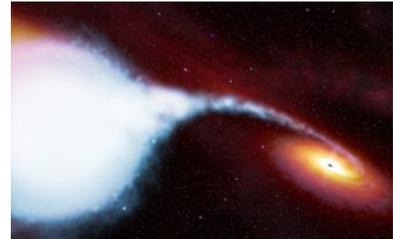
Diameter of Earth:  $1.3 \times 10^4 \text{ km}$ . Length of Manhattan: 21.6 km.

Which of these is closest to the *diameter* of a white dwarf radius? Which to a neutron star?

Arvind Borde

# AST 10: Homework 5

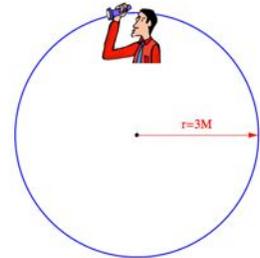
1. Do you think black holes might be detectable by observations? What might be some difficulties? How are they surmounted?
2. What are the different types of black holes we think there are?
3. By the “size” of a black hole astronomers usually mean its Schwarzschild radius. If the mass of a black hole doubles, how does its size change? What if the mass halves?
4. Look at the “galactic center animation” on the course webpage. The animation is based on ongoing observations of the motion of stars at the center of our galaxy (represented by ). There is no visible physical object at that precise location. What does the animation show?
5. One of the smallest known black holes was announced at (link also on course page)  
[http://www.nasa.gov/centers/goddard/news/topstory/2008/smallest\\_blackhole.html](http://www.nasa.gov/centers/goddard/news/topstory/2008/smallest_blackhole.html).  
What is the mass of the black hole? What is its Schwarzschild radius? How does that compare with the value given in the linked article?
6. Here's the Wikipedia list of the most massive black holes (link also on course page):  
[http://en.wikipedia.org/wiki/List\\_of\\_most\\_massive\\_black\\_holes](http://en.wikipedia.org/wiki/List_of_most_massive_black_holes).  
Roughly how many candidates are on the list? What is their range of masses (using  $M_{\odot}$  as the unit and expressing yourself in “millions,” “billions,” etc.)?
7. What are the difficulties with the Newtonian view of black holes?
8. In the 1960s a strong source of x-rays was discovered in a constellation called Cygnus. The x-ray source was named Cygnus X-1. Closer observations revealed a large blue star orbiting every 5 days around an unseen companion. On the right is an artist's rendering. What does the picture show? What might be the source of the x-rays?



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# AST 10: Homework 6

1. According to the theory of relativity, gravity is not a true force. What is it?
2. What attribute of an object is connected to the curvature of spacetime that it causes? (Color? Shape? Charge? Mass?)
3. What are the three effects explained (or predicted) by Einstein when he proposed the final version of the theory of relativity?
4. You are an astronaut orbiting the earth, when catastrophe strikes and the earth goes black hole. How does that affect your orbit?
5. A white dwarf is a solid physical object. A neutron star is a solid physical object. What about a black hole?
6. The sun has an average density of  $1.4 \text{ gm/cm}^3$ . That's 1.4 times the density of water. As mentioned in class, the sun would have to compress to a density of  $2 \times 10^{16} \text{ g/cm}^3$  to "go black hole." That's 20 million billion times the density of water. What would the density be for the formation of a black hole that's (a)  $10M_{\odot}$ ? (b)  $\frac{1}{10}M_{\odot}$ ?
7. We said in class that  $\frac{m}{\frac{4\pi}{3}\left(\frac{2Gm}{c^2}\right)^3} = \frac{3c^6}{32\pi G^3 m^2}$ . Work out the algebra to go from left to right.
8. You're at  $r = 3M$  from a black hole center, with a jet-propulsion pack. Can you safely orbit the black hole? If you shine a flashlight as shown, what might you see? What would happen if you were at  $r = M$ ?



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# AST 10: Homework 6b

1. Einstein calculated a formula predicting the angle that light bends by (for small angles) when it passes a spherical mass:

$$\text{angle} = (2.06 \times 10^5) \frac{4Gm}{bc^2}$$

where  $m$  is the mass of the object that's bending light,  $b$  (called the "impact parameter") is the distance of closest approach.  $G$  and  $c$  are, well,  $G$  and  $c$ . The values are below. The formula gives the angle in "seconds." For a light ray that grazes the sun, we use:

$$b = r_{\odot} = 7 \times 10^8 \text{ m},$$

$$m_{\odot} = 2 \times 10^{30} \text{ kg},$$

$$G = 6.7 \times 10^{-11} \text{ m}^3/\text{kg}\cdot\text{sec}^2,$$

$$c = 3 \times 10^8 \text{ m/s}.$$

What is the angle?

2. In the formula for the angle (above), as the the mass,  $m$ , goes up does the angle go up or down? Does this make sense?

3. In the formula for the angle (above), as the the impact parameter,  $b$ , goes up does the angle go up or down? Does this make sense?

4. I said that the term  $f(r)dt^2$  in the Schwarzschild "metric" (distance formula for spacetime surrounding a spherically symmetric object) determines the "proper time" felt by an observer. Remember that  $f(r) = 1 - \frac{r_s}{r}$ .

For  $r = 1000r_s$ , what is  $f(r)$ ?

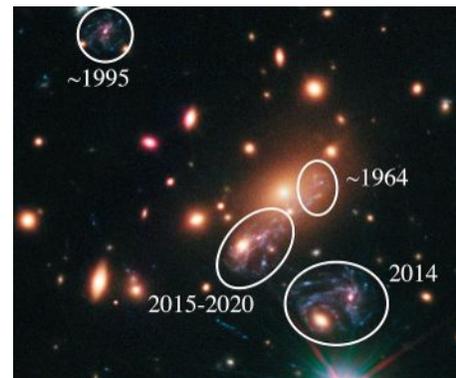
For  $r = 1.001r_s$ , what is  $f(r)$ ?

5. From the previous, does time move more slowly or more quickly close to a black hole compared to far from it?

6. Are gravitational waves hard to detect or easy? Why?

7. Why two detectors for gravitational waves?

8. As we said in class, a supernova that was seen in 2014 seemed to have been seen before in 1995 and 1964, and we expect to see it again by 2020. Why is this supernova flaring up again and again, and how is that even possible, or is something else happening?



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# AST 10: Homework 7

1. If two waves (Wave A and Wave B) have the same speed, but Wave A has three times the wavelength of Wave B, how are their frequencies related?
2. What is the Doppler effect?
3. Is light a wave or a particle?
4. What is a photon?
5. The frequency of a light wave can be measured as the number of waves that go past in a time unit. The speed of light is approximately 300 million meters/sec and the wavelength of orange light is roughly  $6 \times 10^{-7}$  meters. How many waves of orange light will go past you in one second? (Use the relationship between speed of a wave, wavelength and frequency.)
6. Is it possible to identify an element by the radiation it emits? How? What aspect of atomic structure is related to this phenomenon?
7. If the frequency of an electromagnetic wave doubles, how does its energy change?

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# AST 10: Homework 7b

1. Which of the ground-based telescopes that we discussed in class might not have its observations affected by the earth's rotation?

2. How smooth does the surface of the Arecibo telescope seem compared to the surface of the Subaru? (See class notes for pictures.) If the smoothnesses seem different, why might it be OK to have different degrees of smoothness in the two cases? For a radio telescope that's distinctly not smooth, see the picture on the right of a radio telescope at Stanford University. Why might it work as a reflector?



3. If a telescope with focal length 30 cm "sees" an object taking up  $2^\circ$  of its view, how big is the image in the telescope?

4. If an object makes an image that's 0.5 mm on your retina, how many degrees of your view does it occupy?

5. When you zoom into an object optically with your camera (not electronic zoom), does the lens extend or contract? Why?

6. Why is a ground-based x-ray telescope not a great idea?

7. What is the resolution in seconds of telescope with a 0.5 m diameter lens at visible light?

8. We've seen in class that some telescopes are dual-purpose: they detect visible light and infra-red. Were the telescope in the question above capable of this, would it have higher resolution in infra-red or lower (compared to visible light)?

9. What diameter lens do you need on a telescope that can resolve up to a thousandth of a second?

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# AST 10: Homework 8

1. In this portion of the Hubble Ultra Deep Field photograph below, what different types of galaxies do you see?



2. Here are some of the main points made at the Great Debate, between Shapley (“the nebulae are part of the Milky Way”) and Curtis (“the nebulae are separate galaxies”). Choose a winner for each point.

SHAPLEY: The period of Cepheid variables is a good way to tell distance.

CURTIS: Not so.

SHAPLEY: Spectroscopic parallaxes can be trusted as a way of gauging a wide range of distances.

CURTIS: They can only be trusted under 300 ly.

SHAPLEY: The Milky Way is huge with well over a billion stars in it, many obscured by dust.

CURTIS: It’s much smaller; a straightforward count of what you see gives a good estimate of under a billion.

SHAPLEY: The solar system is on the edge of the Milky Way.

CURTIS: It is near the center of the Milky Way.

SHAPLEY: Some nebulae are seen to rotate over a few years. They cannot be too big

CURTIS: More data are needed.

SHAPLEY: Some novae seen in the nebulae are briefly brighter than the entire nebula. How can that be if they are galaxies?

CURTIS: Maybe there’s a special kind of nova.

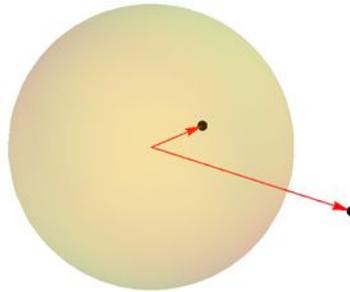
3. Luminosity is measured in watts (W), as with light bulbs, and apparent brightness in watts/meter<sup>2</sup> (W/m<sup>2</sup>). If a 100 W light bulb seems to only be 1 W/m<sup>2</sup>, how far is it from you?

4.  $L_{\odot} \approx 4 \times 10^{26}$  W. The earth is 152 billion meters from the sun. What is the apparent brightness of the sun on earth? Pluto is on an average  $5.9 \times 10^{12}$  m from the sun. What is the apparent brightness of the sun on Pluto?

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# AST 10: Homework 9

1. How does the CfA redshift survey determine distances to the galaxies?
2. What have the CfA redshift and Sloan Digital Sky surveys found about the 3D structure of the Universe?
3. What are the differences among the stellar populations of elliptical and spiral galaxies?
4. Do observations show that stars in galaxies have higher or lower orbital velocities than indicated by the visible matter? What is the implication of this?
5. What's a QSO?
6. Why was the discovery of quasars significant?
7. Can we watch galaxies evolve and interact in "real time"? If not, how do we know the different stages of galactic behavior?
8. Suppose you have a spherical distribution of matter with a fixed density, as shown below:



The "safe" orbital velocity (so that you neither fall to the center nor spiral out) obeys

$$v \propto \begin{cases} r & \text{inside} \\ \frac{1}{\sqrt{r}} & \text{outside} \end{cases}$$

- a) When you are inside the sphere does the orbital velocity go up or down as  $r$  increases?
- b) When you are outside the sphere does the orbital velocity go up or down as  $r$  increases?
- c) Is this consistent with the *expected* rotation curve of galaxies?

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# AST 10: Homework 10

1. What prevents these from undergoing immediate gravitational collapse?
  - a) You.
  - b) Your astronomy professor.
  - c) The Milky Way.
  - d) The Universe.
2. Is a static Universe stable or unstable in
  - a) Newton's theory?
  - b) Einstein's theory?
3. What is the role of  $\Lambda$ , the cosmological constant?
4. Hubble's law says that  $V = HR$ . If  $V$ , the speed of recession of a distant galaxy, is expressed in km/sec, and  $R$ , the distance to the galaxy, is expressed in km, what are the units of  $H$  (called the "Hubble constant")?
5. If galaxy A is twice as far from us as galaxy B, how much faster or slower is it receding from us?

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# AST 10: Homework 10b

- I presented a table in class listing different values for the Hubble constant at present,  $H_0$ , and the age of the Universe in billions of years. But, I engaged in a little standard astronomical hanky-panky with the units. In the numbers presented there, the age of the Universe,  $T$ , is not simply  $1/H_0$ . For example, the first line had Hubble's measured value  $H_0 = 500$  corresponding to an age of  $T = 2$  billion years. But  $2 \neq 1/500$ . You need to divide  $H_0$  by something to get the units to match up so that  $T = 1/H_0$ . What division factor do you need?
- Using the "correct" (i.e., consistent) units, here's a table of recent measurements of  $H_0$ . What are the biggest and smallest ages of the Universe that you get from these values?

 $H_0$  and  $T$  (billions of years)

Year	Mission/Instrument	$H_0$	$T = 1/H_0$
2013	ESA Planck satellite	0.0678	
2012	NASA Explorer 80	0.0693	
2008	Chandra	0.0776	
2005	Hubble	0.0720	

- One of the lines in the Hydrogen spectrum has a wavelength of  $4.861 \times 10^{-7}$  m. The line is observed in the spectrum of a distant galaxy at  $4.923 \times 10^{-7}$  m.
  - What is the change in wavelength,  $\Delta\lambda$ ?
  - What is  $\Delta\lambda/\lambda$ ?
  - Using  $\Delta\lambda/\lambda = v/c$  (and the value  $c = 3 \times 10^5$  km/sec) what is the velocity,  $v$ , of the galaxy?
  - Is the galaxy moving away from us or toward us?
- Let's use Hubble's Law,  $V = H_0 R$ , as a distance-measuring tool to estimate how far away the galaxy in the previous question is.
  - Using a value of  $H = .075$  (in appropriate units – see below), and  $V$  from the answer to the previous question, what is  $R$ ?
  - The answer in (a) is in kpc (kilo parsecs), where the pc is a unit astronomers like to use.  $1 \text{ pc} = 3.26 \text{ ly}$ . Convert your answer to ly and express it in millions of light years.
- The left-hand side of the red-shift equation,  $\Delta\lambda/\lambda = v/c$ , is often denoted by astronomers by the letter  $z$ .
  - Objects with  $z > 1$  are known. What does that say about the velocity of their recession?
  - It is a basic pillar of the theory of relativity that no physical object can travel faster than  $c$ , the speed of light. Is that consistent with part (a)? If not, how do you think this inconsistency might be resolved? (Hint: remember the picture with people riding motorcycles through space as opposed to merely sitting in chairs.)
- We've seen that if the "acceleration" of the scale factor,  $a(t)$ , is negative everywhere, then the graph of  $a(t)$  must be concave down and, therefore,  $a(t) = 0$  at some point in the past. If you don't require the acceleration to be negative, sketch a possible graph for  $a(t)$  that does not have  $a(t) = 0$  anywhere.

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# AST 10: Homework 11

1. In class we pondered (slide/page 9 in printed notes) the  $\diamond$  equation:

$$\frac{3\dot{a}^2(t)}{a^2(t)} + \frac{3k}{a^2(t)} = \frac{8\pi G}{c^4} \rho. \quad \diamond$$

It turns out that the Hubble constant  $H_0 = \dot{a}(t)/a(t)$ .

- Re-express the  $\diamond$  equation using  $H_0$  in place of  $\dot{a}(t)/a(t)$ .
- Then rewrite the equation by solving for  $\rho$ .
- Finally, in the case when  $k = 0$  (zero spatial curvature) what is  $\rho$ ? Call that value  $\rho_c$ .
- When  $k = 1$  do you think the energy density  $\rho$  needs to be greater or less than  $\rho_c$ ?
- When  $k = -1$  do you think the energy density  $\rho$  needs to be greater or less than  $\rho_c$ ?

2. In the  $\heartsuit$  equation,

$$\frac{3\ddot{a}(t)}{a(t)} = -\frac{4\pi G}{c^4}(\rho + 3P) \quad \heartsuit$$

we have seen that  $\ddot{a}(t) < 0$  when  $\rho > 0$ ,  $P > 0$ . Suppose that we have  $\rho > 0$ , but  $P = -\rho$ . Would  $\ddot{a}(t)$  still be negative? What shape would that graph of  $a(t)$  have in this situation? Would there be implications for the *necessity* of a birth for the Universe?

- What is the difference between a static Universe and a steady state Universe?
- Why was the discovery of quasars a problem for steady state cosmology?

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# AST 10: Homework 12

1. Is the “big bang theory” a genuinely new theory or is it derived from other, more fundamental theories? What are those theories? What are the observational successes of the big bang theory?
2. Roughly how old is the Universe according to the big bang theory?
3. What is the horizon problem? Who pointed it out? If the Universe were infinitely old in its present form, would there still be a horizon problem?

4. We have seen before that the  $\diamond$  equation can be written in terms of the Hubble constant  $H_0$  as

$$3H_0^2 + \frac{3k}{a^2(t)} = \frac{8\pi G}{c^4} \rho.$$

Solve this equation for  $H_0$ .

5. What is the flatness problem? Who pointed it out?
6. What problem in particle physics was Alan Guth trying to solve when he discovered cosmic inflation?
7. What is the basic idea of cosmic inflation? Why is it called “inflation”?
8. We have seen that if  $P = -\rho$ , the  $\heartsuit$  equation becomes

$$\frac{3\ddot{a}(t)}{a(t)} = \frac{8\pi G\rho}{c^4}$$

. Solve this for  $\ddot{a}(t)$ .

9. Using the formulas on the slides in class, if a cylinder of dough is originally 50 cm long and 1 cm in radius, what would its length and radius be after 16 “pulls” (as described in class).
10. How does inflation solve the horizon and flatness problems?

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# AST 10: Homework 12b

1. What is “structure formation” and how does cosmic inflation attempt to explain it?
2. When was the first progress made in understanding quantum fluctuations in inflation?
3. What scientific missions are looking for the ripples in the microwave background that inflation predicts? Have we seen the ripples?
4. What are two reasons that inflation re-opens the question of the initial singularity (the beginning of the Universe)?
5. What does it mean to say that inflation is future-eternal?
6. What is a multiverse? Why do we think inflation might predict it?
7. Do we now think that we have proof that inflation, too, must have had a beginning? What expansion condition is this based on?

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# AST 10: Homework 13

1. What is dark matter? How much of the Universe is dark matter?
2. What is dark energy? How much of the Universe is dark energy?
3. What is the evidence for dark matter?
4. What is the evidence for dark energy?
5. Which of dark matter or energy can perhaps be explained by a cosmological constant? How?
6. Does dark matter change the shape of the graph of the scale factor from concave down to up?
7. Does dark energy change the shape of the graph of the scale factor from concave down to up?
8. What are some things that dark matter is not?
9. What are some things that dark matter might be?
10. What standard candles were used by the teams that found the accelerating expansion of the Universe?
11. What are Type 1a supernovae?