Arvind Borde / MTH 675, Unit 16: Special Relativity II

Einstein's 1905 paper had two parts:

I. Kinematical ________________________

II. Electrodynamical

It was the kinematical part that contained the basic new ideas of what we now call the ____________ ________________

1 It had 5 sections:

§1. Definition of Simultaneity

§2. On the Relativity of Lengths and Times

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§3. Theory of the Transformation of Co-ordinates...

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§4. Physical Meaning of the Equations Obtained...

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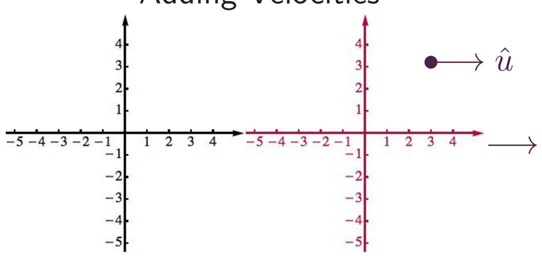
§5. The Composition of Velocities

2 ________________

Adding Velocities



Hatted frame initially coincides with unhatted, and is moving with speed v relative to it; a blob is moving with speed $\hat{u}$, *as measured in the hatted frame*, starting from origin at

3 $\hat{t} = 0 = t$.

What is the speed, u, of the blob as measured in the unhatted frame?

Galilean commonsense says...

(1) What? ________________.

4

We turn to our new bff's, the Lorentz transformations:

$$\hat{t} = \gamma(t - vx/c^2)$$

$$\hat{x} = \gamma(x - vt)$$

A bit of algebra (or a symmetry argument) gives the reverse Lorentz transformations:

$$t = \gamma(\hat{t} + v\hat{x}/c^2)$$

5 $$x = \gamma(\hat{x} + v\hat{t})$$

(2) What is the $\hat{x}$ coordinate of the blob?

$\hat{x} =$ __________

(3) What do you get when you plug this into the reverse Lorentz transformations?

$t =$ ______________________________

$x =$ ________________________

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ADDITIONAL NOTES

(4) What's u (speed of blob in unhatted frame) in unhatted coordinates? ____________

(5) Using the results of Q3, what does this ratio work out to?

$$u = \frac{x}{t} =$$

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§ 5. The Composition of Velocities

In the system k moving along the axis of X of the system K with velocity v, let a point move in accordance with the equations

y, z, t and the corresponding quantities of k', which differ from the equations found in § 3 only in that the place of "v" is taken by the quantity

$$\frac{v+w}{1+vw/c^2};$$

from which we see that such parallel transformations—necessarily—form a group.

We have now deduced the requisite laws of the theory of kinematics corresponding to our two principles, and we proceed to show their application to electrodynamics.

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(6) If the hatted frame and the blob are both moving at speeds much smaller than that of light ($v \ll c$, $\hat{u} \ll c$), show that the relativistic addition of velocities reduces to our discarded commonsense.

9

If the blob were a blob of light,

(7) What would $\hat{u}$ be? ____________

(8) What would u be (from the formula)?

$$u =$$

10

Momentum, Mass and Energy

It follows that our concepts of momentum and energy have to change if we want the "laws of mechanics" to be the same for all observers moving uniformly with respect to each other.

What are these laws that we wish to save?

The most important are ________________.

11

(9) What's a conservation law?

____

____

(10) What are examples of conservation laws?

- ____________________________

____

- ____________________________

____

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ADDITIONAL NOTES

Momentum

In order to preserve the conservation of momentum, we need to redefine it in relativity.

(11) What is the usual (Newtonian) definition of momentum for an object whose mass is m_0 and velocity is $\vec{v}$? ______________

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If momentum has to be conserved, the relativistic law of addition of velocities dictates that we redefine it as

Does it make more sense to view the γ factor as "belonging" to the velocity or to the mass?

It's possible to make a case either way.

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Mass

If you view it as attached to the mass, then an object whose mass at rest (______________) is m_0 will have an effective mass when moving at speed v of

____

15

(12) How does $m(v)$ behave as $v \to c$?

____

____

____

(13) At low speeds what does the relativistic momentum formula, $\vec{p} = \gamma m_0 \vec{v}$, reduce to?

____

____

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Energy

(14) What's the Newtonian formula for the kinetic energy of an object (mass m_0, speed v)?

____

(15) What (do you think) is the relativistic formula for KE?

____

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What does the relativistic kinetic energy formula reduce to at low speeds?
We can't simply use $\gamma \approx 1$. Rewrite the definition:

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = (1 - v^2/c^2)^{\_\_\_\_}$$

This can be expanded via the binomial theorem.

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ADDITIONAL NOTES

Binomial Theorem

$$(x+y)^r = x^r + rx^{r-1}y^1 + \frac{r(r-1)}{2!}x^{r-2}y^2 + \frac{r(r-1)(r-2)}{3!}x^{r-3}y^3 + \dots$$

where

$$k! = k(k-1)\cdots 1 \quad \text{with} \quad 0! \equiv 1.$$

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(16) What are these

$(x+y)^1 =$

$(x+y)^2 =$

20

(17) What are the first two terms in

$$\gamma = (1 - v^2/c^2)^{-1/2}?$$

21

(18) Plugging these two terms into the relativistic kinetic energy formula, what do you get?

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The equation

$$E_{\text{kin}} = m_0c^2(\gamma - 1)$$

may be rewritten as

$$\gamma m_0 c^2 = \underline{\hspace{6cm}}$$

Defining the left-hand side of the equation as the total energy, E, for a particle at rest we get . . .

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DOES THE INERTIA OF A BODY DEPEND UPON ITS ENERGY-CONTENT?

By A. Einstein

September 27, 1905

The results of the previous investigation lead to a very interesting conclusion, which is here to be deduced.

I based that investigation on the Maxwell-Hertz equations for empty space, together with the Maxwellian expression for the electromagnetic energy of space, and in addition the principle that:—

The laws by which the states of physical systems alter are independent of the alternative, to which of two systems of coordinates, in uniform motion of parallel translation relatively to each other, these alterations of state are referred (principle of relativity).

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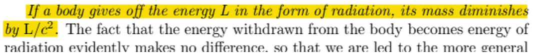
ADDITIONAL NOTES

It was a short paper (three pages) at the end of which Einstein concluded


If a body gives off the energy L in the form of radiation, its mass diminishes by L/c². The fact that the energy withdrawn from the body becomes energy of radiation evidently makes no difference, so that we are led to the more general

In other words, $m = L/c^2$, where L is the energy.

Or, as we know it, $E = mc^2$.

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(19) You're 6 feet tall and are diving (at high speed) on a race course toward a finish line equipped with a sensor. The moment you cross the line, a blade falls six feet behind the finish line. The organizers say that you'll be length-contracted as you dive, and your feet will be well past where the blade falls. Your mother, taking your point of view, says that the race course will be contracted relative to you and the blade will fall on you. Who's right?

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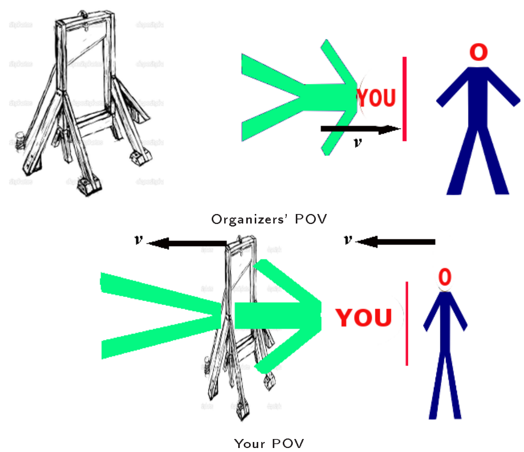

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(20) *Einstein's Train*: A train passes by you with a guard in the middle. Just as the guard passes you, flashes of light emitted from the front and the back of the train reach both you and the guard. Both of you agree on this. Do you agree on when the flashes were emitted?

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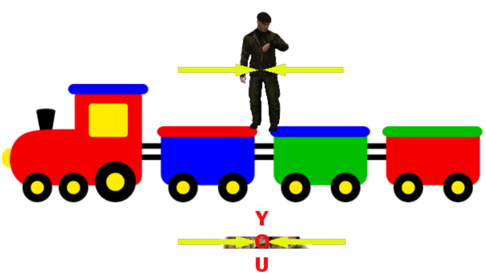

Simultaneous arrival of the signals.

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(21) Read about and resolve the "twin paradox."

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ADDITIONAL NOTES

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