(1) What's differential geometry?

(2) Why calculus in geometry?

1

Brief, sketchy history of geometry

- First(0ld), "Pythagorean theorem":
  - ∼500–1800 BCE: Examples. Egypt/Mesopotamia, Plimpton Tablets.
  - ∼800 BCE: Statement. India, *Shubha Shastra*, Baudhayan.
  - ∼600 BCE: Proof. China, *Zhou Bi Suan Jing*.
  - ∼500 BCE: Statement. Pythagoras.
  - ∼300 BCE: Proof. Greece. *Elements*, Euclid.

2

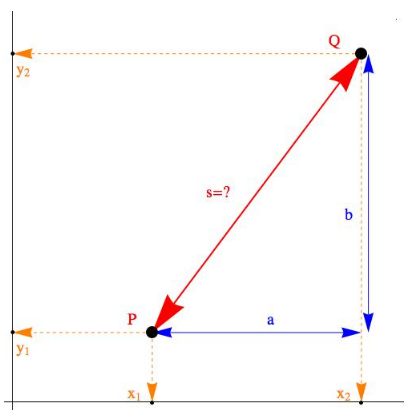
- Next (Newish):
  - 1827: *General Investigation of Curved Surfaces*, Gauss. 2d surfaces in 3d space.
  - 1854: *On the Hypotheses which lie at the bases of Geometry*, Riemann.
  - 1887–1912: Ricci, Tensor Calculus.
  - 1913–1916: General Relativity

General what? Before that ...

3

(3) What's the Pythagorean theorem?

4



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(4) What's that in terms of the distance between the two points (x_1, y_1) and (x_2, y_2)?

6

ADDITIONAL NOTES

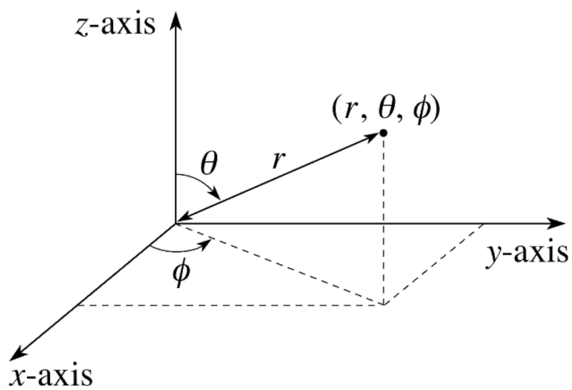
We'll need the notation that "dx" means a (*very* small) difference in the variable x. The squared distance formula for flat space is then just the sum of squares of coordinate differences:

7

Every geometry, curved or flat, has a characteristic distance formula expressible via squares of the appropriate coordinate differences.

If you're discussing the geometry of a sphere you use a distance formula that defines *that* geometry. The standard coordinates here are basically latitude (θ) and longitude (ϕ).

8



Spherical coordinates

9

The squared distance formula on a sphere is

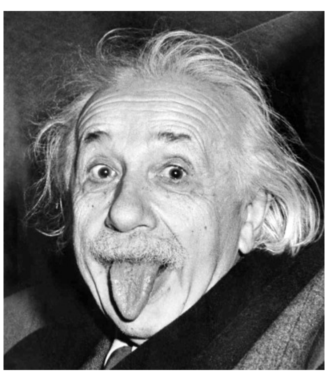
$$ds^2 = r^2 d\theta^2 + r^2\, sin^2\, \theta d\phi^2$$

This _____ allows us to discuss the geometry of a sphere from properties of the sphere itself, not its embedding in a larger space.

OK, what about relativity?

10

The story starts with him . . .



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http://www.npr.org/blogs/13.7/2011/09/28/140839445/is-einstein-wrong

(5) Who's he?

____

(6) What's he most known for?

____

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ADDITIONAL NOTES

Summary of the Theory of Relativity

Theory developed between 1905 and 1916, primarily by Albert Einstein.

First version (1905), called Special Relativity. Einstein worked for a decade on extending it, till he succeeded in 1915 (published in 1916) with the General Theory. General Relativity has four main ingredients:

13

1. ________________
2. ________________________
3. ______________________________________
   ____
   ____
4. ______________________________________
   ____

14

It's not all words:

Einstein's theory, expressed via equations,

$$G_{ab} \equiv R_{ab} - \frac{1}{2} g_{ab} R = \frac{8\pi G}{c^4} T_{ab}$$

$\downarrow$ Spacetime Geometry — Ricci Curvature, R_{ab}; Curvature Scalar, R; Metric, g_{ab}

$\downarrow$ Matter — Energy-Momentum

$$2R_{ab} = \partial_c[g^{cd}(\partial_a g_{bd} + \partial_b g_{ad} - \partial_d g_{ab})] - \partial_b[g^{cd}(\partial_a g_{cd} + \partial_c g_{ad} - \partial_d g_{ac})] - [g^{cd}(\partial_a g_{ed} + \partial_e g_{ad} - \partial_d g_{ae})] \times [g^{ed}(\partial_c g_{bd} + \partial_b g_{cd} - \partial_d g_{cb})] + [g^{cd}(\partial_a g_{bd} + \partial_b g_{ad} - \partial_d g_{ab})] \times [g^{ed}(\partial_e g_{cd} + \partial_c g_{ed} - \partial_d g_{ec})]$$

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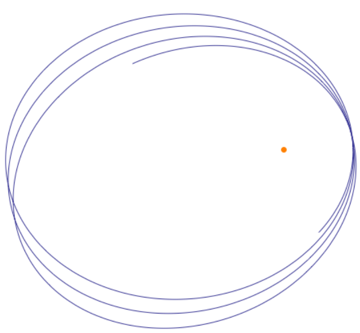
Einstein Test 3

Motion of the perihelion of a planet

The ________________ of a planetary orbit is ____

A planet (Mercury, e.g.) goes around the sun on an elliptical path. But, the path does not close: the perihelion is not at the same point every year.

16 This is called ___________________



A precessing ellipse

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Till Einstein, we could explain most of the precession of Mercury, except for a small amount:

0.012° – every hundred years!

Einstein's proposal was that the matter of the sun warps surrounding spacetime geometry. Mercury moves on a straight line on this curved background.

Sounds weird, but you get exactly the extra 0.012° that you need.

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ADDITIONAL NOTES

How do we describe the warped geometry of space-time caused by a (roughly) spherical object such as the sun with mass m?

Through a spacetime metric:

$$ds^2 = f(r)dt^2 - \frac{1}{f(r)}dr^2 - r^2(d\theta^2 + sin^2\,\theta\,d\phi^2)$$

where

$$f(r) = 1 - \frac{2Gm/c^2}{r} = 1 - \frac{r_s}{r}.$$

19

ADDITIONAL NOTES