ON THE ELECTRODYNAMICS OF MOVING BODIES

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It is known that Maxwell's electrodynamics—as usually understood at the present time—when applied to moving bodies, leads to asymmetries which do not appear to be inherent in the phenomena. Take, for example, the reciprocal electrodynamic action of a magnet and a conductor. The observable phenomenon here depends only on the relative motion of the conductor and the

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In this 1905 paper, Einstein had *no references to previous work*, and he thanked nobody other than Michele Besso, an old friend:

to which, by the theory here advanced, the electron must move.

In conclusion I wish to say that in working at the problem here dealt with I have had the loyal assistance of my friend and colleague M. Besso, and that I am indebted to him for several valuable suggestions.

2

Discussions with Michele Besso appear to have led Einstein to write to an ex-classmate, Marcel Grossman, in September 1901:

. . . *A considerably simpler method of investigating the relative motion of matter with respect to luminiferous ether. . . has just sprung to my mind. If only for once, relentless Fate gave me the necessary time and peace!*. . .

Doc. 122, Collected Papers of AE, Vol. 1.

After that, comments on the ether dwindle. . .

3

Somewhere between 1901 and 1905 he had, apparently on his own, abandoned the idea that the ether was necessary. Relying only on constructs that he deemed necessary, he introduced a new approach.

What did Einstein present whole in 1905, after ten years of thought, as something entirely original?

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It's presented in paragraph 2 of his paper:

phenomena of electrodynamics as well as of mechanics possess no properties corresponding to the idea of absolute rest. They suggest rather that, as has already been shown to the first order of small quantities, the same laws of electrodynamics and optics will be valid for all frames of reference for which the equations of mechanics hold good.[1] We will raise this conjecture (the purport of which will hereafter be called the "Principle of Relativity") to the status of a postulate, and also introduce another postulate, which is only apparently irreconcilable with the former, namely, that light is always propagated in empty space with a definite velocity c which is independent of the state of motion of the emitting body. These two postulates suffice for the attainment of a simple and consistent theory of the electrodynamics of moving bodies based on Maxwell's

Developing these ideas requires us to understand how a "hatted" system, moving with respect to an unhatted, are mathematically related.

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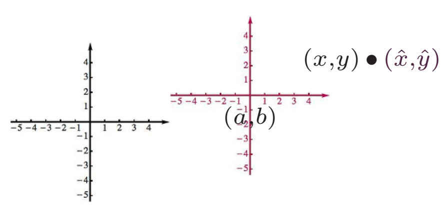
(**1**) If the origin of the hatted coordinates is at (a, b) in the original (unhatted) coordinates, how are the hatted coordinates, $(\hat{x}, \hat{y})$, of a point related to its unhatted coordinates, (x, y)?



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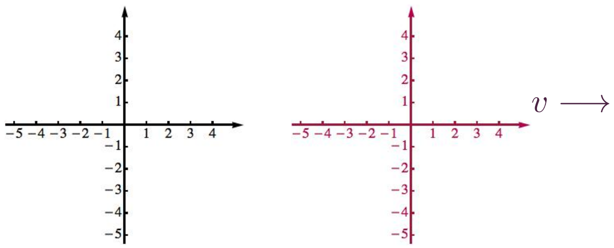
ADDITIONAL NOTES

Suppose the hatted coordinates initially coincide with the unhatted, but are now moving away from them at a fixed speed v in the positive x direction:



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At a time t,

(2) Looking at Q1 what's a here? ________

(3) How are $\hat{x}$ and x related? ________

(4) How are $\hat{y}$ and y related? ________

These relations between the hatted and unhatted coordinates are called ________

The assumption here is that ________

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We'll study the transformation of coordinates without this assumption, but guided by the two "postulates" that Einstein states at the start of his paper and repeats on page 4:

1. The laws by which the states of physical systems undergo change are not affected, whether these changes of state be referred to the one or the other of two systems of co-ordinates in uniform translatory motion.

2. Any ray of light moves in the "stationary" system of co-ordinates with the determined velocity c, whether the ray be emitted by a stationary or by a moving body. Hence

$$\text{velocity} = \frac{\text{light path}}{\text{time interval}}$$

where time interval is to be taken in the sense of the definition in § 1.

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Assume that the hatted coordinates initially coincide with the unhatted and are moving away from them at a fixed speed v in the positive x direction. The simplest transformation between the two sets of coordinates is a ________:

10 (α, β, γ, δ are fixed quantities that we'll determine.)

(5) Under the assumption of the previous slide, when $x = vt$, $\hat{x} =$ ________

(6) Plugging that into the $\hat{x}$ transformation equation, what do we get for δ?

____

____

(7) Is it OK to cancel t?

11

(8) Use this to kick δ out of the $\hat{x}$ equation:

____

(9) Now, from the p.o.v. of the hatted frame, the unhatted is moving away from *it* with velocity ________

(10) If the same laws apply (postulate 1), we have

$$x = \text{________}$$

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ADDITIONAL NOTES

Using postulate 2 (speed of light, c, is the same in all coordinate systems), we have for a ray of light sent out in the $x/\hat{x}$ direction from the origin at the initial instant

$$x/t = c = \hat{x}/\hat{t}$$

(11) Solve these for t and $\hat{t}$.

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(12) Plugging these expressions for t and $\hat{t}$ into

$$\hat{x} = \gamma(x - vt)$$
$$x = \gamma(\hat{x} + v\hat{t}),$$

you get:

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(13) Multiply the two equations together and get a formula for γ.

15

So, the space transformation equation becomes

$$\hat{x} = \gamma(x - vt) = \frac{(x - vt)}{\sqrt{1 - v^2/c^2}}. \qquad \text{[LT}x\text{]}$$

We can also figure out what α and β must be in the time transformation equation

$$\hat{t} = \alpha x + \beta t.$$

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We again use postulate 2 (speed of light, c, is the same in all coordinate systems)

$$\frac{x}{t} = c = \frac{\hat{x}}{\hat{t}}$$

and re-express it as $x = ct$ and $\hat{x} = c\hat{t}$. The space equation on the previous slide becomes

$$c\hat{t} = \gamma(ct - vx/c).$$

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Dividing by c, we get

$$\hat{t} = \gamma(t - vx/c^2). \qquad \text{[LT}t\text{]}$$

(14) Comparing this with the time transformation equation, identify α and β.

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ADDITIONAL NOTES

The equations

$$\hat{t} = \gamma(t - vx/c^2) \qquad \text{[LT}t\text{]}$$

$$\hat{x} = \gamma(x - vt) \qquad \text{[LT}x\text{]}$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

are called the Lorentz Transformations, in honor of Lorentz, who got there a year before Einstein. But Einstein got there independently and more deeply.

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the transformation equations which have been found become

$$\tau = \beta(t - vx/c^2),$$
$$\xi = \beta(x - vt),$$
$$\eta = y,$$
$$\zeta = z,$$

where

$$\beta = 1/\sqrt{1 - v^2/c^2}.$$

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Understanding γ

(15) By looking at the structure of the formula,

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

and keeping things real, is there a maximum speed for the hatted system? ____________

(16) Are there minimum/maximum values for γ?

____

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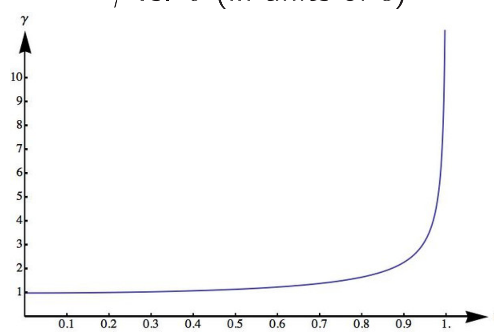
γ vs. v (in units of c)



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"Relativistic effects" are important only at high values of γ, which kick in as $v \to c$.

(17) Calculate γ for v equal to

(a) $0.25c$: ____________,

(b) $0.5c$: ____________,

(c) $0.75c$: ____________, and

(d) $0.9999c$: ____________.

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The Lorentz Transformations

$$\hat{t} = \gamma(t - vx/c^2)$$

$$\hat{x} = \gamma(x - vt)$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

Our little algebraic excursion has important and odd consequences.

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ADDITIONAL NOTES

Odd Consequence 1: Simultaneity

(18) Suppose two events occur at the same time (say $t = 0$) in the unhatted coordinates, but at different places: $x = +1$ for event 1 and $x = -1$ for event 2. When will they seem to occur in the hatted coordinates?

____

____

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(19) What's the big deal?

____

____

As Einstein observed

So we see that we cannot attach any *absolute* signification to the concept of simultaneity, but that two events which, viewed from a system of co-ordinates, are simultaneous, can no longer be looked upon as simultaneous events when envisaged from a system which is in motion relatively to that system.

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Odd Consequence 2: Time Dilation

Suppose you have a clock at a fixed place in the hatted coordinates, and you record two times with it $\hat{t}_1$ and $\hat{t}_2$. How long will the interval $t_2 - t_1$ be in the unhatted coordinates?

We have
$$\hat{x}_2 = \gamma(x_2 - vt_2)$$
$$\hat{x}_1 = \gamma(x_1 - vt_1)$$

So
$$\hat{x}_2 - \hat{x}_1 = \gamma((x_2 - x_1) - v(t_2 - t_1))$$

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Because the clock is at a fixed place in the hatted system, $\hat{x}_2 = \hat{x}_1$. So

$$0 = \gamma((x_2 - x_1) - v(t_2 - t_1))$$

or

$$(x_2 - x_1) = v(t_2 - t_1).$$

Put this into memory.

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From the time equations
$$\hat{t}_2 = \gamma(t_2 - vx_2/c^2)$$
$$\hat{t}_1 = \gamma(t_1 - vx_1/c^2)$$

(20) Find a formula for $\hat{t}_2 - \hat{t}_1$.

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So
$$t_2 - t_1 = \gamma(\hat{t}_2 - \hat{t}_1)$$

This means that the time interval in the unhatted frame will be longer than in the hatted.

(21) Why? ________________________

____

____

____

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ADDITIONAL NOTES

Gains from time dilation

Picking three modes of travel, say walking, driving and flying, let's assign each a plausible speed in km/sec. Using $c = 300,000$ km/sec, we'll calculate how much time you gain on a friend stationary on earth if you travel for an earth-year at that speed.

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$$t_2 - t_1 = \gamma(\hat{t}_2 - \hat{t}_1)$$

or

$$\Delta\hat{t} = \Delta t/\gamma$$

where you're the hatted traveler, and the unhatted frame is "stationary."

The amount of time you "gain" is

Gain $= \Delta t - \Delta\hat{t} =$ ________

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($\mathbf{22}$) To the nearest $1{,}000$, how many seconds are there in a non-leap year?

____

So we'll use ____________________

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Time gains at different speeds

Activity	Speed (km/s)	$1 - 1/\gamma$	Gain (sec)
Walking	6/3600	$1.5 \cdot 10^{-17}$	$4.9 \cdot 10^{-10}$
Driving	60/3600		
Flying	900/3600		

If flying time were 14 hrs (from ground pov), what would the time gain be in nanoseconds?

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Tests of time dilation

1) In 1971 Keating and Hafele flew four caesium atomic clocks around the world. The results of the experiment confirmed relativistic predictions within 10%. The experiment was repeated in 1996 on a trip from London to Washington and back, a 14 hour journey. The result, a 16.1 ns time gain from motion, was within 2 ns of the prediction.

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2) Muon lifetime [Bailey,J. et al. Nature **268**, 301 (1977)]: Muons with "rest lifetime" of $2.198\,\mu$s were sped to high speed ($\gamma = 29.33$). The measured lifetimes at those speeds were found to be $64.368\,\mu$s.

($\mathbf{23}$) Is that consistent with relativity?

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ADDITIONAL NOTES

Odd Consequence 3: Length Contraction

From $\hat{x}_2 = \gamma(x_2 - vt_2), \quad \hat{x}_1 = \gamma(x_1 - vt_1)$
we see that

$$\hat{x}_2 - \hat{x}_1 = \gamma((x_2 - x_1) - v(t_2 - t_1))$$

If two position measurements are made at the same time in the unhatted frame ($t_1 = t_2$), we will have

$$\hat{x}_2 - \hat{x}_1 = \gamma(x_2 - x_1)$$

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So

$$x_2 - x_1 = (\hat{x}_2 - \hat{x}_1)/\gamma$$

The length of an object at rest in the hatted frame will be shorter in the unhatted frame.

(24) Why? ____________________

____

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Paradoxes and Puzzles of Relativity

Relativity is an entirely consistent theory, but people often speak of “paradoxes” in it because it can sometimes lead to apparent self-contradictions.

They can always be resolved.

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ADDITIONAL NOTES

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