1. Introduction

RAUM UND ZEIT

VORTRAG, GEHALTEN AUF DER 80. NATURFORSCHER-VERSAMMLUNG ZU KÖLN AM 21. SEPTEMBER 1908

VON

HERMANN MINKOWSKI

"Space and Time"

1

Minkowski said:

> Gentlemen! The views of space and time which I want to present to you arose from the domain of experimental physics, and therein lies their strength. Their tendency is radical. From now onwards space by itself and time by itself will recede completely to become mere shadows and only a type of union of the two will still stand independently on its own.

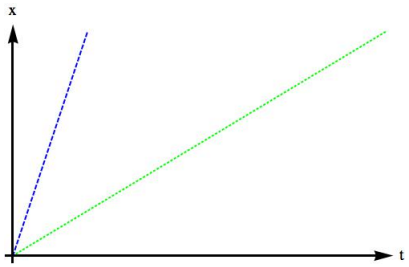
Lecture delivered before the Naturforscher Versammlung (Congress of Natural Philosophers) at Cologne, September 21, 1908.

2

In this paper, Minkowski introduced diagrams, we now name after him, which represent the profound idea of _____, and, through it, a visual, geometrical approach to relativity.

3

Consider two objects moving in one space dimension, and plot their positions as a function of time:



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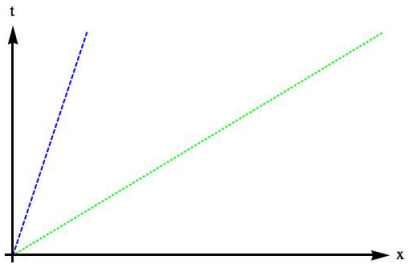
(1) Which of the two plots represents a higher speed? _____

_____

(2) What geometrical aspect of the previous plots represents the speed? _____

5

In the diagrams that Minkowski introduced, the time axis is represented vertically:



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ADDITIONAL NOTES

(3) *Now*, which of the two plots represents a higher speed? _____

_____

_____

(4) What geometrical aspect of the previous plots represents the speed? _____

7

Spacetime

Minkowski unified space and time into spacetime.

An _____ is an entity (aka $\mathbb{R}^n$) with coordinates $(t, x_1, x_2, x_3, \ldots, x_{n-1})$.

You may think of t as "time" and $x_1, x_2, \ldots x_{n-1}$ as "space."

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We'll mostly look at $4d$ space-time with coordinates such as (t, x, y, z) or (t, x_1, x_2, x_3). We'll use other coordinates (such as polar) as needed.

In order to be able to draw pictures, many of our examples will be drawn in 2d [coordinates (t, x)] or 3d [coordinates (t, x, y)].

These pictures are drawn with the t axis vertical.

9

A single spacetime point, p, is an _____. It's _____

_____.

A curve in spacetime is a _____. Examples include (a) events that may be thought of as occurring sequentially in time, or (b) events that may be thought of as occurring at different places at the same time.

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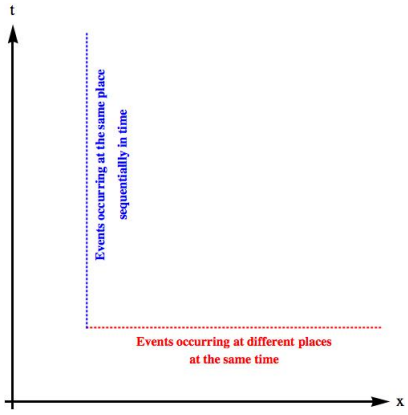

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The vertical curve on the previous diagram is called _____ and it represents events that occur sequentially in time (they are in _____ _____ because prior events can send signals to or influence later ones).

The horizontal curve is called _____ and it represents _____.

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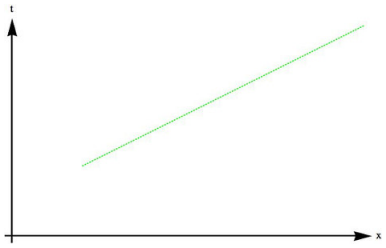
ADDITIONAL NOTES



What about a curve that's neither vertical nor horizontal such as this one? Is it "timelike" or "spacelike"? Does the question even make sense?

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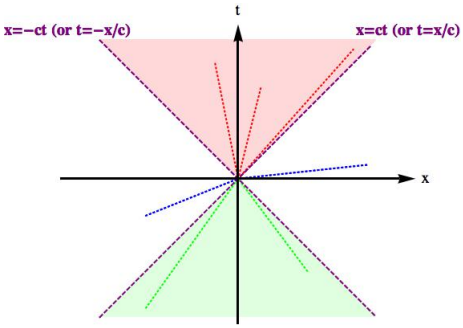
That's where _____ help.

_____

_____.

The straight lines emanating from P on the light cone are called _____ or _____.

14



Null lines from the origin have equations _____.

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(5) What is their slope? _____

Straight lines from P _____ (e.g., within the shaded regions in the diagram) are called _____.

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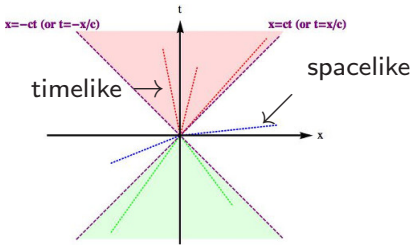
Straight lines from P outside the light cone are called _____.



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The |spacetime slope| criteria for a straightline segment are:

timelike: _____

spacelike:_____

lightlike: _____

where "spacetime slope" in 2d is _____.

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ADDITIONAL NOTES

Relative to the speed of light, c, what speed would you have to be traveling to traverse

(6) a lightlike (null) line? _____.

(7) a timelike line? _____.

(8) a spacelike line? _____.

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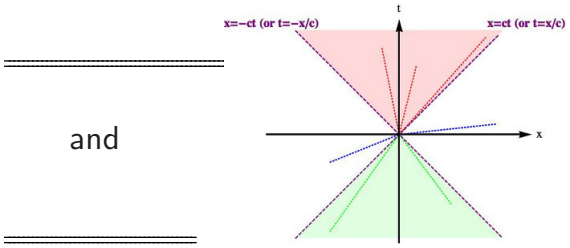
(9) Why are there two light cones at P?

_____

and

_____



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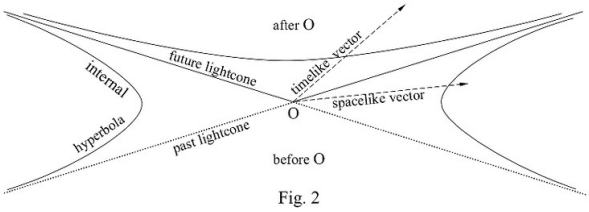

Fig. 2

Minkowski's terms were:

Vorkegel – "in front cone" (past cone)

Nachkegel – "after cone" (future cone)

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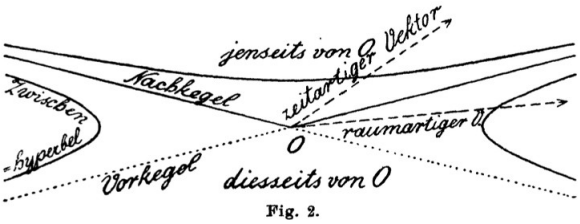

Fig. 2.

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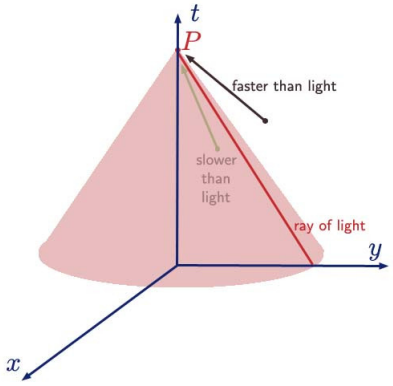

Why "light cones"?

_____

_____

_____

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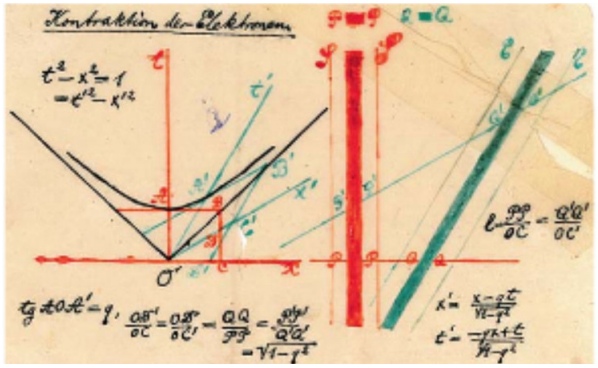

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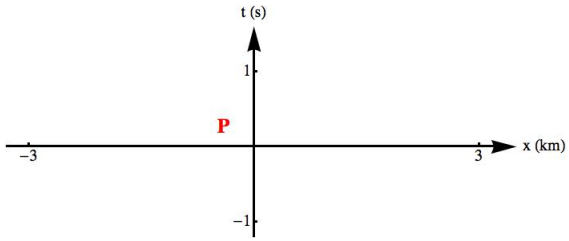
ADDITIONAL NOTES

(**10**) $c \approx 3\times10^5$ km/s. What would the light cone of point P look like on a graph that has distance plotted between ±3 km and time between ±1 sec?



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(**11**) If t is measured in hundred-thousandths of a second and x in km, what value would c have?

In these units classify the line segments between these points as timelike, spacelike or null:

(**12**) $(1,5)$ and $(2,7)$: _____

(**13**) $(-1,4)$ and $(1,-5)$: _____

(**14**) $(2,1)$ and $(3,4)$: _____

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The Distance Formula

If we label our coordinates as $(x_1, x_2, \ldots x_n)$ and use the notation that Δx is the difference in x, then we can think of the Pythagorean distance formula we've used earlier as

_____

This is _____

27

What does this mean? If we transform to new coordinates moving at velocity $\vec{v} = (v_1, v_2, \ldots v_n)$ with respect to the old, we'll have

28

"Invariance" means both coordinate systems give the same Pythagorean distance.

(**15**) Show this.

.

29

This means that two coordinate systems that are moving uniformly relative to each other will measure the same distances/sizes, *as long as Galilean transformations are the correct ones to use*.

But we know they are not the correct transformations, and that lengths are not invariant.

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ADDITIONAL NOTES

The Lorentz Transformations

$$\hat{t} = \gamma(t - vx/c^2) \qquad \hat{x} = \gamma(x - vt)$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

Therefore

$\Delta\hat{t} =$ _____

$\Delta\hat{x} =$ _____

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The Pythagorean distance is not invariant under these transformations. But, ~~we~~ you can

(16) show that $c^2\Delta\hat{t}^2 - \Delta\hat{x}^2 = c^2\Delta t^2 - \Delta x^2$.

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33

The preceding is analogous to the invariance of the Pythagorean distance under Galilean transformations.

The quantity $c^2\Delta t^2 - \Delta x^2$ is very important. It is called the squared _____. This concept will be with us for the rest of the course, from now to the beginning (of the Universe).

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Proper Time

The squared _____ between two points is given by

_____

We've seen (in 2d at least) that this is invariant under Lorentz transformations.

35 The _____ is $\sqrt{\Delta s^2}/c = \Delta s/c$.

With $c = 3$, as before, what is the squared proper interval between each of these?

(17) $(1, 5)$ and $(2, 7)$.

_____.

(18) $(-1, 4)$ and $(1, -5)$

_____.

(19) $(2, 1)$ and $(3, 4)$

_____.

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ADDITIONAL NOTES

(**20**) How do we get a proper time out of these answers, and what are its units? _____

(**21**) What connection do you guess between type of line (timelike, null, etc.) and proper time?

37

When an observer follows a spacetime curve, that curve is called his/her _____. The proper time along a worldline is the actual time experienced and measured by the observer on it.

This concept can be used to clarify several "paradoxes," such as the twin paradox.

38

(**22**) If $c = 3 \times 10^5$ km/s, how many km does light travel in a year?

(**23**) What's that in the language of gazillions?

(**24**) Another name for this? _____

(**25**) What's c in units of ly and y? _____

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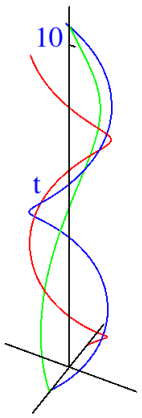

The different worldlines shown on the left will "experience" different physical times even though they all start at $t = 0$ and stop at $t = 10$.

The physical time is obtained by calculating the proper time along each curve.

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The Lorentz Transformations: Geometry

$$\hat{t} = \gamma(t - vx/c^2)$$
$$\hat{x} = \gamma(x - vt)$$
$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

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What do the $\hat{x}$ and $\hat{t}$ coordinate lines look like in the x-t coordinate system? To be concrete, suppose we use units where $c = 1$ and $v = 1/2$. Then

$\hat{t}$ axis: _____

$\hat{x}$ axis: _____

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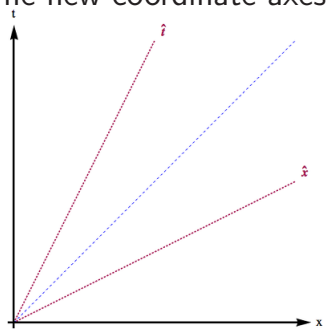
ADDITIONAL NOTES

The new coordinate axes look like this:



Slope of $\hat{x}$ axis: v.

Slope of $\hat{t}$ axis: $1/v$

(units where $c = 1$).

Can also study this dynamically.

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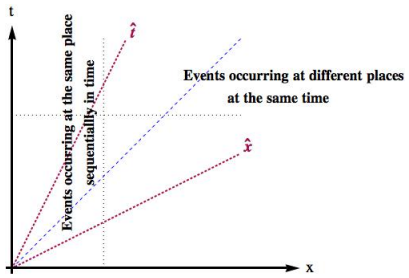
Lorentz Transformations: The Unhatted POV



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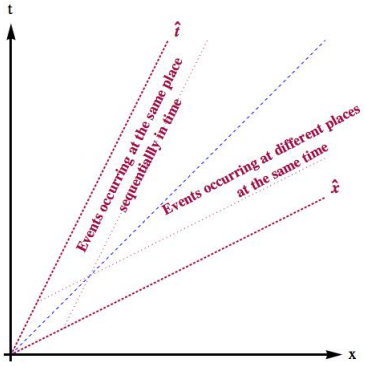
Lorentz Transformations: The Hatted POV



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ADDITIONAL NOTES